APPENDIX C

SALIENT-POLE THEORY OF SYNCHRONOUS MACHINES

The equivalent circuit for a synchronous generator derived in Chapter 5 is in fact valid only for machines built with cylindrical rotors, and not for machines built with salient-pole rotors. Likewise, the expression for the relationship between the torque angle δ and the power supplied by the generator [Equation (5–20)] is valid only for cylindrical rotors. In Chapter 5, we ignored any effects due to the saliency of rotors and assumed that the simple cylindrical theory applied. This assumption is in fact not too bad for steady-state work, but it is quite poor for examining the transient behavior of generators and motors.

The problem with the simple equivalent circuit of induction motors is that it ignores the effect of the *reluctance torque* on the generator. To understand the idea of reluctance torque, refer to Figure C–1. This figure shows a salient-pole rotor with no windings inside a three-phase stator. If a stator magnetic field is produced as shown in the figure, it will induce a magnetic field in the rotor. Since it is *much* easier to produce a flux along the axis of the rotor than it is to produce a flux across the axis, the flux induced in the rotor will line up with the axis of the rotor. Since there is an angle between the stator magnetic field and the rotor magnetic field, a torque will be induced in the rotor which will tend to line up the rotor with the stator field. The magnitude of this torque is proportional to the sine of twice the angle between the two magnetic fields ($\sin 2\delta$).

Since the cylindrical rotor theory of synchronous machines ignores the fact that it is easier to establish a magnetic field in some directions than in others (i.e., ignores the effect of reluctance torques), it is inaccurate when salient-pole rotors are involved.

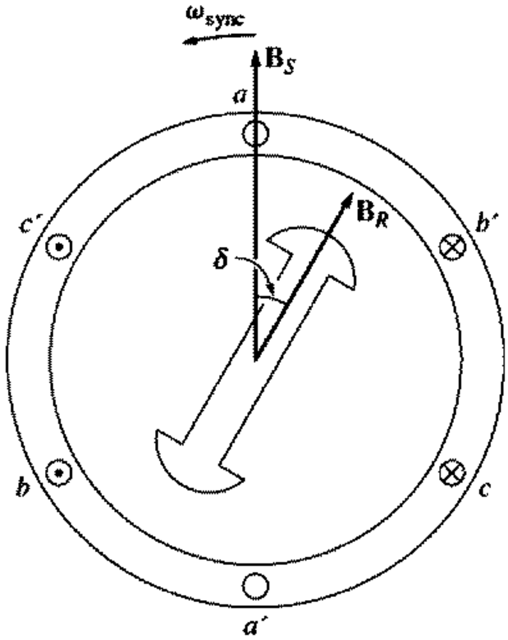

FIGURE C–1
A salient-pole rotor, illustrating the idea of reluctance torque. A magnetic field is induced in the rotor by the stator magnetic field, and a torque is produced on the rotor that is proportional to the sine of twice the angle between the two fields.

C.1 DEVELOPMENT OF THE EQUIVALENT CIRCUIT OF A SALIENT-POLE SYNCHRONOUS GENERATOR

As was the case for the cylindrical rotor theory, there are four elements in the equivalent circuit of a synchronous generator:

1. The internal generated voltage of the generator $\mathbf{E}_A$
2. The armature reaction of the synchronous generator
3. The stator winding's self-inductance
4. The stator winding's resistance

The first, third, and fourth elements are unchanged in the salient-pole theory of synchronous generators, but the armature-reaction effect must be modified to explain the fact that it is easier to establish a flux in some directions than in others.

This modification of the armature-reaction effects is accomplished as explained below. Figure C–2 shows a two-pole salient-pole rotor rotating counterclockwise within a two-pole stator. The rotor flux of this rotor is called $\mathbf{B}_R$, and it points upward. By the equation for the induced voltage on a moving conductor in the presence of a magnetic field,

$$e_{ind} = (\mathbf{v} \times \mathbf{B}) \cdot \mathbf{l} \qquad (1\text{–}45)$$

the voltage in the conductors in the upper part of the stator will be positive out of the page, and the voltage in the conductors in the lower part of the stator will be into the page. The plane of maximum induced voltage will lie directly under the rotor pole at any given time.

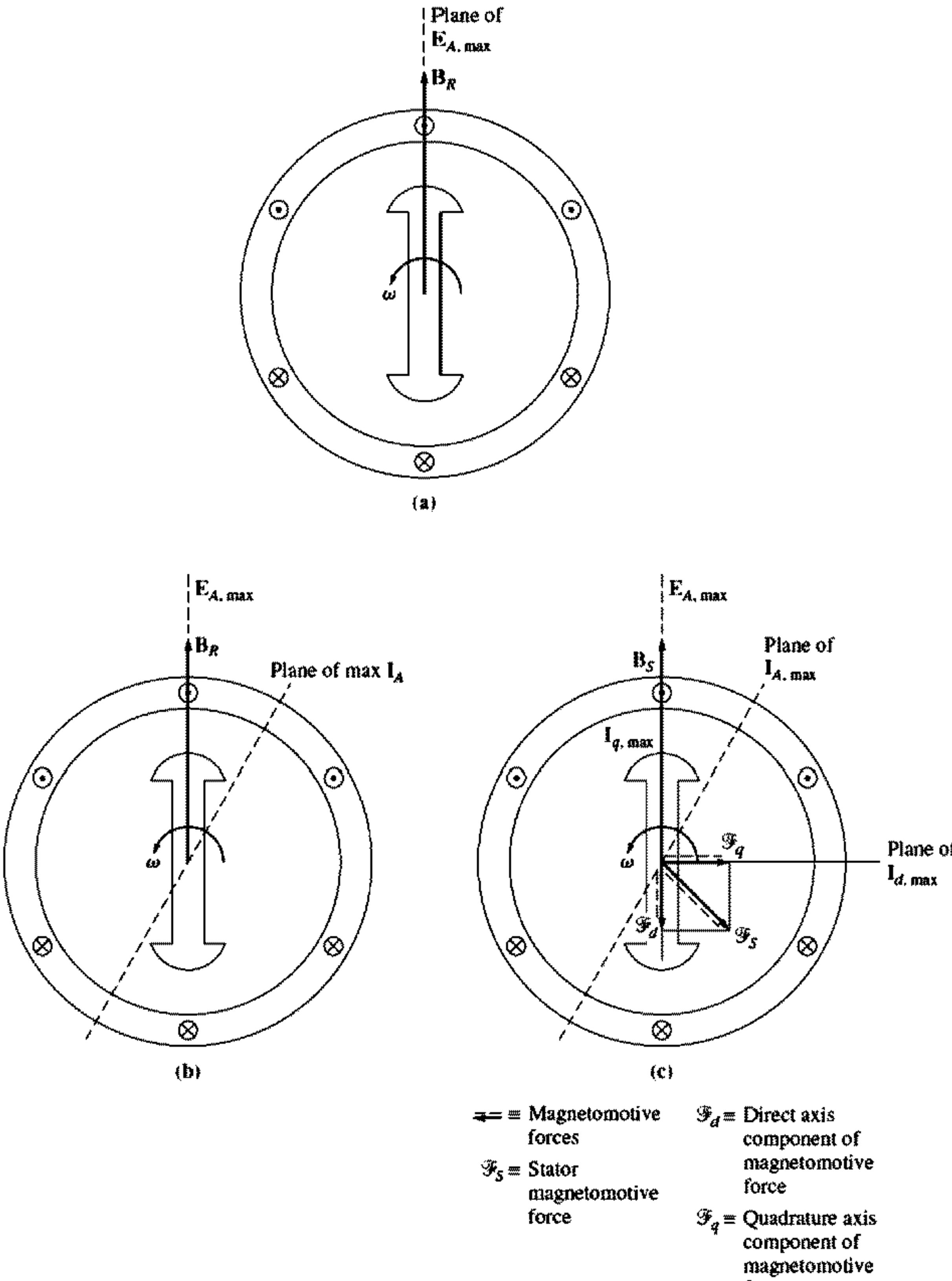

FIGURE C–2
The effects of armature reaction in a salient-pole synchronous generator. (a) The rotor magnetic field induces a voltage in the stator which peaks in the wires directly under the pole faces. (b) If a lagging load is connected to the generator, a stator current will flow that peaks at an angle behind $\mathbf{E}_A$. (c) This stator current $\mathbf{I}_A$ produces a stator magnetomotive force in the machine.

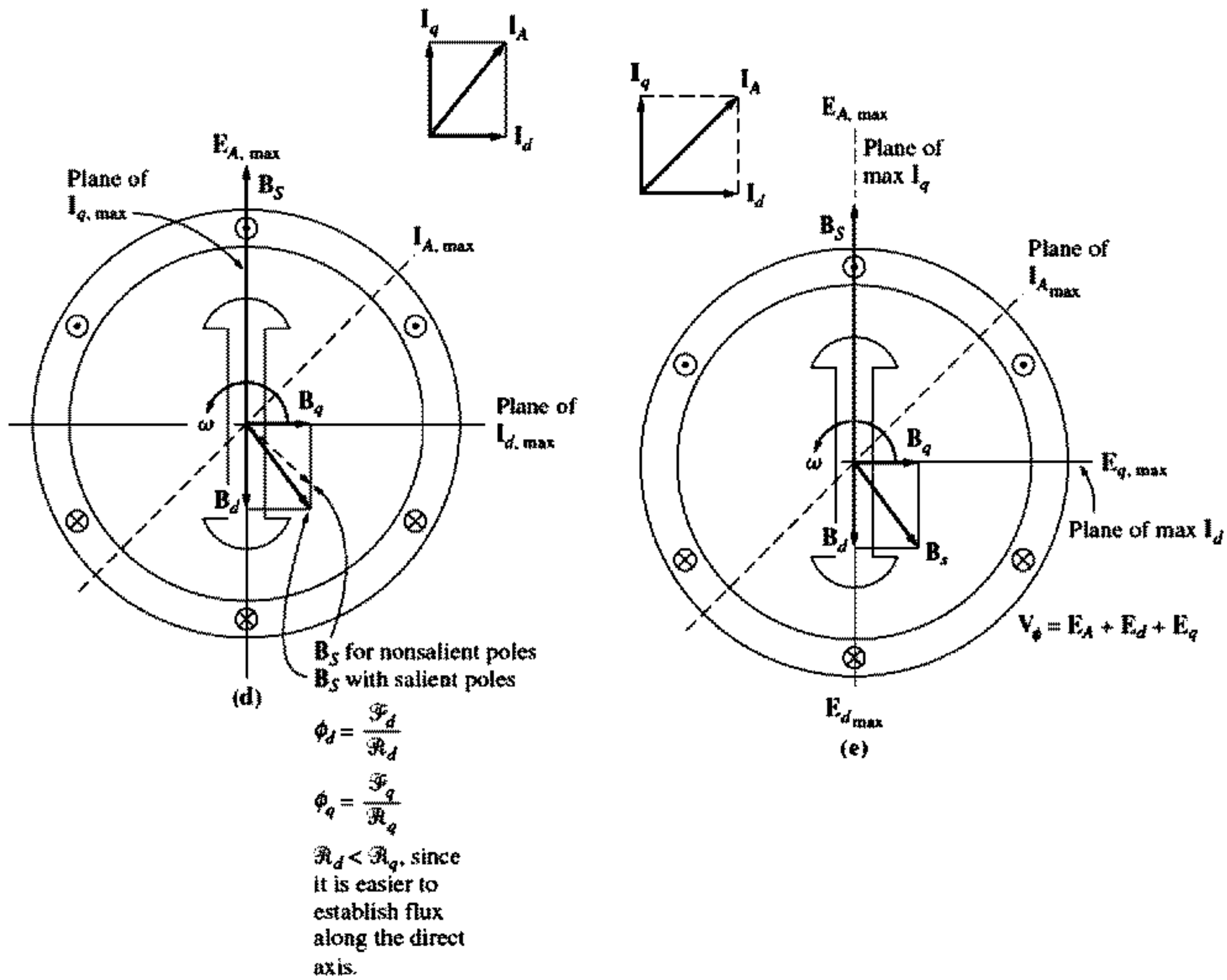

FIGURE C–2 (concluded)
(d) The stator magnetomotive force produces a stator flux $\mathbf{B}_S$. However, the direct-axis component of magnetomotive force produces more flux per ampere-turn than the quadrature-axis component does, since the reluctance of the direct-axis flux path is lower than the reluctance of the quadrature-axis flux path. (e) The direct- and quadrature-axis stator fluxes produce armature reaction voltages in the stator of the machine.

If a lagging load is now connected to the terminals of this generator, then a current will flow whose peak is delayed behind the peak voltage. This current is shown in Figure C–2b.

The stator current flow produces a magnetomotive force that lags 90° behind the plane of peak stator current, as shown in Figure C–2c. In the cylindrical theory, this magnetomotive force then produces a stator magnetic field $\mathbf{B}_S$ that lines up with the stator magnetomotive force. However, it is actually easier to produce a magnetic field in the direction of the rotor than it is to produce one in the direction perpendicular to the rotor. Therefore, we will break down the stator magnetomotive force into components parallel to and perpendicular to the rotor's axis. Each of these magnetomotive forces produces a magnetic field, but more flux is produced per ampere-turn along the axis than is produced perpendicular (*in quadrature*) to the axis.

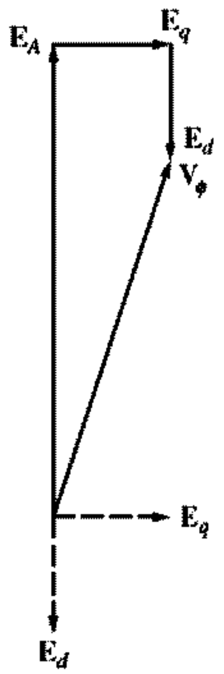

FIGURE C–3
The phase voltage of the generator is just the sum of its internal generated voltage and its armature reaction voltages.

The resulting stator magnetic field is shown in Figure C–2d, compared to the field predicted by the cylindrical rotor theory.

Now, each component of the stator magnetic field produces a voltage of its own in the stator winding by armature reaction. These armature-reaction voltages are shown in Figure C–2e.

The total voltage in the stator is thus

$$\mathbf{V}_\phi = \mathbf{E}_A + \mathbf{E}_d + \mathbf{E}_q \tag{C–1}$$

where $\mathbf{E}_d$ is the direct-axis component of the armature-reaction voltage and $\mathbf{E}_q$ is the quadrature-axis component of armature reaction voltage (see Figure C–3). As in the case of the cylindrical rotor theory, each armature-reaction voltage is *directly proportional to its stator current* and *delayed 90°* behind the stator current. Therefore, each armature-reaction voltage can be modeled by

$$\mathbf{E}_d = -jx_d\mathbf{I}_d \tag{C–2}$$

$$\mathbf{E}_q = -jx_q\mathbf{I}_q \tag{C–3}$$

and the total stator voltage becomes

$$\mathbf{V}_\phi = \mathbf{E}_A - jx_d\mathbf{I}_d - jx_q\mathbf{I}_q \tag{C–4}$$

The armature resistance and self-reactance must now be included. Since the armature self-reactance X_A is independent of the rotor angle, it is normally added to the direct and quadrature armature-reaction reactances to produce the *direct synchronous reactance* and the *quadrature synchronous reactance* of the generator:

$$\boxed{X_d = x_d + X_A} \tag{C–5}$$

$$\boxed{X_q = x_q + X_A} \tag{C–6}$$

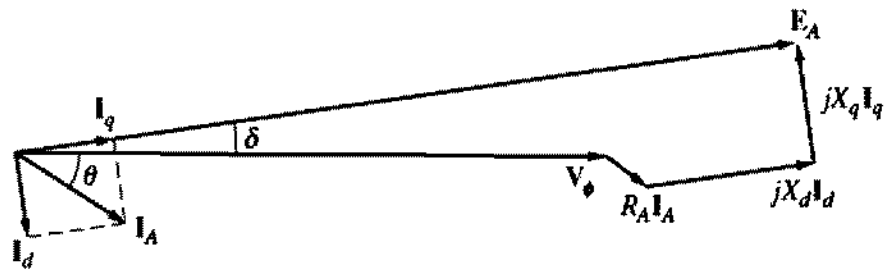

FIGURE C–4
The phasor diagram of a salient-pole synchronous generator.

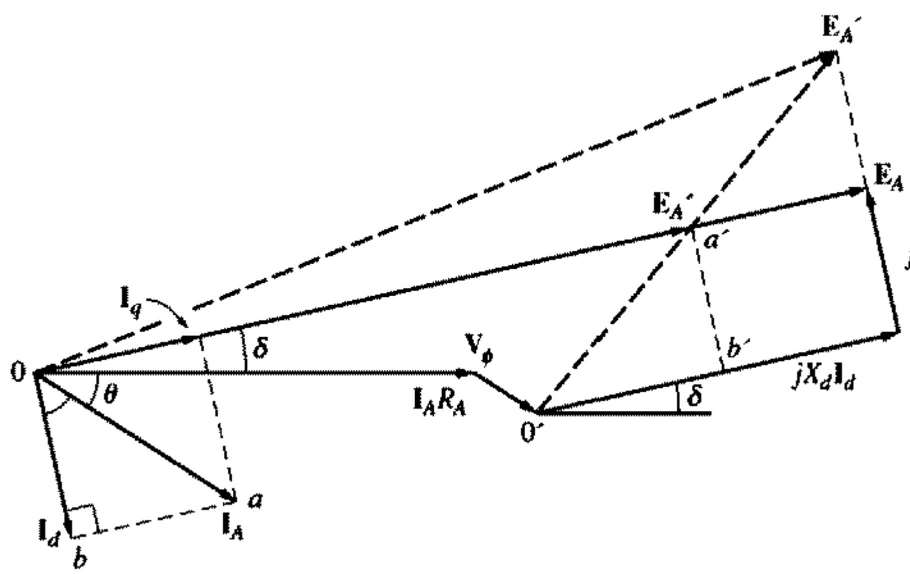

FIGURE C–5
Constructing the phasor diagram with no prior knowledge of δ. E_A'' lies at the same angle as E_A, and E_A'' may be determined exclusively from information at the terminals of the generator. Therefore, the angle δ may be found, and the current can be divided into d and q components.

The armature resistance voltage drop is just the armature resistance times the armature current I_A.

Therefore, the final expression for the phase voltage of a salient-pole synchronous motor is

$$\boxed{V_\phi = E_A - jX_dI_d - jX_qI_q - R_AI_A} \qquad \text{(C–7)}$$

and the resulting phasor diagram is shown in Figure C–4.

Note that this phasor diagram requires that the armature current be resolved into components in parallel with E_A and in quadrature with E_A. However, the *angle* between E_A and I_A is $\delta + \theta$, which is *not usually known* before the diagram is constructed. Normally, only the power-factor angle θ is known in advance.

It is possible to construct the phasor diagram without advance knowledge of the angle δ, as shown in Figure C–5. The solid lines in Figure C–5 are the same as the lines shown in Figure C–4, while the dotted lines present the phasor diagram as though the machine had a cylindrical rotor with synchronous reactance X_d.

The angle δ of $\mathbf{E}_A$ can be found by using information known at the terminals of the generator. Notice that the phasor $\mathbf{E}_A''$, which is given by

$$\boxed{\mathbf{E}_A'' = \mathbf{V}_\phi + R_A\mathbf{I}_A + jX_q\mathbf{I}_A} \tag{C–8}$$

is collinear with the internal generated voltage $\mathbf{E}_A$. Since $\mathbf{E}_A''$ is determined by the current at the terminals of the generator, the angle δ can be determined with a knowledge of the armature current. Once the angle δ is known, the armature current can be broken down into direct and quadrature components, and the internal generated voltage can be determined.

Example C–1. A 480-V, 60-Hz, Δ-connected, four-pole synchronous generator has a direct-axis reactance of 0.1 Ω, and a quadrature-axis reactance of 0.075 Ω. Its armature resistance may be neglected. At full load, this generator supplies 1200 A at a power factor of 0.8 lagging.

(a) Find the internal generated voltage $\mathbf{E}_A$ of this generator at full load, assuming that it has a cylindrical rotor of reactance X_d.
(b) Find the internal generated voltage $\mathbf{E}_A$ of this generator at full load, assuming it has a salient-pole rotor.

Solution

(a) Since this generator is Δ-connected, the armature current at full load is

$$I_A = \frac{1200\text{ A}}{\sqrt{3}} = 693\text{ A}$$

The power factor of the current is 0.8 lagging, so the impedance angle θ of the load is

$$\theta = \cos^{-1} 0.8 = 36.87°$$

Therefore, the internal generated voltage is

$$\begin{aligned}\mathbf{E}_A &= \mathbf{V}_\phi + jX_S\mathbf{I}_A \\ &= 480\angle 0°\text{ V} + j(0.1\ \Omega)(693\angle -36.87°\text{ A}) \\ &= 480\angle 0° + 69.3\angle 53.13° = 524.5\angle 6.1°\text{ V}\end{aligned}$$

Notice that the torque angle δ is 6.1°.

(b) Assume that the rotor is salient. To break down the current into direct- and quadrature-axis components, it is necessary to know the *direction* of $\mathbf{E}_A$. This direction may be determined from Equation (C–8):

$$\mathbf{E}_A'' = \mathbf{V}_\phi + R_A\mathbf{I}_A + jX_q\mathbf{I}_A \tag{C–8}$$

$$= 480\angle 0°\text{ V} + 0\text{ V} + j(0.075\ \Omega)(693\angle -36.87°\text{ A})$$

$$= 480\angle 0° + 52\angle 53.13° = 513\angle 4.65°\text{ V}$$

The direction of $\mathbf{E}_A$ is $\delta = 4.65°$. The magnitude of the direct-axis component of current is thus

$$\begin{aligned}I_d &= I_A \sin(\theta + \delta) \\ &= (693\text{ A}) \sin(36.87 + 4.65) = 459\text{ A}\end{aligned}$$

and the magnitude of the quadrature-axis component of current is

$$I_q = I_A \cos(\theta + \delta)$$
$$= (693 \text{ A}) \cos(36.87 + 4.65) = 519 \text{ A}$$

Combining magnitudes and angles yields

$$\mathbf{I}_d = 459 \angle -85.35° \text{ A}$$
$$\mathbf{I}_q = 519 \angle 4.65° \text{ A}$$

The resulting internal generated voltage is

$$\mathbf{E}_A = \mathbf{V}_\phi + R_A\mathbf{I}_A + jX_d\mathbf{I}_d + jX_q\mathbf{I}_q$$
$$= 480 \angle 0° \text{ V} + 0 \text{ V} + j(0.1\ \Omega)(459 \angle -85.35° \text{ A}) + j(0.075\ \Omega)(519 \angle 4.65° \text{ A})$$
$$= 524.3 \angle 4.65° \text{ V}$$

Notice that the *magnitude of* $\mathbf{E}_A$ is not much affected by the salient poles, but the *angle* of $\mathbf{E}_A$ is considerably different with salient poles than it is without salient poles.

C.2 TORQUE AND POWER EQUATIONS OF SALIENT-POLE MACHINE

The power output of a synchronous generator with a cylindrical rotor as a function of the torque angle was given in Chapter 5 as

$$\boxed{P = \frac{3V_\phi E_A \sin\delta}{X_S}} \tag{5–20}$$

This equation assumed that the armature resistance was negligible. Making the same assumption, what is the output power of a salient-pole generator as a function of torque angle? To find out, refer to Figure C–6. The power out of a synchronous generator is the sum of the power due to the direct-axis current and the power due to the quadrature-axis current:

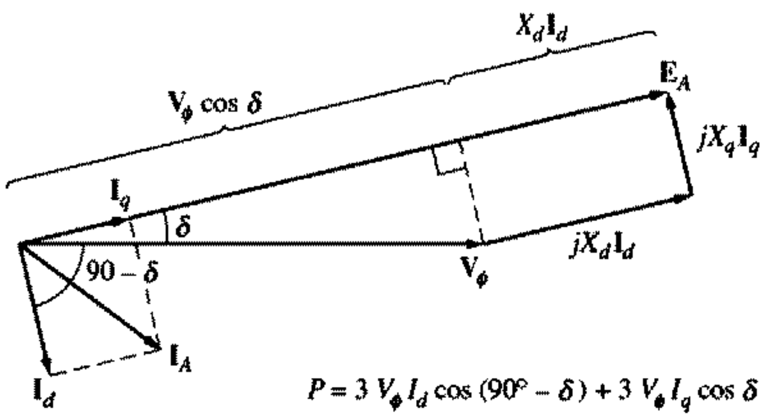

FIGURE C–6
Determining the power output of a salient-pole synchronous generator. Both $\mathbf{I}_d$ and $\mathbf{I}_q$ contribute to the output power, as shown.

$$\begin{aligned} P &= P_d + P_q \\ &= 3V_\phi I_d \cos(90° - \delta) + 3V_\phi I_q \cos\delta \\ &= 3V_\phi I_d \sin\delta + 3V_\phi I_q \cos\delta \end{aligned} \tag{C–9}$$

From Figure C–6, the direct-axis current is given by

$$I_d = \frac{E_A - V_\phi \cos\delta}{X_d} \tag{C–10}$$

and the quadrature-axis current is given by

$$I_q = \frac{V_\phi \sin\delta}{X_q} \tag{C–11}$$

Substituting Equations (C–10) and (C–11) into Equation (C–9) yields

$$\begin{aligned} P &= 3V_\phi\left(\frac{E_A - V_\phi \cos\delta}{X_d}\right)\sin\delta + 3V_\phi\left(\frac{V_\phi \sin\delta}{X_q}\right)\cos\delta \\ &= \frac{3V_\phi E_A}{X_d}\sin\delta + 3V_\phi^2\left(\frac{1}{X_q} - \frac{1}{X_d}\right)\sin\delta\cos\delta \end{aligned}$$

Since, $\sin\delta\cos\delta = \frac{1}{2}\sin 2\delta$, this expression reduces to

$$\boxed{P = \frac{3V_\phi E_A}{X_d}\sin\delta + \frac{3V_\phi^2}{2}\left(\frac{X_d - X_q}{X_d X_q}\right)\sin 2\delta} \tag{C–12}$$

The first term of this expression is the same as the power in a cylindrical rotor machine, and the second term is the additional power due to the reluctance torque in the machine.

Since the induced torque in the generator is given by $\tau_{ind} = P_{conv}/\omega_m$, the induced torque in the motor can be expressed as

$$\boxed{\tau_{ind} = \frac{3V_\phi E_A}{\omega_m X_d}\sin\delta + \frac{3V_\phi^2}{2\omega_m}\left(\frac{X_d - X_q}{X_d X_q}\right)\sin 2\delta} \tag{C–13}$$

The induced torque out of a salient-pole generator as a function of the torque angle δ is plotted in Figure C–7.

PROBLEMS

C–1. A 480-V, 200-kVA, 0.8-PF-lagging, 60-Hz, four-pole, Y-connected synchronous generator has a direct-axis reactance of 0.25 Ω, a quadrature-axis reactance of 0.18 Ω and an armature resistance of 0.03 Ω. Friction, windage, and stray losses may be assumed negligible. The generator's open-circuit characteristic is given by Figure P5–1.

(a) How much field current is required to make V_T equal to 480 V when the generator is running at no load?

(b) What is the internal generated voltage of this machine when it is operating at rated conditions? How does this value of $\mathbf{E}_A$ compare to that of Problem 5–2b?

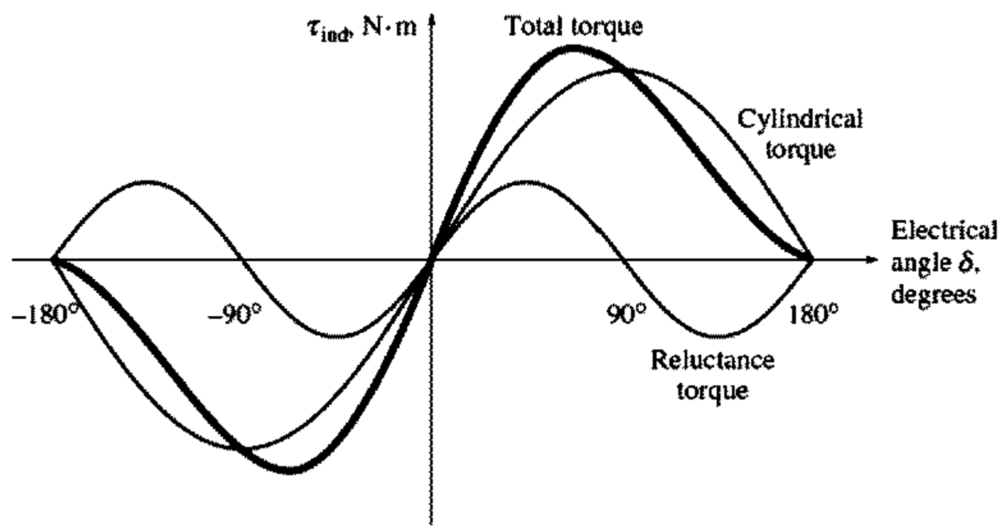

FIGURE C–7
Plot of torque versus torque angle for a salient-pole synchronous generator. Note the component of torque due to rotor reluctance.

(*c*) What fraction of this generator's full-load power is due to the reluctance torque of the rotor?

C–2. A 14-pole, Y-connected, three-phase, water-turbine-driven generator is rated at 120 MVA, 13.2 kV, 0.8 PF lagging, and 60 Hz. Its direct-axis reactance is 0.62 Ω and its quadrature-axis reactance is 0.40 Ω. All rotational losses may be neglected.

(*a*) What internal generated voltage would be required for this generator to operate at the rated conditions?

(*b*) What is the voltage regulation of this generator at the rated conditions?

(*c*) Sketch the power-versus-torque-angle curve for this generator. At what angle δ is the power of the generator maximum?

(*d*) How does the maximum power out of this generator compare to the maximum power available if it were of cylindrical rotor construction?

C–3. Suppose that a salient-pole machine is to be used as a motor.

(*a*) Sketch the phasor diagram of a salient-pole synchronous machine used as a motor.

(*b*) Write the equations describing the voltages and currents in this motor.

(*c*) Prove that the torque angle δ between $\mathbf{E}_A$ and $\mathbf{V}_\phi$ on this motor is given by

$$\delta = \tan^{-1} \frac{I_A X_q \cos\theta - I_A R_A \sin\theta}{V_\phi + I_A X_q \sin\theta + I_A R_A \cos\theta}$$

C–4. If the machine in Problem C–1 is running as a *motor* at the rated conditions, what is the maximum torque that can be drawn from its shaft without it slipping poles *when the field current is zero?*