# CHAPTER 10

# SINGLE-PHASE AND SPECIAL-PURPOSE MOTORS

Chapters 4 through 7 were devoted to the operation of the two major classes of ac machines (synchronous and induction) on *three-phase* power systems. Motors and generators of these types are by far the most common ones in larger commercial and industrial settings. However, most homes and small businesses do not have three-phase power available. For such locations, all motors must run from single-phase power sources. This chapter deals with the theory and operation of two major types of single-phase motors: the universal motor and the single-phase induction motor. The universal motor, which is a straightforward extension of the series dc motor, is described in Section 10.1.

The single-phase induction motor is described in Sections 10.2 to 10.5. The major problem associated with the design of single-phase induction motors is that, unlike three-phase power sources, a single-phase source does *not* produce a rotating magnetic field. Instead, the magnetic field produced by a single-phase source remains stationary in position and *pulses* with time. Since there is no net rotating magnetic field, conventional induction motors cannot function, and special designs are necessary.

In addition, there are a number of special-purpose motors which have not been previously covered. These include reluctance motors, hysteresis motors, stepper motors, and brushless dc motors. They are included in Section 10.6.

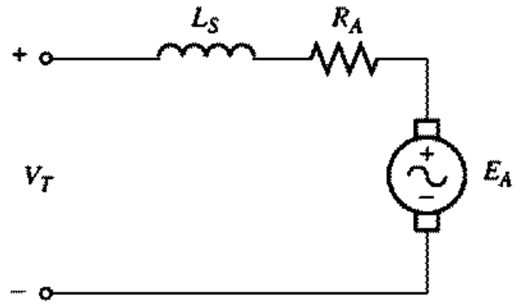


**FIGURE 10–1**
Equivalent circuit of a universal motor.

## 10.1 THE UNIVERSAL MOTOR

Perhaps the simplest approach to the design of a motor that will operate on a single-phase ac power source is to take a dc machine and run it from an ac supply. Recall from Chapter 8 that the induced torque of a dc motor is given by

$$\tau_{\text{ind}} = K\phi I_A \tag{8–49}$$

If the polarity of the voltage applied to a shunt or series dc motor is reversed, *both* the direction of the field flux *and* the direction of the armature current reverse, and the resulting induced torque continues in the same direction as before. Therefore, it should be possible to achieve a pulsating but unidirectional torque from a dc motor connected to an ac power supply.

Such a design is practical only for the series dc motor (see Figure 10–1), since the armature current and the field current in the machine must reverse at exactly the same time. For shunt dc motors, the very high field inductance tends to delay the reversal of the field current and thus to unacceptably reduce the average induced torque of the motor.

In order for a series dc motor to function effectively on ac, its field poles and stator frame must be completely laminated. If they were not completely laminated, their core losses would be enormous. When the poles and stator are laminated, this motor is often called a ***universal motor***, since it can run from either an ac or a dc source.

When the motor is running from an ac source, the commutation will be much poorer than it would be with a dc source. The extra sparking at the brushes is caused by transformer action inducing voltages in the coils undergoing commutation. These sparks significantly shorten brush life and can be a source of radio-frequency interference in certain environments.

A typical torque–speed characteristic of a universal motor is shown in Figure 10–2. It differs from the torque–speed characteristic of the same machine operating from a dc voltage source for two reasons:

1. The armature and field windings have quite a large reactance at 50 or 60 Hz. A significant part of the input voltage is dropped across these reactances, and therefore $E_A$ is *smaller* for a given input voltage during ac operation than it is during dc operation. Since $E_A = K\phi\omega$, the motor is *slower* for a given

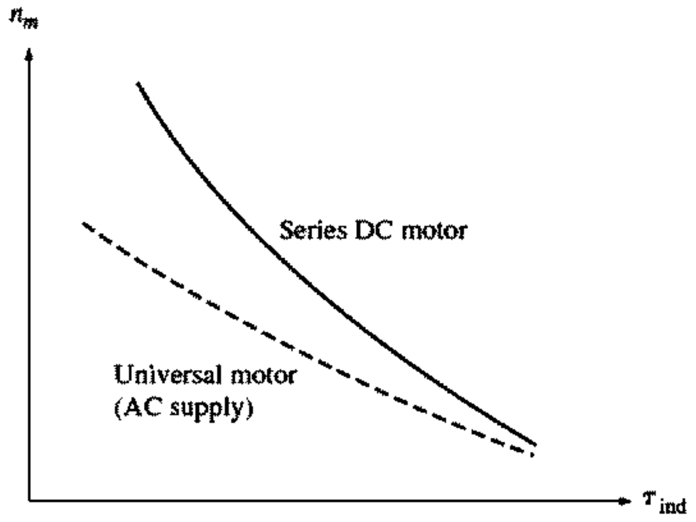


**FIGURE 10–2**
Comparison of the torque–speed characteristic of a universal motor when operating from ac and dc power supplies.

armature current and induced torque on alternating current than it would be on direct current.

2. In addition, the peak voltage of an ac system is $\sqrt{2}$ times its rms value, so magnetic saturation could occur near the peak current in the machine. This saturation could significantly lower the rms flux of the motor for a given current level, tending to reduce the machine's induced torque. Recall that a decrease in flux increases the speed of a dc machine, so this effect may partially offset the speed decrease caused by the first effect.

## Applications of Universal Motors

The universal motor has the sharply drooping torque–speed characteristic of a dc series motor, so it is not suitable for constant-speed applications. However, it is compact and gives more torque per ampere than any other single-phase motor. It is therefore used where light weight and high torque are important.

Typical applications for this motor are vacuum cleaners, drills, similar portable tools, and kitchen appliances.

## Speed Control of Universal Motors

As with dc series motors, the best way to control the speed of a universal motor is to vary its rms input voltage. The higher the rms input voltage, the greater the resulting speed of the motor. Typical torque–speed characteristics of a universal motor as a function of voltage are shown in Figure 10–3.

In practice, the average voltage applied to such a motor is varied with one of the SCR or TRIAC circuits introduced in Chapter 3. Two such speed control circuits are shown in Figure 10–4. The variable resistors shown in these figures are the speed adjustment knobs of the motors (e.g., such a resistor would be the trigger of a variable-speed drill).

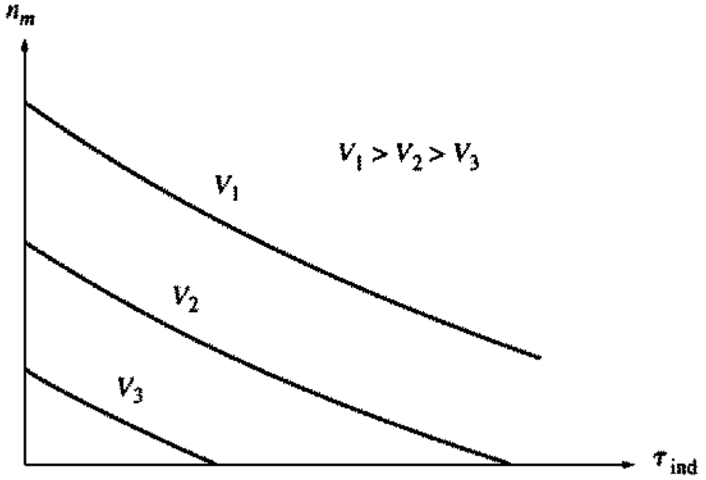
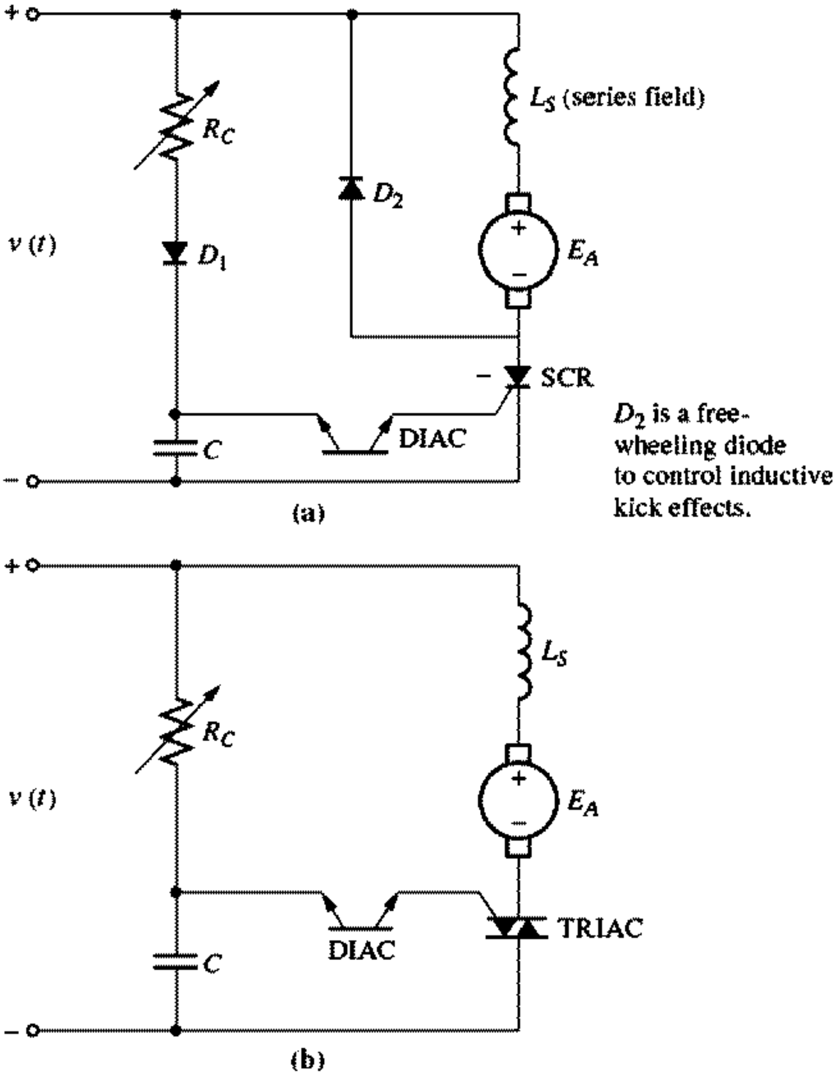


**FIGURE 10–3**
The effect of changing terminal voltage on the torque–speed characteristic of a universal motor.



$D_2$ is a free-wheeling diode to control inductive kick effects.

**FIGURE 10–4**
Sample universal motor speed-control circuits. (a) Half-wave; (b) full-wave.

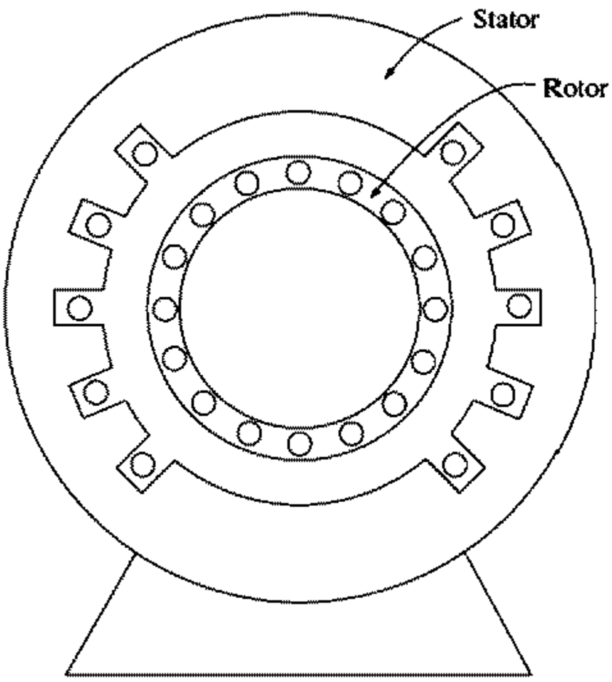


**FIGURE 10–5**
Construction of a single-phase induction motor. The rotor is the same as in a three-phase induction motor, but the stator has only a single distributed phase.

## 10.2 INTRODUCTION TO SINGLE-PHASE INDUCTION MOTORS

Another common single-phase motor is the single-phase version of the induction motor. An induction motor with a squirrel-cage rotor and a single-phase stator is shown in Figure 10–5.

Single-phase induction motors suffer from a severe handicap. Since there is only one phase on the stator winding, the magnetic field in a single-phase induction motor does not rotate. Instead, it *pulses*, getting first larger and then smaller, but always remaining in the same direction. Because there is no rotating stator magnetic field, a single-phase induction motor has *no starting torque*.

This fact is easy to see from an examination of the motor when its rotor is stationary. The stator flux of the machine first increases and then decreases, but it always points in the same direction. Since the stator magnetic field does not rotate, there is *no relative motion* between the stator field and the bars of the rotor. Therefore, there is no induced voltage due to relative motion in the rotor, no rotor current flow due to relative motion, and no induced torque. Actually, a voltage is induced in the rotor bars by transformer action ($d\phi/dt$), and since the bars are short-circuited, current flows in the rotor. However, this magnetic field is lined up with the stator magnetic field, and it produces no net torque on the rotor,

$$\begin{aligned} \tau_{\text{ind}} &= k\mathbf{B}_R \times \mathbf{B}_S \qquad (4\text{–}58) \\ &= kB_RB_S \sin \gamma \\ &= kB_RB_S \sin 180^\circ = 0 \end{aligned}$$

At stall conditions, the motor looks like a transformer with a short-circuited secondary winding (see Figure 10–6).

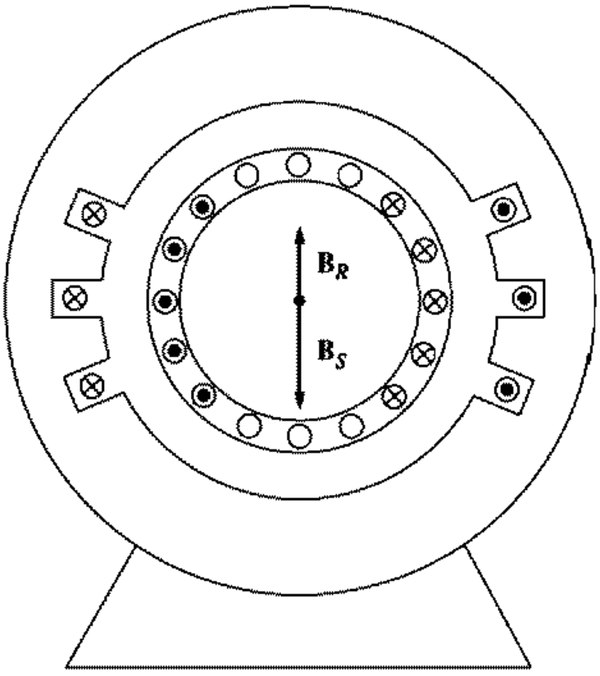


**FIGURE 10–6**
The single-phase induction motor at starting conditions. The stator winding induces opposing voltages and currents into the rotor circuit, resulting in a rotor magnetic field *lined up* with the stator magnetic field. $\tau_{\text{ind}} = 0$.

The fact that single-phase induction motors have no intrinsic starting torque was a serious impediment to early development of the induction motor. When induction motors were first being developed in the late 1880s and early 1890s, the first available ac power systems were 133-Hz, single-phase. With the materials and techniques then available, it was impossible to build a motor that worked well. The induction motor did not become an off-the-shelf working product until three-phase, 25-Hz power systems were developed in the mid-1890s.

However, *once the rotor begins to turn, an induced torque will be produced in it.* There are two basic theories which explain why a torque is produced in the rotor once it is turning. One is called the *double-revolving-field theory* of single-phase induction motors, and the other is called the *cross-field theory* of single-phase induction motors. Each of these approaches will be described below.

## The Double-Revolving-Field Theory of Single-Phase Induction Motors

The double-revolving-field theory of single-phase induction motors basically states that a stationary pulsating magnetic field can be resolved into two *rotating* magnetic fields, each of equal magnitude but rotating in opposite directions. The induction motor responds to each magnetic field separately, and the net torque in the machine will be the sum of the torques due to each of the two magnetic fields.

Figure 10–7 shows how a stationary pulsating magnetic field can be resolved into two equal and oppositely rotating magnetic fields. The flux density of the stationary magnetic field is given by

$$\mathbf{B}_S(t) = (\mathbf{B}_{\max} \cos \omega t)\,\hat{\mathbf{j}} \qquad (10\text{–}1)$$

A clockwise-rotating magnetic field can be expressed as

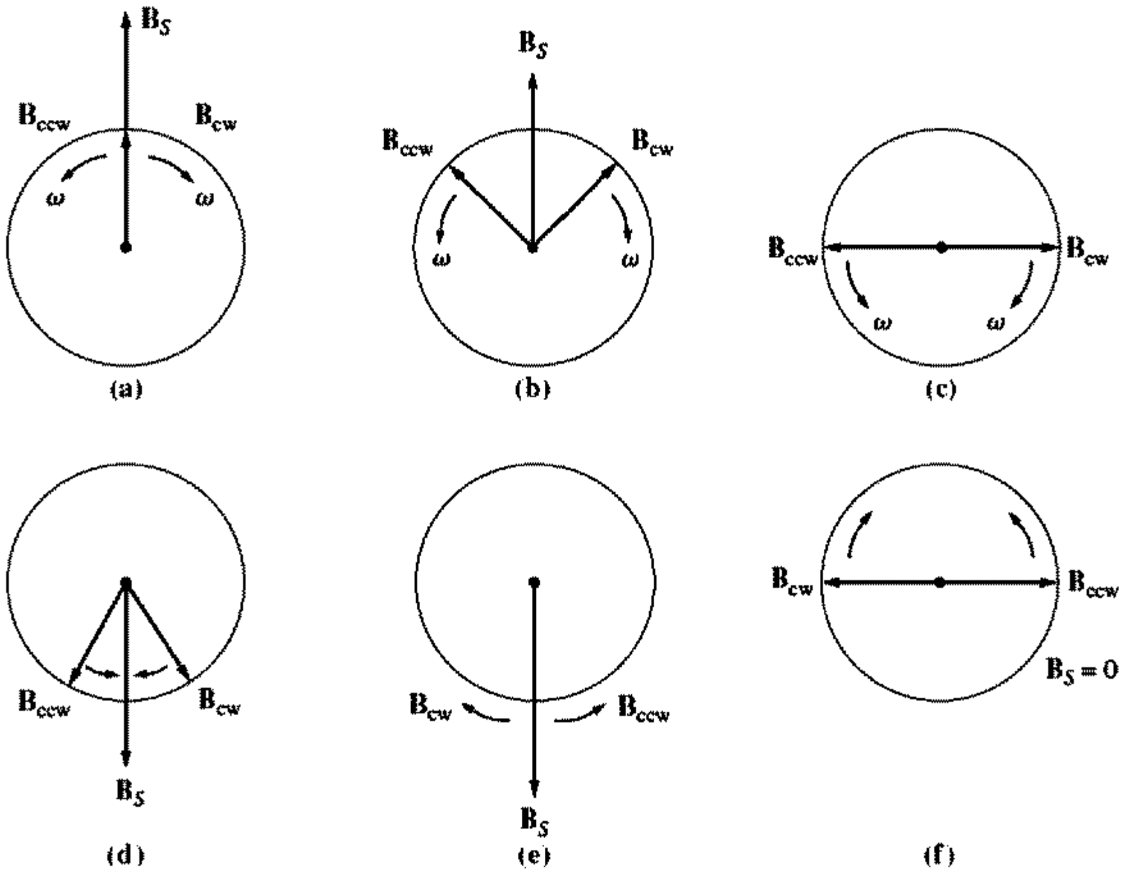


**FIGURE 10–7**

The resolution of a single pulsating magnetic field into two magnetic fields of equal magnitude by rotation in opposite directions. Notice that at all times the vector sum of the two magnetic fields lies in the vertical plane.

$$\mathbf{B}_{CW}(t) = \left(\frac{1}{2}B_{\max}\cos\omega t\right)\hat{\mathbf{i}} - \left(\frac{1}{2}B_{\max}\sin\omega t\right)\hat{\mathbf{j}} \tag{10–2}$$

and a counterclockwise-rotating magnetic field can be expressed as

$$\mathbf{B}_{CCW}(t) = \left(\frac{1}{2}B_{\max}\cos\omega t\right)\hat{\mathbf{i}} + \left(\frac{1}{2}B_{\max}\sin\omega t\right)\hat{\mathbf{j}} \tag{10–3}$$

Notice that the sum of the clockwise and counterclockwise magnetic fields is equal to the stationary pulsating magnetic field $\mathbf{B}_S$:

$$\mathbf{B}_S(t) = \mathbf{B}_{CW}(t) + \mathbf{B}_{CCW}(t) \tag{10–4}$$

The torque–speed characteristic of a three-phase induction motor in response to its single rotating magnetic field is shown in Figure 10–8a. A single-phase induction motor responds to each of the two magnetic fields present within it, so the net induced torque in the motor is the *difference* between the two torque–speed curves. This net torque is shown in Figure 10–8b. Notice that there is no net torque at zero speed, so this motor has no starting torque.

The torque–speed characteristic shown in Figure 10–8b is not quite an accurate description of the torque in a single-phase motor. It was formed by the

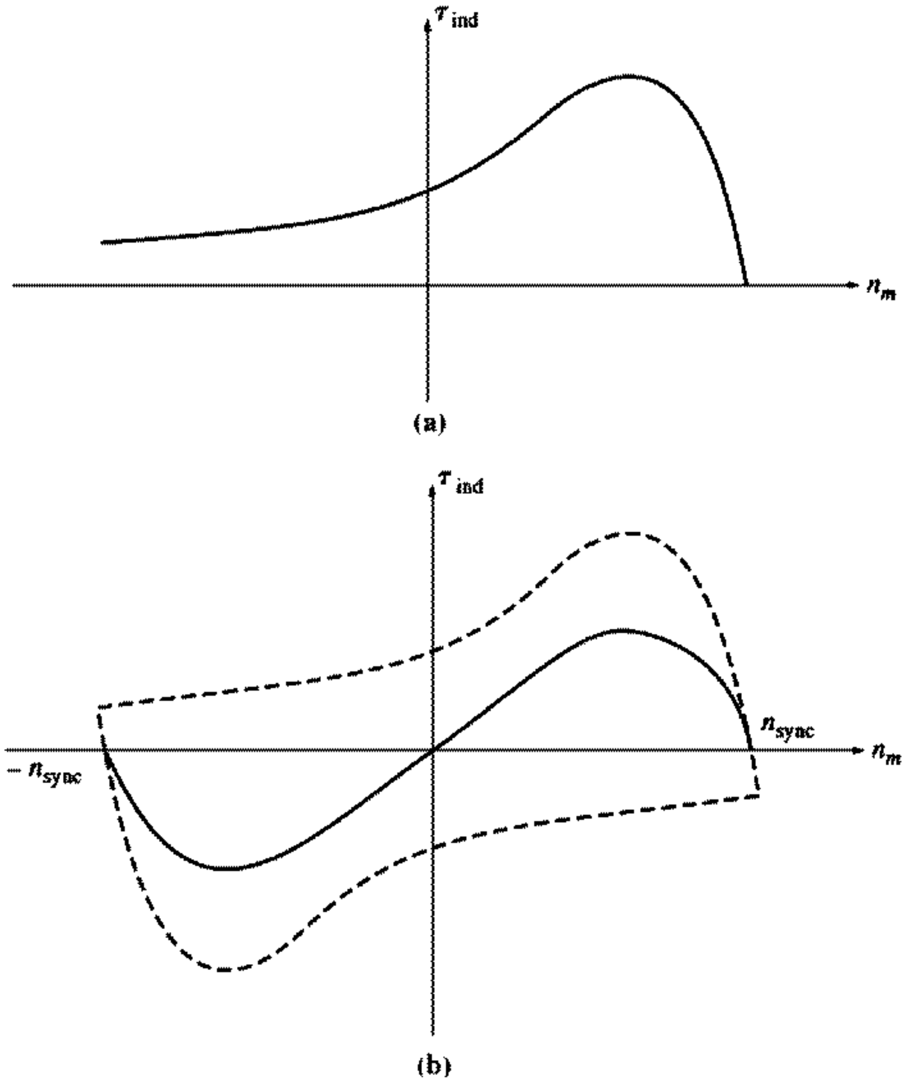


**FIGURE 10–8**
(a) The torque–speed characteristic of a three-phase induction motor. (b) The torque–speed characteristic curves of the two equal and oppositely rotating stator magnetic fields.

superposition of two three-phase characteristics and ignored the fact that both magnetic fields are present *simultaneously* in the single-phase motor.

If power is applied to a three-phase motor while it is forced to turn backward, its rotor currents will be very high (see Figure 10–9a). However, the rotor frequency is also very high, making the rotor's reactance much much larger than its resistance. Since the rotor's reactance is so very high, the rotor current lags the rotor voltage by almost 90°, producing a magnetic field that is nearly 180° from the stator magnetic field (see Figure 10–10). The induced torque in the motor is proportional to the sine of the angle between the two fields, and the sine of an angle near 180° is a very small number. The motor's torque would be very small, except that the extremely high rotor currents partially offset the effect of the magnetic field angles (see Figure 10–9b).

On the other hand, in a single-phase motor, both the forward and the reverse magnetic fields are present and both are produced by the *same* current. The forward

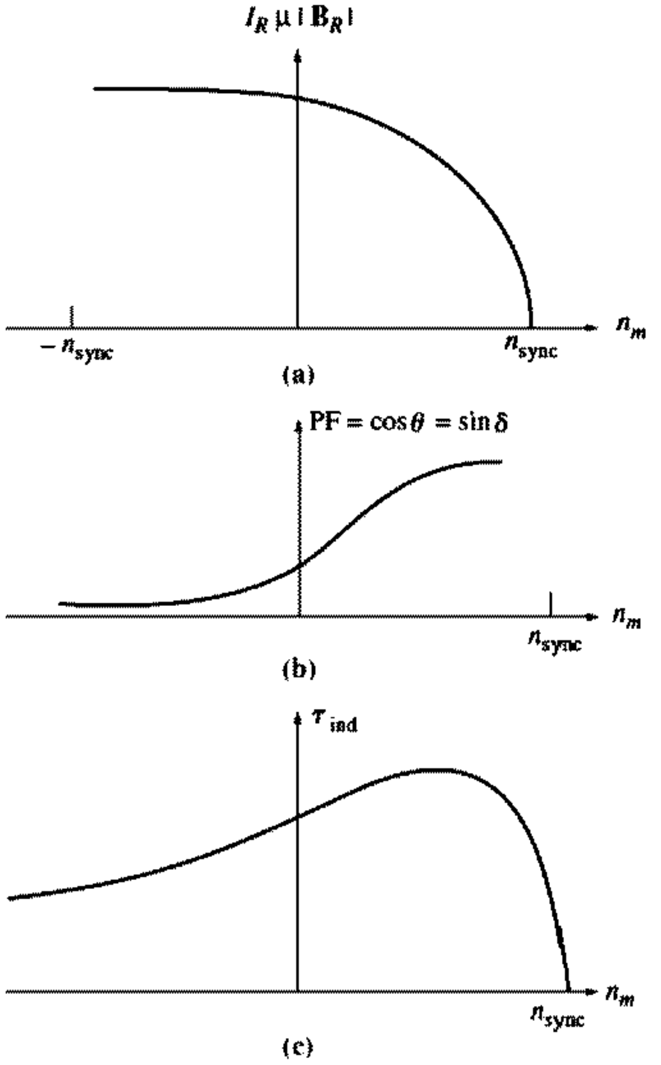


**FIGURE 10–9**
The torque–speed characteristic of a three-phase induction motor is proportional to both the strength of the rotor magnetic field and the sine of the angle between the fields. When the rotor is turned backward, $I_R$ and $I_S$ are very high, but the angle between the fields is very large, and that angle limits the torque of the motor.

and reverse magnetic fields in the motor each contribute a component to the total voltage in the stator and, in a sense, are in series with each other. Because both magnetic fields are present, the forward-rotating magnetic field (which has a high effective rotor resistance $R_2/s$) will limit the stator current flow in the motor (which produces both the forward and reverse fields). Since the current supplying the reverse stator magnetic field is limited to a small value and since the reverse rotor magnetic field is at a very large angle with respect to the reverse stator magnetic field, the torque due to the reverse magnetic fields is *very* small near synchronous speed. A more accurate torque–speed characteristic for the single-phase induction motor is shown in Figure 10–11.

In addition to the average net torque shown in Figure 10–11, there are torque pulsations at twice the stator frequency. These torque pulsations are caused when the forward and reverse magnetic fields cross each other twice each cycle. Although these torque pulsations produce no average torque, they do increase vibration, and they make single-phase induction motors noisier than three-phase motors of the same size. There is no way to eliminate these pulsations, since instantaneous power always comes in pulses in a single-phase circuit. A motor designer must allow for this inherent vibration in the mechanical design of single-phase motors.

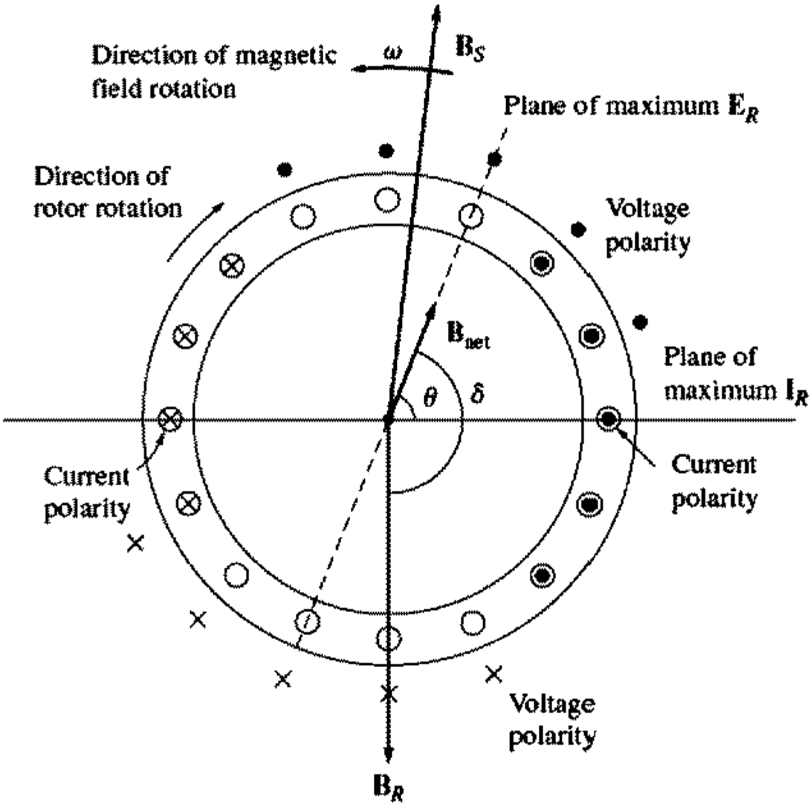


**FIGURE 10–10**
When the rotor of the motor is forced to turn backward, the angle $\gamma$ between $\mathbf{B}_R$ and $\mathbf{B}_S$ approaches 180°.

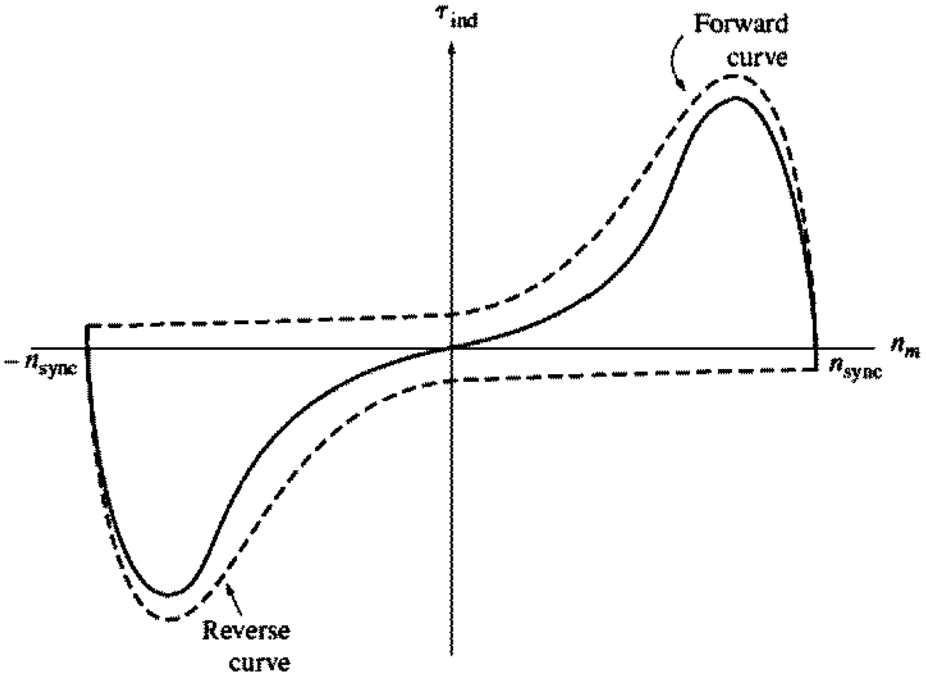


**FIGURE 10–11**
The torque–speed characteristic of a single-phase induction motor, taking into account the current limitation on the backward-rotating magnetic field caused by the presence of the forward-rotating magnetic field.

## The Cross-Field Theory of Single-Phase Induction Motors

The cross-field theory of single-phase induction motors looks at the induction motor from a totally different point of view. This theory is concerned with the voltages and currents that the stationary stator magnetic field can induce in the bars of the rotor when the rotor is moving.

Consider a single-phase induction motor with a rotor which has been brought up to speed by some external method. Such a motor is shown in Figure 10–12a. Voltages are induced in the bars of this rotor, with the peak voltage occurring in the windings passing directly under the stator windings. These rotor voltages produce a current flow in the rotor, but because of the rotor's high reactance, the current lags the voltage by almost 90°. Since the rotor is rotating at nearly synchronous speed, that 90° time lag in current produces an almost 90° *angular* shift between the plane of peak rotor voltage and the plane of peak current. The resulting rotor magnetic field is shown in Figure 10–12b.

The rotor magnetic field is somewhat smaller than the stator magnetic field, because of the losses in the rotor, but they differ by nearly 90° in *both space and*

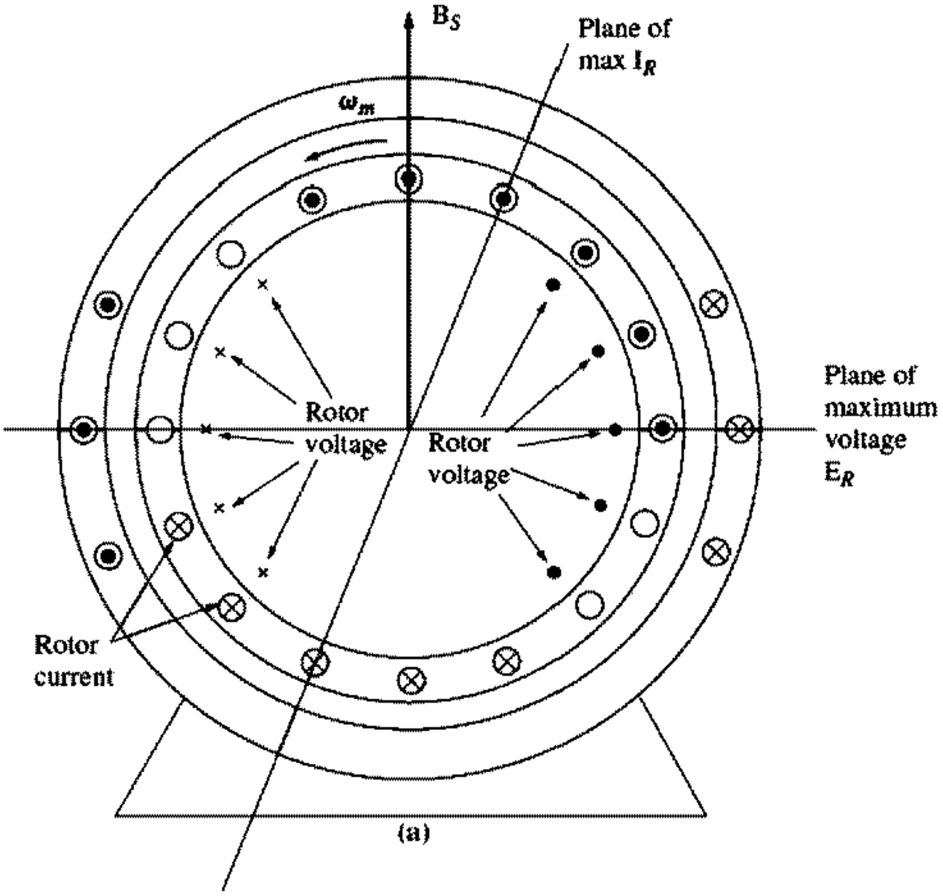


**FIGURE 10–12**
(a) The development of induced torque in a single-phase induction motor, as explained by the cross-field theory. If the stator field is pulsing, it will induce voltages in the rotor bars, as shown by the marks inside the rotor. However, the rotor current is delayed by nearly 90° behind the rotor voltage, and if the rotor is turning, the rotor current will peak at an angle different from that of the rotor voltage.

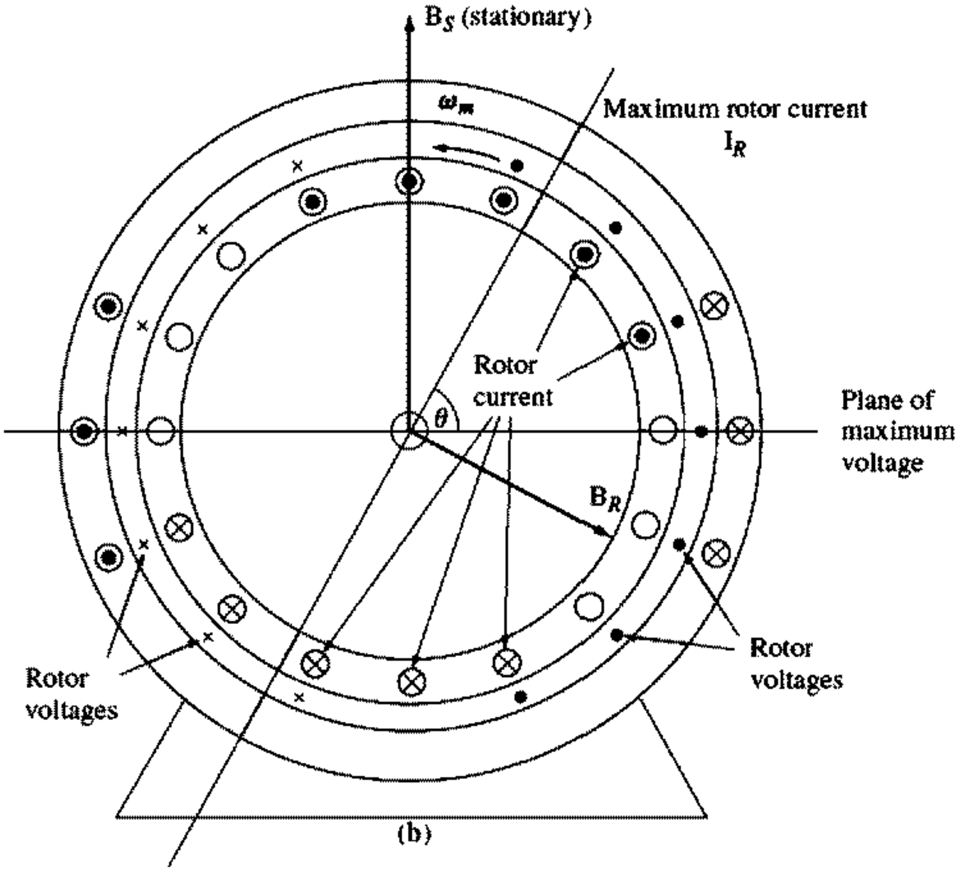


**FIGURE 10–12 (concluded)**
(b) This delayed rotor current produces a rotor magnetic field at an angle different from the angle of the stator magnetic field.

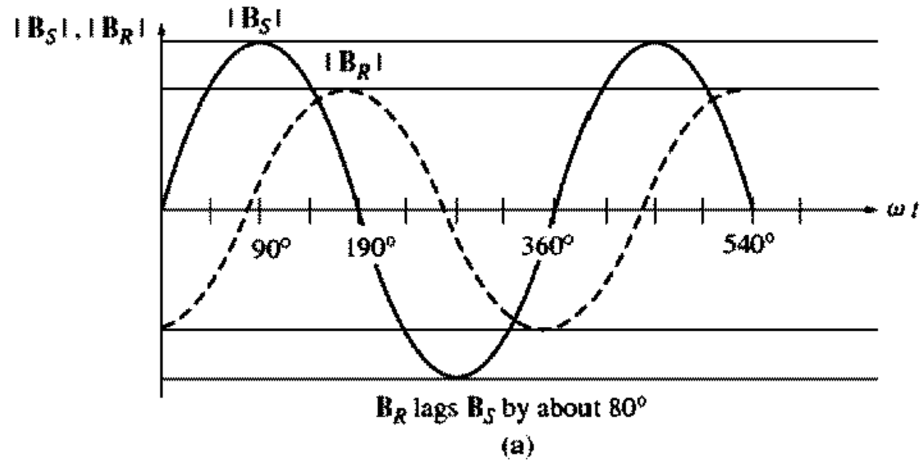


**FIGURE 10–13**
(a) The *magnitudes* of the magnetic fields as a function of time.

*time*. If these two magnetic fields are added at different times, one sees that the total magnetic field in the motor is rotating in a counterclockwise direction (see Figure 10–13). With a rotating magnetic field present in the motor, the induction

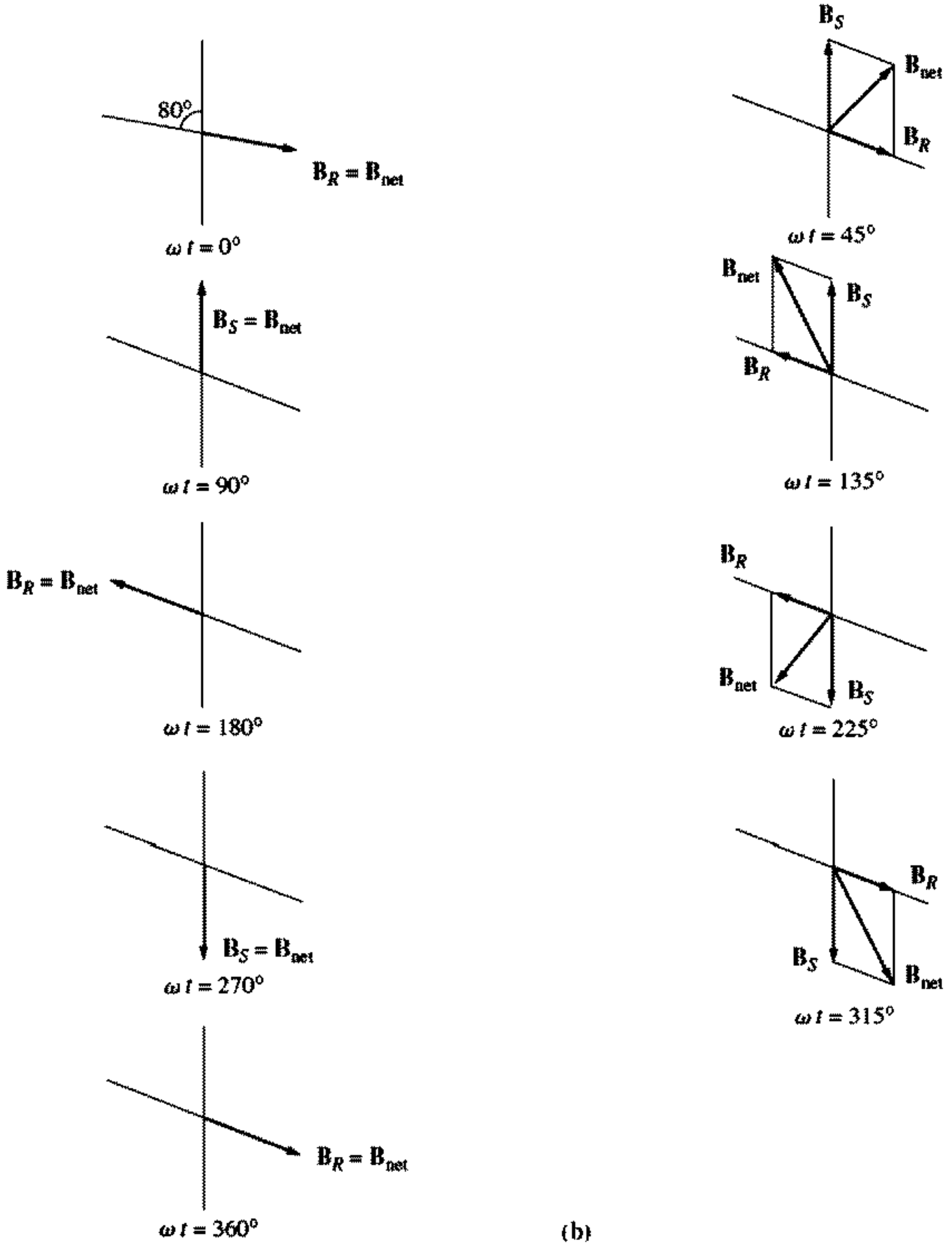


**FIGURE 10–13** (concluded)
(b) The vector sum of the rotor and stator magnetic fields at various times, showing a net magnetic field which rotates in a counterclockwise direction.

motor will develop a net torque in the direction of motion, and that torque will keep the rotor turning.

If the motor's rotor had originally been turned in a clockwise direction, the resulting torque would be clockwise and would again keep the rotor turning.

## 10.3 STARTING SINGLE-PHASE INDUCTION MOTORS

As previously explained, a single-phase induction motor has no intrinsic starting torque. There are three techniques commonly used to start these motors, and single-phase induction motors are classified according to the methods used to produce their starting torque. These starting techniques differ in cost and in the amount of starting torque produced, and an engineer normally uses the least expensive technique that meets the torque requirements in any given application. The three major starting techniques are

1. Split-phase windings
2. Capacitor-type windings
3. Shaded stator poles

All three starting techniques are methods of making one of the two revolving magnetic fields in the motor stronger than the other and so giving the motor an initial nudge in one direction or the other.

### Split-Phase Windings

A split-phase motor is a single-phase induction motor with two stator windings, a main stator winding ($M$) and an auxiliary starting winding ($A$) (see Figure 10–14). These two windings are set 90 electrical degrees apart along the stator of the motor, and the auxiliary winding is designed to be switched out of the circuit at some set speed by a centrifugal switch. The auxiliary winding is designed to have

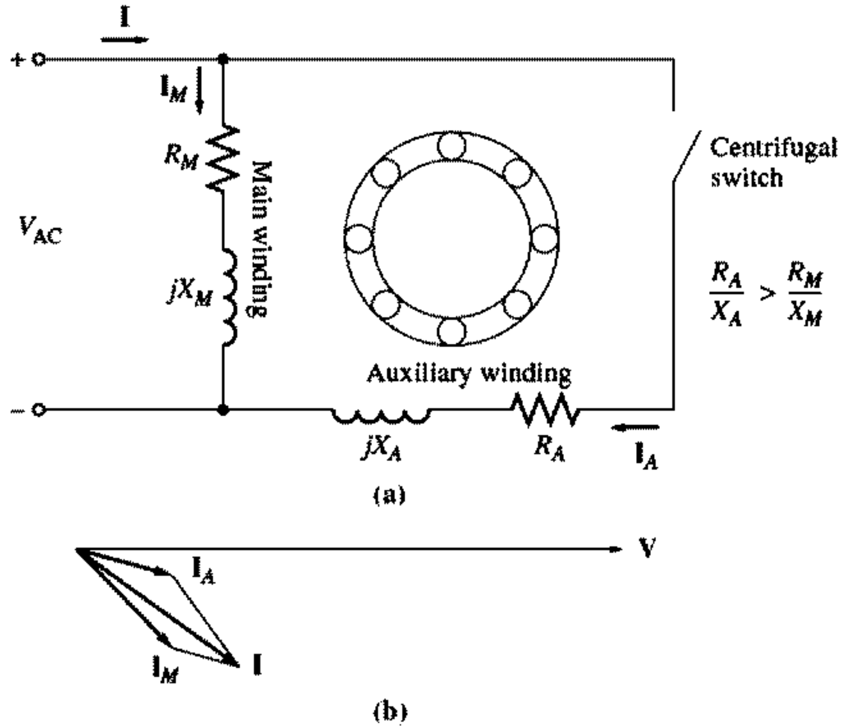


**FIGURE 10–14**
(a) A split-phase induction motor. (b) The currents in the motor at starting conditions.

a higher resistance/reactance ratio than the main winding, so that the current in the auxiliary winding *leads* the current in the main winding. This higher *R/X* ratio is usually accomplished by using smaller wire for the auxiliary winding. Smaller wire is permissible in the auxiliary winding because it is used only for starting and therefore does not have to take full current continuously.

To understand the function of the auxiliary winding, refer to Figure 10–15. Since the current in the auxiliary winding leads the current in the main winding,

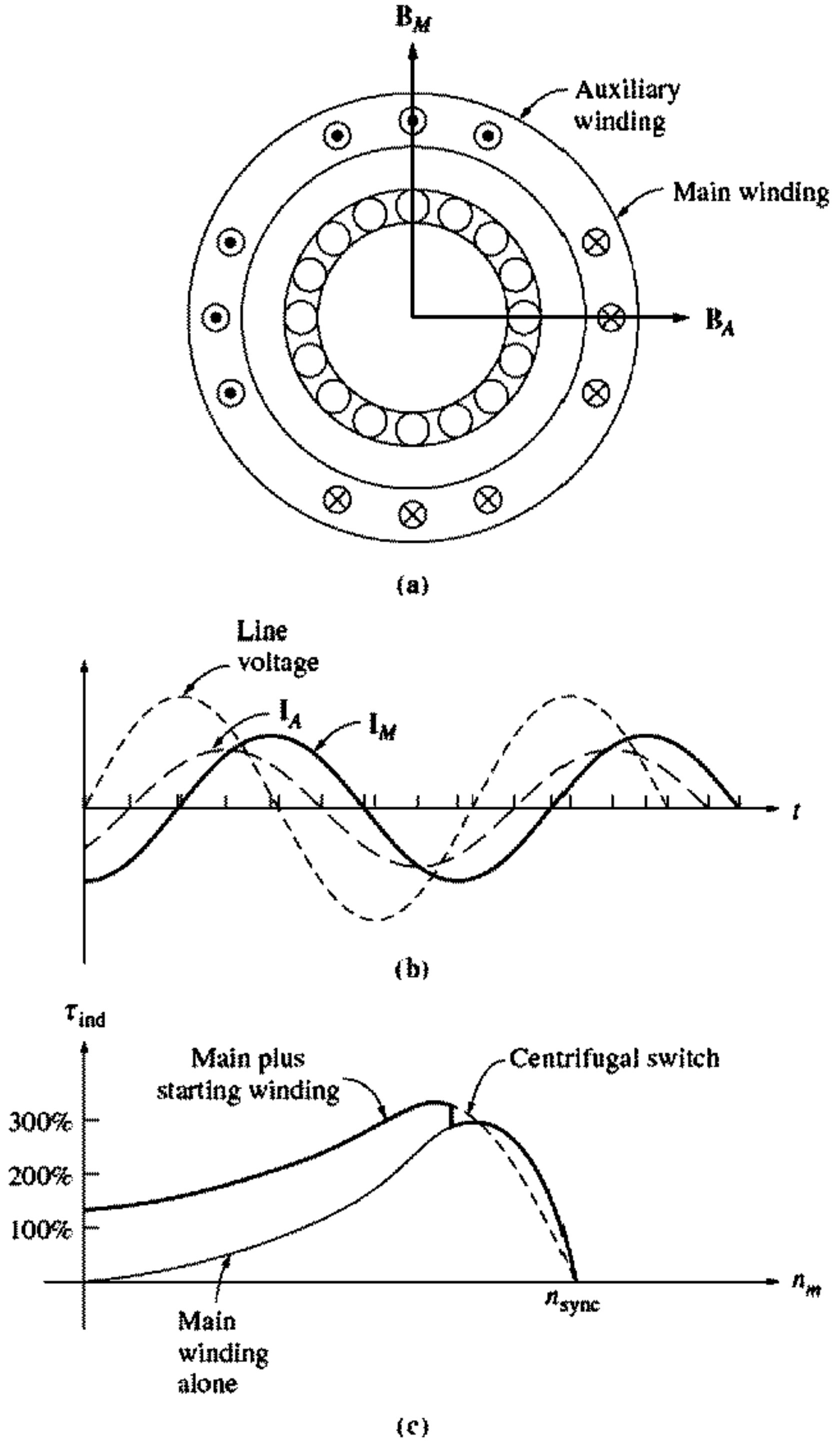


**FIGURE 10–15**
(a) Relationship of main and auxiliary magnetic fields. (b) $\mathbf{I}_A$ peaks before $\mathbf{I}_M$, producing a net counterclockwise rotation of the magnetic fields. (c) The resulting torque–speed characteristic.

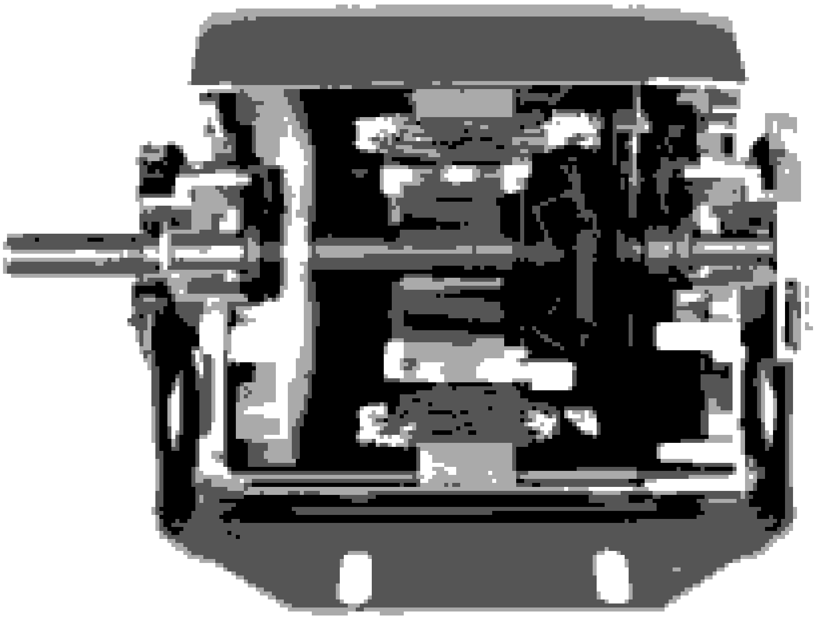


**FIGURE 10–16**
Cutaway view of a split-phase motor, showing the main and auxiliary windings and the centrifugal switch. (*Courtesy of Westinghouse Electric Corporation.*)

the magnetic field $\mathbf{B}_A$ peaks before the main magnetic field $\mathbf{B}_M$. Since $\mathbf{B}_A$ peaks first and then $\mathbf{B}_M$, there is a net counterclockwise rotation in the magnetic field. In other words, the auxiliary winding makes one of the oppositely rotating stator magnetic fields larger than the other one and provides a net starting torque for the motor. A typical torque–speed characteristic is shown in Figure 10–15c.

A cutaway diagram of a split-phase motor is shown in Figure 10–16. It is easy to see the main and auxiliary windings (the auxiliary windings are the smaller-diameter wires) and the centrifugal switch that cuts the auxiliary windings out of the circuit when the motor approaches operating speed.

Split-phase motors have a moderate starting torque with a fairly low starting current. They are used for applications which do not require very high starting torques, such as fans, blowers, and centrifugal pumps. They are available for sizes in the fractional-horsepower range and are quite inexpensive.

In a split-phase induction motor, the current in the auxiliary windings always peaks before the current in the main winding, and therefore the magnetic field from the auxiliary winding always peaks before the magnetic field from the main winding. The direction of rotation of the motor is determined by whether the space angle of the magnetic field from the auxiliary winding is 90° ahead or 90° behind the angle of the main winding. Since that angle can be changed from 90° ahead to 90° behind just by switching the connections on the auxiliary winding, *the direction of rotation of the motor can be reversed by switching the connections of the auxiliary winding* while leaving the main winding's connections unchanged.

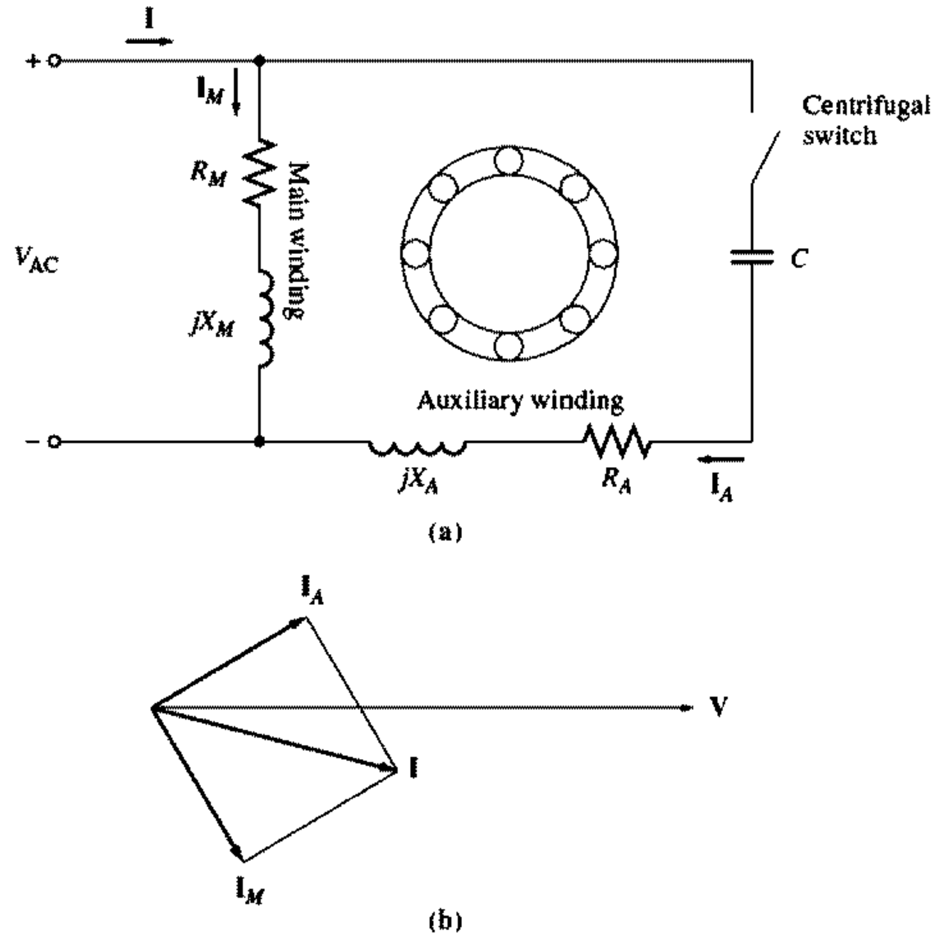


**FIGURE 10–17**
(a) A capacitor-start induction motor. (b) Current angles at starting in this motor.

## Capacitor-Start Motors

For some applications, the starting torque supplied by a split-phase motor is insufficient to start the load on a motor's shaft. In those cases, capacitor-start motors may be used (Figure 10–17). In a capacitor-start motor, a capacitor is placed in series with the auxiliary winding of the motor. By proper selection of capacitor size, the magnetomotive force of the starting current in the auxiliary winding can be adjusted to be equal to the magnetomotive force of the current in the main winding, and the phase angle of the current in the auxiliary winding can be made to lead the current in the main winding by 90°. Since the two windings are physically separated by 90°, a 90° phase difference in current will yield a single uniform rotating stator magnetic field, and the motor will behave just as though it were starting from a three-phase power source. In this case, the starting torque of the motor can be more than 300 percent of its rated value (see Figure 10–18).

Capacitor-start motors are more expensive than split-phase motors, and they are used in applications where a high starting torque is absolutely required. Typical applications for such motors are compressors, pumps, air conditioners, and other pieces of equipment that must start under a load. (See Figure 10–19.)

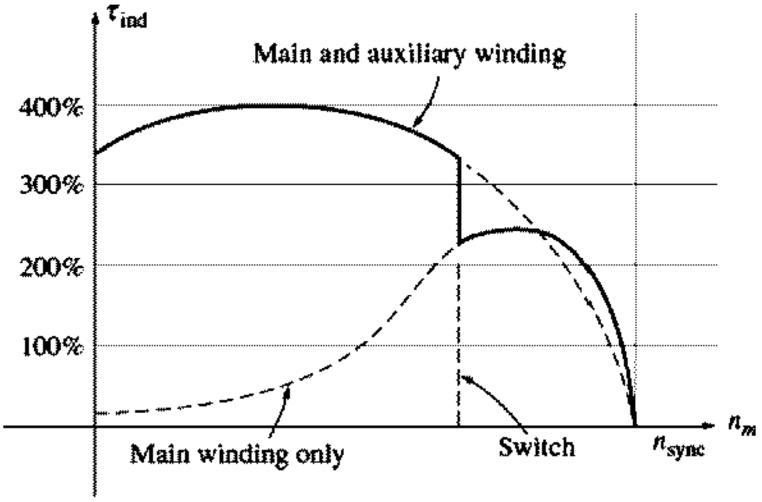


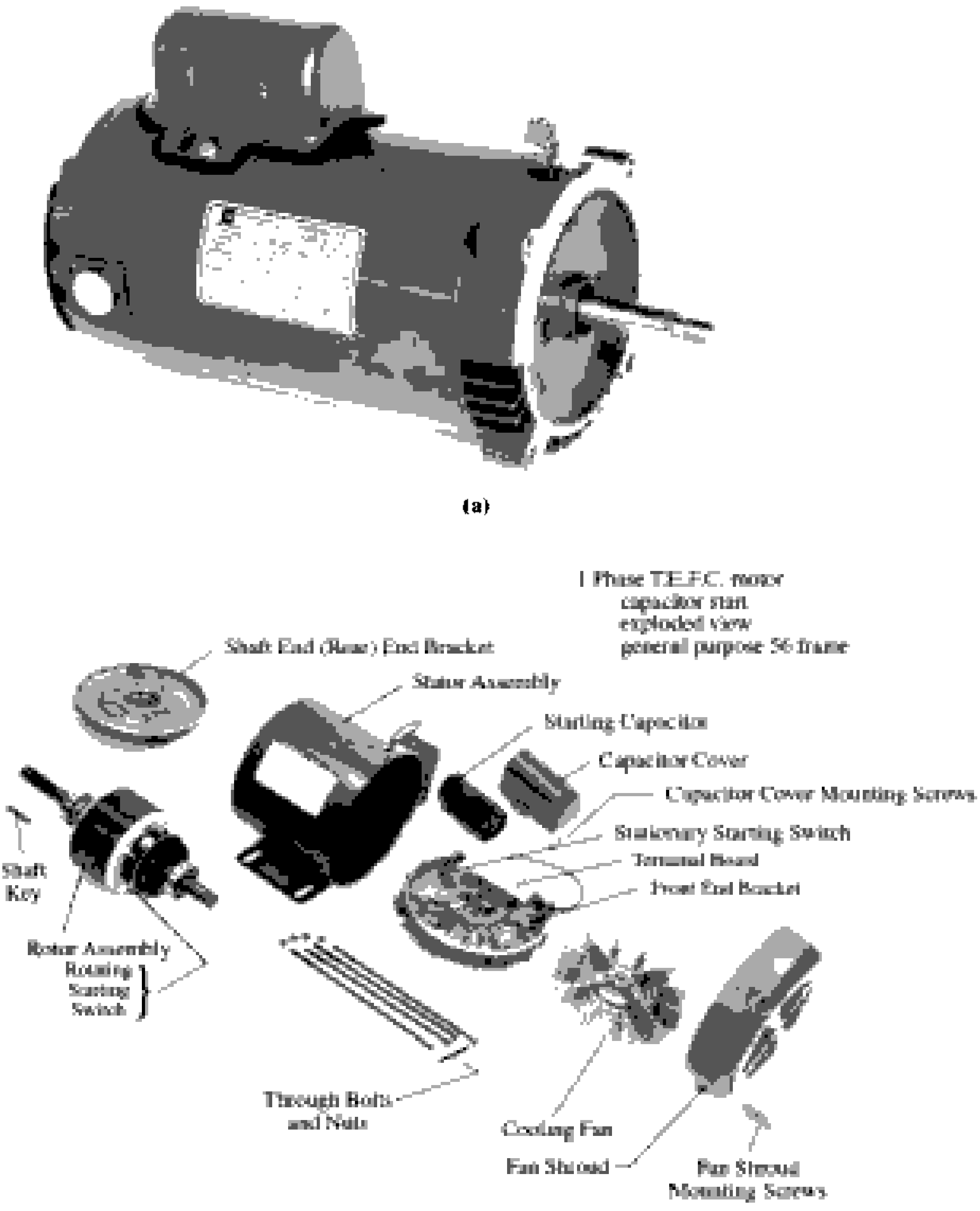
**FIGURE 10–18**
Torque–speed characteristic of a capacitor-start induction motor.

## Permanent Split-Capacitor and Capacitor-Start, Capacitor-Run Motors

The starting capacitor does such a good job of improving the torque–speed characteristic of an induction motor that an auxiliary winding with a smaller capacitor is sometimes left permanently in the motor circuit. If the capacitor's value is chosen correctly, such a motor will have a perfectly uniform rotating magnetic field at some specific load, and it will behave just like a three-phase induction motor at that point. Such a design is called a *permanent split-capacitor* or *capacitor-start-and-run* motor (Figure 10–20). Permanent split-capacitor motors are simpler than capacitor-start motors, since the starting switch is not needed. At normal loads, they are more efficient and have a higher power factor and a smoother torque than ordinary single-phase induction motors.

However, permanent split-capacitor motors have a *lower starting torque* than capacitor-start motors, since the capacitor must be sized to balance the currents in the main and auxiliary windings at normal-load conditions. Since the starting current is much greater than the normal-load current, a capacitor that balances the phases under normal loads leaves them very unbalanced under starting conditions.

If both the largest possible starting torque and the best running conditions are needed, two capacitors can be used with the auxiliary winding. Motors with two capacitors are called *capacitor-start, capacitor-run*, or *two-value capacitor* motors (Figure 10–21). The larger capacitor is present in the circuit only during starting, when it ensures that the currents in the main and auxiliary windings are roughly balanced, yielding very high starting torques. When the motor gets up to speed, the centrifugal switch opens, and the permanent capacitor is left by itself in the auxiliary winding circuit. The permanent capacitor is just large enough to balance the currents at normal motor loads, so the motor again operates efficiently with a high torque and power factor. The permanent capacitor in such a motor is typically about 10 to 20 percent of the size of the starting capacitor.



(a)

(b)

**FIGURE 10–19**
(a) A capacitor-start induction motor. (*Courtesy of Emerson Electric Company.*) (b) Exploded view of a capacitor-start induction motor. (*Courtesy of Westinghouse Electric Corporation.*)

The direction of rotation of any capacitor-type motor may be reversed by switching the connections of its auxiliary windings.

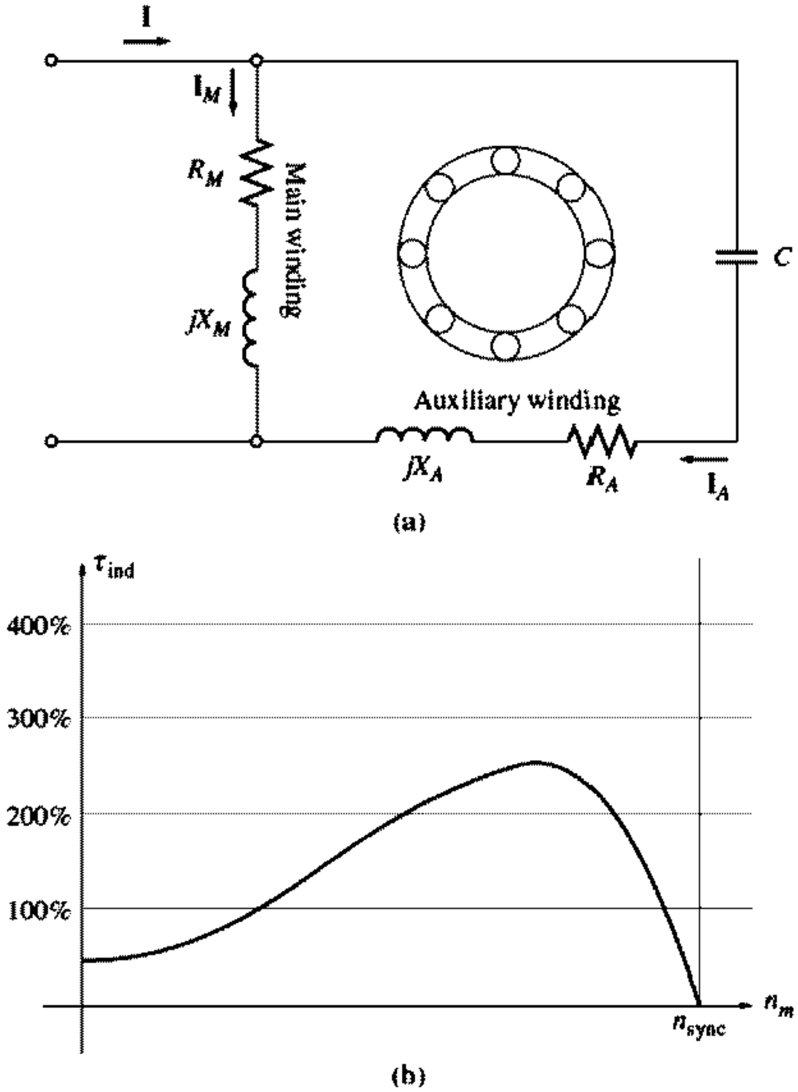


**FIGURE 10–20**
(a) A permanent split-capacitor induction motor. (b) Torque–speed characteristic of this motor.

## Shaded-Pole Motors

A shaded-pole induction motor is an induction motor with only a main winding. Instead of having an auxiliary winding, it has salient poles, and one portion of each pole is surrounded by a short-circuited coil called a *shading coil* (see Figure 10–22a). A time-varying flux is induced in the poles by the main winding. When the pole flux varies, it induces a voltage and a current in the shading coil which *opposes* the original change in flux. This opposition *retards* the flux changes under the shaded portions of the coils and therefore produces a slight imbalance between the two oppositely rotating stator magnetic fields. The net rotation is in the direction from the unshaded to the shaded portion of the pole face. The torque–speed characteristic of a shaded-pole motor is shown in Figure 10–22b.

Shaded poles produce less starting torque than any other type of induction motor starting system. They are much less efficient and have a much higher slip

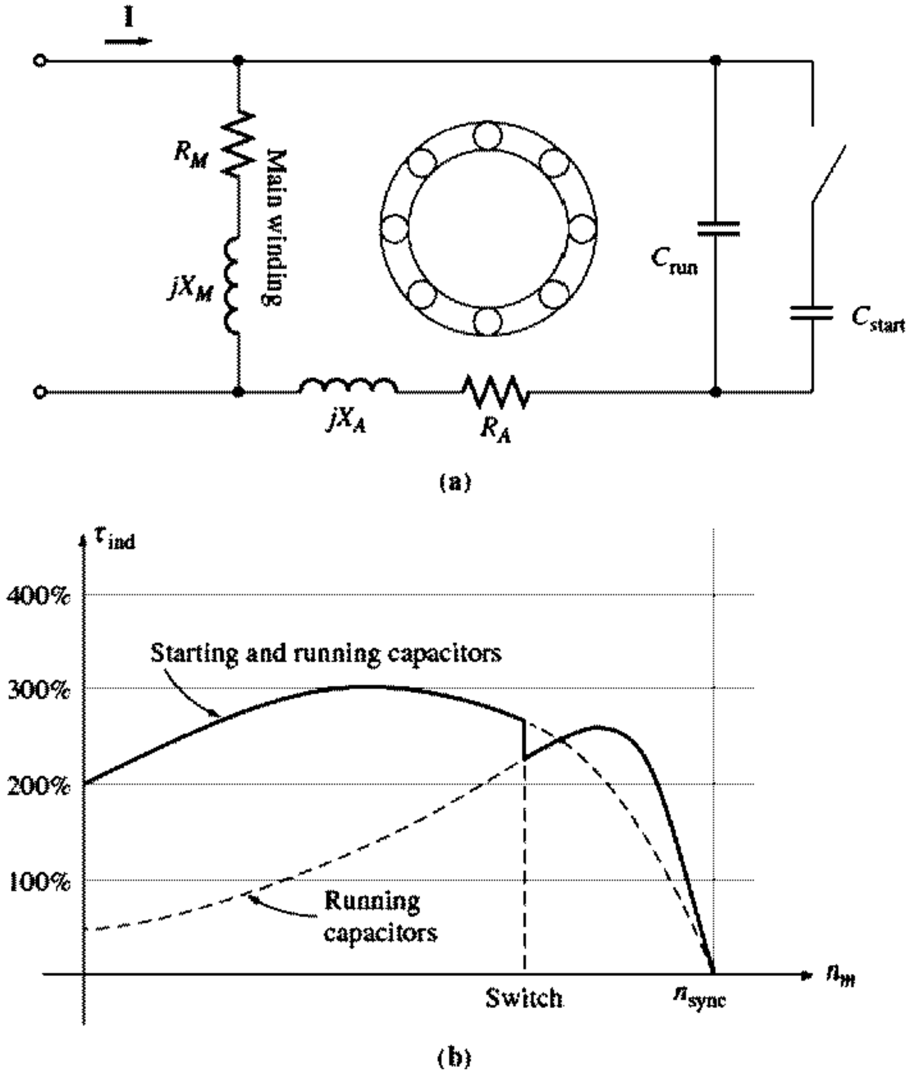


**FIGURE 10–21**
(a) A capacitor-start, capacitor-run induction motor. (b) Torque–speed characteristic of this motor.

than other types of single-phase induction motors. Such poles are used only in very small motors ($\frac{1}{20}$ hp and less) with very low starting torque requirements. Where it is possible to use them, shaded-pole motors are the cheapest design available.

Because shaded-pole motors rely on a shading coil for their starting torque, there is no easy way to reverse the direction of rotation of such a motor. To achieve reversal, it is necessary to install two shading coils on each pole face and to selectively short one or the other of them. See Figures 10–23 and 10–24.

## Comparison of Single-Phase Induction Motors

Single-phase induction motors may be ranked from best to worst in terms of their starting and running characteristics:

1. Capacitor-start, capacitor-run motor
2. Capacitor-start motor

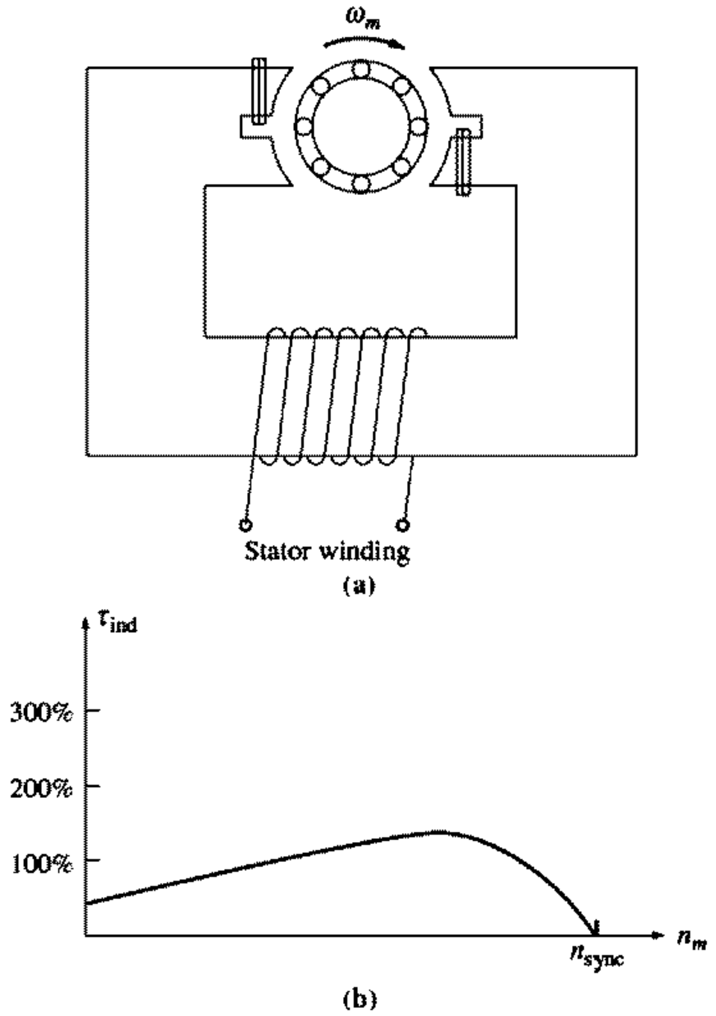


**FIGURE 10–22**
(a) A basic shaded-pole induction motor. (b) The resulting torque–speed characteristic.

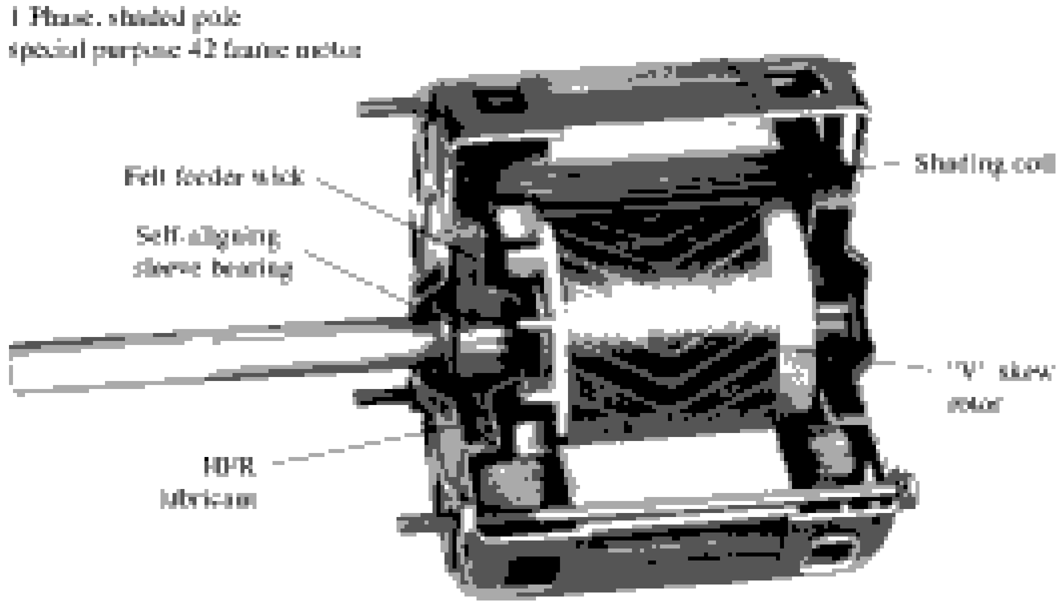


**FIGURE 10–23**
Cutaway view of a shaded-pole induction motor. (*Courtesy of Westinghouse Electric Corporation.*)

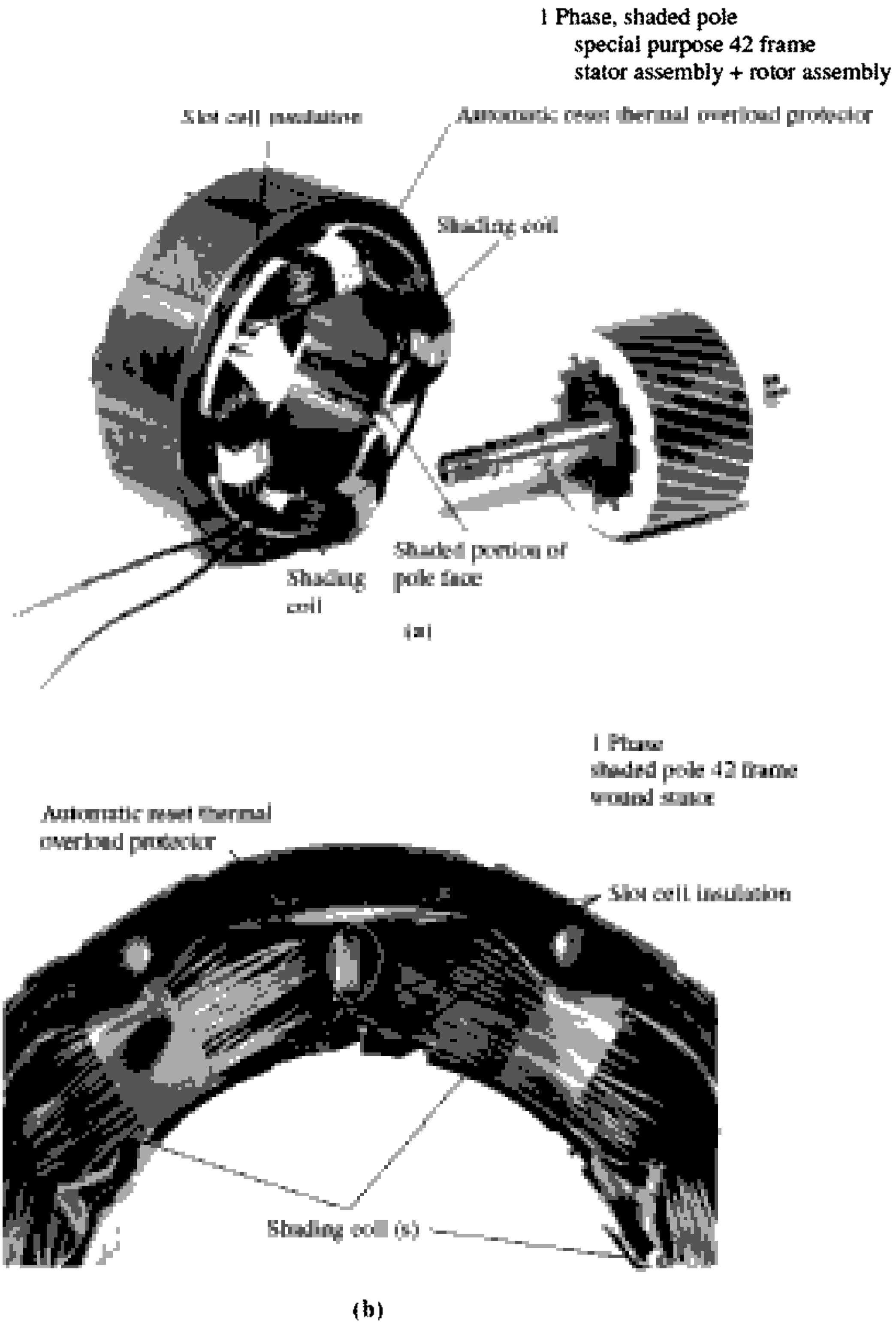


**FIGURE 10–24**
Close-up views of the construction of a four-pole shaded-pole induction motor. (*Courtesy of Westinghouse Electric Corporation.*)

3. Permanent split-capacitor motor
4. Split-phase motor
5. Shaded-pole motor

Naturally, the best motor is also the most expensive, and the worst motor is the least expensive. Also, not all these starting techniques are available in all motor size ranges. It is up to the design engineer to select the cheapest available motor for any given application that will do the job.

## 10.4 SPEED CONTROL OF SINGLE-PHASE INDUCTION MOTORS

In general, the speed of single-phase induction motors may be controlled in the same manner as the speed of polyphase induction motors. For squirrel-cage rotor motors, the following techniques are available:

1. Vary the stator frequency.
2. Change the number of poles.
3. Change the applied terminal voltage $V_T$.

In practical designs involving fairly high-slip motors, the usual approach to speed control is to vary the terminal voltage of the motor. The voltage applied to a motor may be varied in one of three ways:

1. An autotransformer may be used to continually adjust the line voltage. This is the most expensive method of voltage speed control and is used only when very smooth speed control is needed.
2. An SCR or TRIAC circuit may be used to reduce the rms voltage applied to the motor by ac phase control. This approach chops up the ac waveform as described in Chapter 3 and somewhat increases the motor's noise and vibration. Solid-state control circuits are considerably cheaper than autotransformers and so are becoming more and more common.
3. A resistor may be inserted in series with the motor's stator circuit. This is the cheapest method of voltage control, but it has the disadvantage that considerable power is lost in the resistor, reducing the overall power conversion efficiency.

Another technique is also used with very high-slip motors such as shaded-pole motors. Instead of using a separate autotransformer to vary the voltage applied to the stator of the motor, *the stator winding itself* can be used as an autotransformer. Figure 10–25 shows a schematic representation of a main stator winding, with a number of taps along its length. Since the stator winding is wrapped about an iron core, it behaves as an autotransformer.

When the full line voltage $V$ is applied across the entire main winding, then the induction motor operates normally. Suppose instead that the full line voltage is applied to tap 2, the center tap of the winding. Then an identical voltage will be induced in the upper half of the winding by transformer action, and the total winding

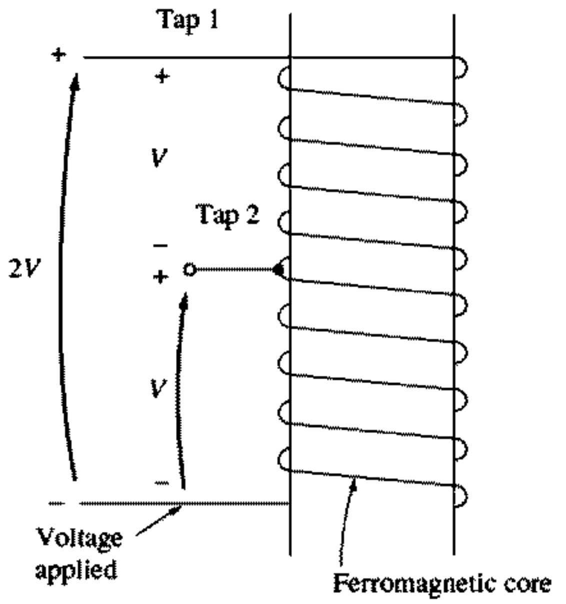


**FIGURE 10–25**
The use of a stator winding as an autotransformer. If voltage $V$ is applied to the winding at the center tap, the total winding voltage will be $2V$.

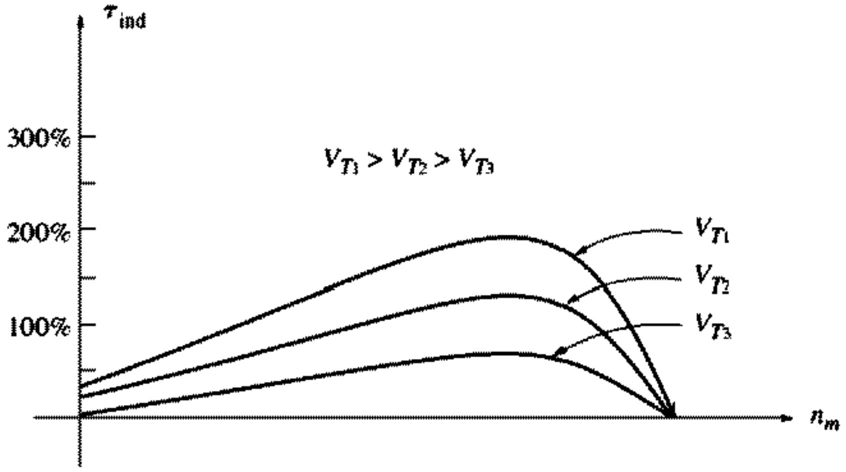


**FIGURE 10–26**
The torque–speed characteristic of a shaded-pole induction motor as the terminal voltage is changed. Increases in $V_T$ may be accomplished either by actually raising the voltage across the whole winding or by switching to a lower tap on the stator winding.

voltage will be twice the applied line voltage. The total voltage applied to the winding has effectively been doubled.

Therefore, the smaller the fraction of the total coil that the line voltage is applied across, the greater the total voltage will be across the whole winding, and the higher the speed of the motor will be for a given load (see Figure 10–26).

This is the standard approach used to control the speed of single-phase motors in many fan and blower applications. Such speed control has the advantage that it is quite inexpensive, since the only components necessary are taps on the main motor winding and an ordinary multiposition switch. It also has the advantage that the autotransformer effect does not consume power the way series resistors would.

## 10.5 THE CIRCUIT MODEL OF A SINGLE-PHASE INDUCTION MOTOR

As previously described, an understanding of the induced torque in a single-phase induction motor can be achieved through either the double-revolving-field theory or the cross-field theory of single-phase motors. Either approach can lead to an equivalent circuit of the motor, and the torque–speed characteristic can be derived through either method.

This section is restricted to an examination of an equivalent circuit based on the double-revolving-field theory—in fact, to only a special case of that theory. We will develop an equivalent circuit of the *main winding* of a single-phase induction motor when it is operating alone. The technique of symmetrical components is necessary to analyze a single-phase motor with both main and auxiliary windings present, and since symmetrical components are beyond the scope of this book, that case will not be discussed. For a more detailed analysis of single-phase motors, see Reference 4.

The best way to begin the analysis of a single-phase induction motor is to consider the motor when it is stalled. At that time, the motor appears to be just a single-phase transformer with its secondary circuit shorted out, and so its equivalent circuit is that of a transformer. This equivalent circuit is shown in Figure 10–27a. In this figure, $R_1$ and $X_1$ are the resistance and reactance of the stator winding, $X_M$ is the magnetizing reactance, and $R_2$ and $X_2$ are the referred values of the rotor's resistance and reactance. The core losses of the machine are not shown and will be lumped together with the mechanical and stray losses as a part of the motor's rotational losses.

Now recall that the pulsating air-gap flux in the motor at stall conditions can be resolved into two equal and opposite magnetic fields within the motor. Since these fields are of equal size, each one contributes an equal share to the resistive and reactive voltage drops in the rotor circuit. It is possible to split the rotor equivalent circuit into two sections, each one corresponding to the effects of one of the magnetic fields. The motor equivalent circuit with the effects of the forward and reverse magnetic fields separated is shown in Figure 10–27b.

Now suppose that the motor's rotor begins to turn with the help of an auxiliary winding and that the winding is switched out again after the motor comes up to speed. As derived in Chapter 7, the effective rotor resistance of an induction motor depends on the amount of relative motion between the rotor and the stator magnetic fields. However, there are two magnetic fields in this motor, and the amount of relative motion differs for each of them.

For the *forward* magnetic field, the per-unit difference between the rotor speed and the speed of the magnetic field is the slip $s$, where slip is defined in the same manner as it was for three-phase induction motors. The rotor resistance in the part of the circuit associated with the forward magnetic field is thus $0.5R_2/s$.

The forward magnetic field rotates at speed $n_{\text{sync}}$ and the reverse magnetic field rotates at speed $-n_{\text{sync}}$. Therefore, the total per-unit difference in speed (on a base of $n_{\text{sync}}$) between the forward and reverse magnetic fields is 2. Since the rotor

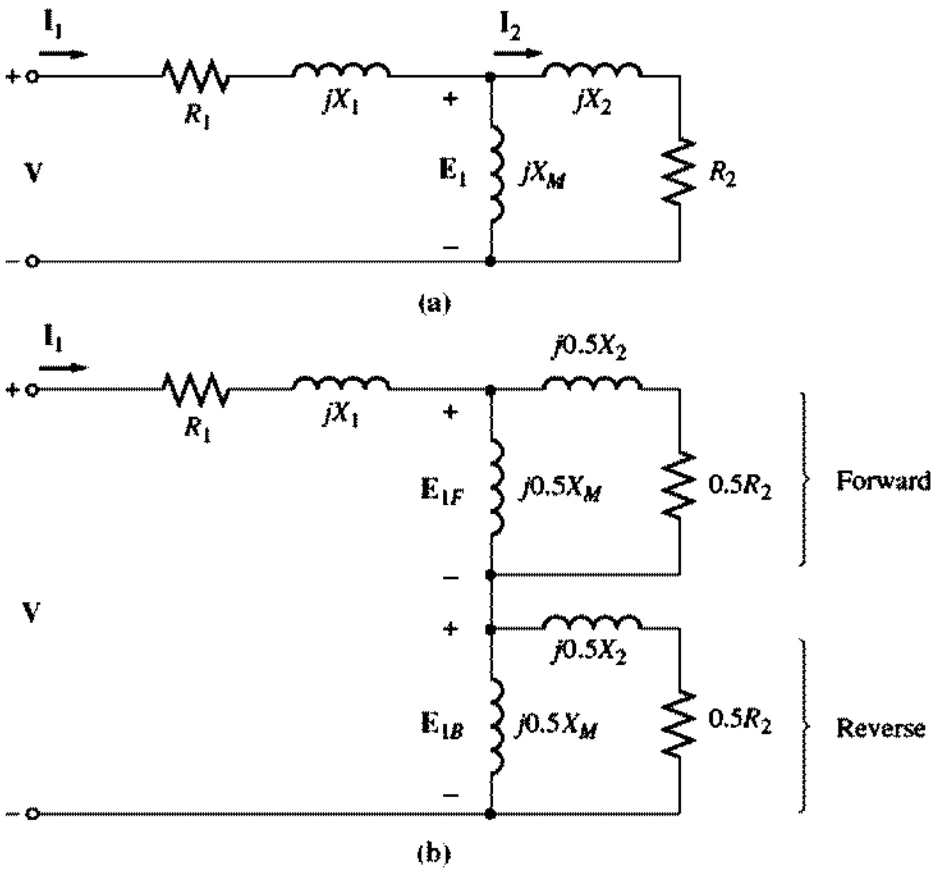


**FIGURE 10–27**
(a) The equivalent circuit of a single-phase induction motor at standstill. Only its main windings are energized. (b) The equivalent circuit with the effects of the forward and reverse magnetic fields separated.

is turning at a speed $s$ slower than the forward magnetic field, the total per-unit difference in speed between the rotor and the reverse magnetic field is $2 - s$. Therefore, the effective rotor resistance in the part of the circuit associated with the reverse magnetic field is $0.5R_2/(2 - s)$.

The final induction motor equivalent circuit is shown in Figure 10–28.

## Circuit Analysis with the Single-Phase Induction Motor Equivalent Circuit

The single-phase induction motor equivalent circuit in Figure 10–28 is similar to the three-phase equivalent circuit, except that there are both forward and backward components of power and torque present. The same general power and torque relationships that applied for three-phase motors also apply for either the forward or the backward components of the single-phase motor, and the net power and torque in the machine is the *difference* between the forward and reverse components.

The power-flow diagram of an induction motor is repeated in Figure 10–29 for easy reference.

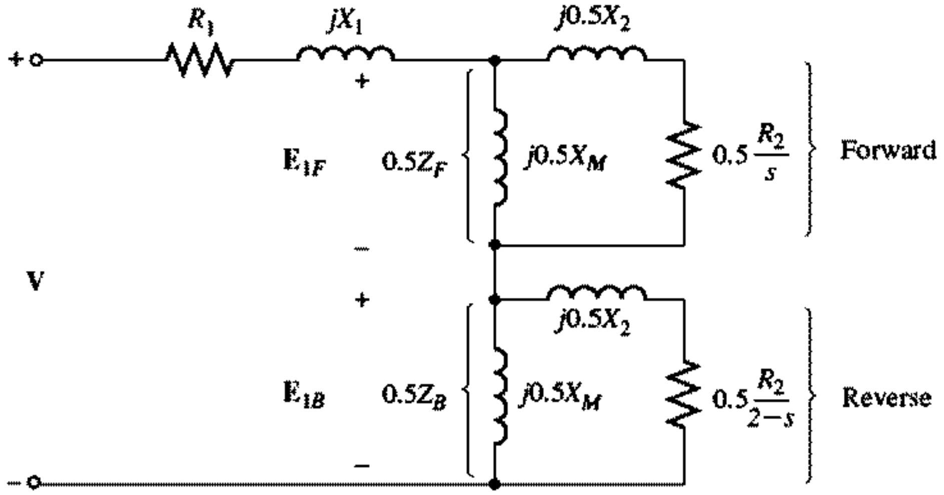


**FIGURE 10–28**
The equivalent circuit of a single-phase induction motor running at speed with only its main windings energized.

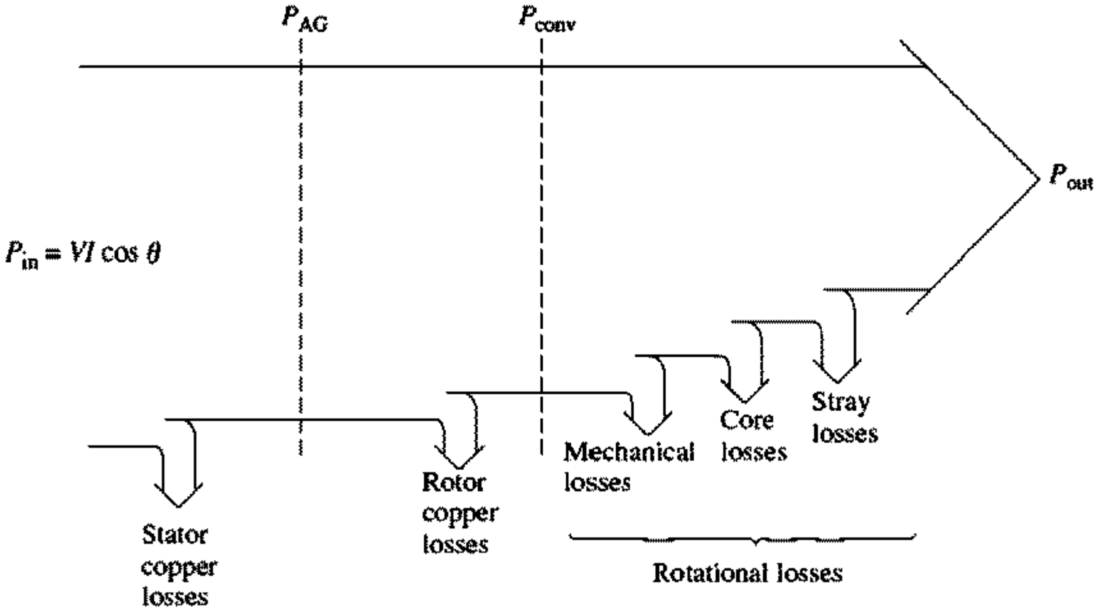


**FIGURE 10–29**
The power-flow diagram of a single-phase induction motor.

To make the calculation of the input current flow into the motor simpler, it is customary to define impedances $Z_F$ and $Z_B$, where $Z_F$ is a single impedance equivalent to all the forward magnetic field impedance elements and $Z_B$ is a single impedance equivalent to all the backward magnetic field impedance elements (see Figure 10–30). These impedances are given by

$$Z_F = R_F + jX_F = \frac{(R_2/s + jX_2)(jX_M)}{(R_2/s + jX_2) + jX_M} \tag{10–5}$$

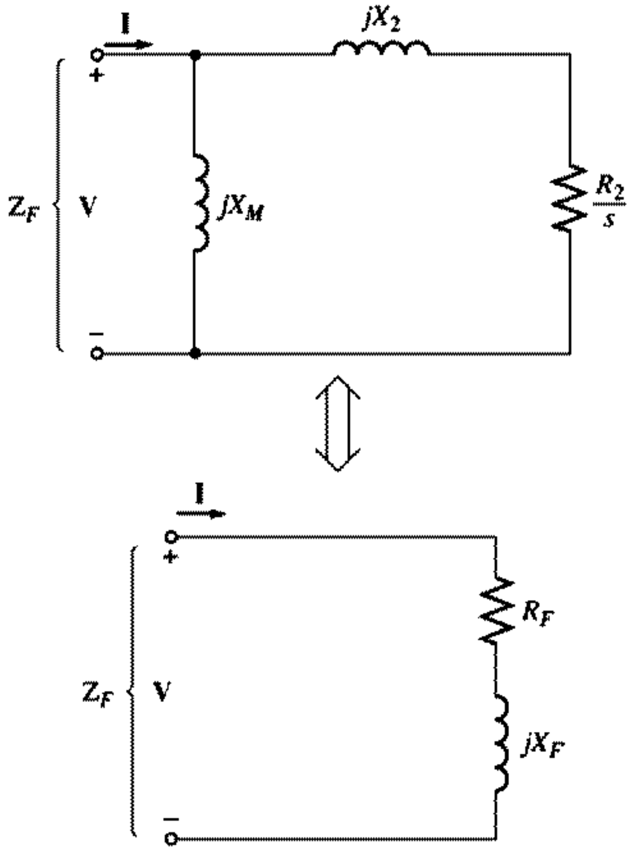


**FIGURE 10–30**
A series combination of $R_F$ and $jX_F$ is the Thevenin equivalent of the forward-field impedance elements, and therefore $R_F$ must consume the same power from a given current as $R_2/s$ would.

$$Z_B = R_B + jX_B = \frac{[R_2/(2 - s) + jX_2](jX_M)}{[R_2/(2 - s) + jX_2] + jX_M} \tag{10–6}$$

In terms of $Z_F$ and $Z_B$, the current flowing in the induction motor's stator winding is

$$\mathbf{I}_1 = \frac{\mathbf{V}}{R_1 + jX_1 + 0.5\,Z_F + 0.5\,Z_B} \tag{10–7}$$

The per-phase air-gap power of a three-phase induction motor is the power consumed in the rotor circuit resistance $0.5R_2/s$. Similarly, the forward air-gap power of a single-phase induction motor is the power consumed by $0.5R_2/s$, and the reverse air-gap power of the motor is the power consumed by $0.5R_2/(2 - s)$. Therefore, the air-gap power of the motor could be calculated by determining the power in the forward resistor $0.5R_2/s$, determining the power in the reverse resistor $0.5R_2/(2 - s)$, and subtracting one from the other.

The most difficult part of this calculation is the determination of the separate currents flowing in the two resistors. Fortunately, a simplification of this calculation is possible. Notice that the *only* resistor within the circuit elements composing the equivalent impedance $Z_F$ is the resistor $R_2/s$. Since $Z_F$ is equivalent to that circuit, any power consumed by $Z_F$ must also be consumed by the original circuit, and since $R_2/s$ is the only resistor in the original circuit, its power consumption must equal that of impedance $Z_F$. Therefore, the air-gap power for the forward magnetic field can be expressed as

$$P_{\mathrm{AG},F} = I_1^2(0.5\,R_F) \tag{10–8}$$

Similarly, the air-gap power for the reverse magnetic field can be expressed as

$$P_{\mathrm{AG},B} = I_1^2(0.5\,R_B) \tag{10–9}$$

The advantage of these two equations is that only the one current $I_1$ needs to be calculated to determine both powers.

The total air-gap power in a single-phase induction motor is thus

$$P_{\mathrm{AG}} = P_{\mathrm{AG},F} - P_{\mathrm{AG},B} \tag{10–10}$$

The induced torque in a three-phase induction motor can be found from the equation

$$\tau_{\mathrm{ind}} = \frac{P_{\mathrm{AG}}}{\omega_{\mathrm{sync}}} \tag{10–11}$$

where $P_{\mathrm{AG}}$ is the net air-gap power given by Equation (10–10).

The rotor copper losses can be found as the sum of the rotor copper losses due to the forward field and the rotor copper losses due to the reverse field.

$$P_{\mathrm{RCL}} = P_{\mathrm{RCL},F} + P_{\mathrm{RCL},B} \tag{10–12}$$

The rotor copper losses in a three-phase induction motor were equal to the per-unit relative motion between the rotor and the stator field (the slip) times the air-gap power of the machine. Similarly, the forward rotor copper losses of a single-phase induction motor are given by

$$P_{\mathrm{RCL},F} = sP_{\mathrm{AG},F} \tag{10–13}$$

and the reverse rotor copper losses of the motor are given by

$$P_{\mathrm{RCL},B} = sP_{\mathrm{AG},B} \tag{10–14}$$

Since these two power losses in the rotor are at different frequencies, the total rotor power loss is just their sum.

The power converted from electrical to mechanical form in a single-phase induction motor is given by the same equation as $P_{\mathrm{conv}}$ for three-phase induction motors. This equation is

$$P_{\mathrm{conv}} = \tau_{\mathrm{ind}}\omega_m \tag{10–15}$$

Since $\omega_m = (1 - s)\omega_{\mathrm{sync}}$, this equation can be reexpressed as

$$P_{\mathrm{conv}} = \tau_{\mathrm{ind}}(1 - s)\omega_m \tag{10–16}$$

From Equation (10–11), $P_{\mathrm{AG}} = \tau_{\mathrm{ind}}\omega_{\mathrm{sync}}$, so $P_{\mathrm{conv}}$ can also be expressed as

$$P_{\mathrm{conv}} = (1 - s)\,P_{\mathrm{AG}} \tag{10–17}$$

As in the three-phase induction motor, the shaft output power is not equal to $P_{\mathrm{conv}}$, since the rotational losses must still be subtracted. In the single-phase induction motor model used here, the core losses, mechanical losses, and stray losses must be subtracted from $P_{\mathrm{conv}}$ in order to get $P_{\mathrm{out}}$.

**Example 10–1.** A $\frac{1}{3}$-hp, 110-V, 60-Hz, six-pole, split-phase induction motor has the following impedances:

$$R_1 = 1.52\ \Omega \qquad X_1 = 2.10\ \Omega \qquad X_M = 58.2\ \Omega$$

$$R_2 = 3.13\ \Omega \qquad X_2 = 1.56\ \Omega$$

The core losses of this motor are 35 W, and the friction, windage, and stray losses are 16 W. The motor is operating at the rated voltage and frequency with its starting winding open, and the motor's slip is 5 percent. Find the following quantities in the motor at these conditions:

- *(a)* Speed in revolutions per minute
- *(b)* Stator current in amperes
- *(c)* Stator power factor
- *(d)* $P_{\text{in}}$
- *(e)* $P_{\text{AG}}$
- *(f)* $P_{\text{conv}}$
- *(g)* $\tau_{\text{ind}}$
- *(h)* $P_{\text{out}}$
- *(i)* $\tau_{\text{load}}$
- *(j)* Efficiency

***Solution***

The forward and reverse impedances of this motor at a slip of 5 percent are

$$Z_F = R_F + jX_F = \frac{(R_2/s + jX_2)(jX_M)}{(R_2/s + jX_2) + jX_M} \tag{10–5}$$

$$= \frac{(3.13\ \Omega/0.05 + j1.56\ \Omega)(j58.2\ \Omega)}{(3.13\ \Omega/0.05 + j1.56\ \Omega) + j58.2\ \Omega}$$

$$= \frac{(62.6\angle 1.43^\circ\ \Omega)(j58.2\ \Omega)}{(62.6\ \Omega + j1.56\ \Omega) + j58.2\ \Omega}$$

$$= 39.9\angle 50.5^\circ\ \Omega = 25.4 + j30.7\ \Omega$$

$$Z_B = R_B + jX_B = \frac{[R_2/(2 - s) + jX_2](jX_M)}{[R_2/(2 - s) + jX_2] + jX_M} \tag{10–6}$$

$$= \frac{(3.13\ \Omega/1.95 + j1.56\ \Omega)(j58.2\ \Omega)}{(3.13\ \Omega/1.95 + j1.56\ \Omega) + j58.2\ \Omega}$$

$$= \frac{(2.24\angle 44.2^\circ\ \Omega)(j58.2\ \Omega)}{(1.61\ \Omega + j1.56\ \Omega) + j58.2\ \Omega}$$

$$= 2.18\angle 45.9^\circ\ \Omega = 1.51 + j1.56\ \Omega$$

These values will be used to determine the motor current, power, and torque.

*(a)* The synchronous speed of this motor is

$$n_{\text{sync}} = \frac{120f_e}{P} = \frac{120(60\text{ Hz})}{6\text{ pole}} = 1200\text{ r/min}$$

Since the motor is operating at 5 percent slip, its mechanical speed is

$$n_m = (1 - s)n_{\text{sync}}$$

$$n_m = (1 - 0.05)(1200\text{ r/min}) = 1140\text{ r/min}$$

*(b)* The stator current in this motor is

$$\mathbf{I}_1 = \frac{\mathbf{V}}{R_1 + jX_1 + 0.5Z_F + 0.5Z_B} \tag{10–7}$$

$$= \frac{110\angle 0^\circ\text{ V}}{1.52\ \Omega + j2.10\ \Omega + 0.5(25.4\ \Omega + j30.7\ \Omega) + 0.5(1.51\ \Omega + j1.56\ \Omega)}$$

$$= \frac{110\angle 0^\circ\text{ V}}{14.98\ \Omega + j18.23\ \Omega} = \frac{110\angle 0^\circ\text{ V}}{23.6\angle 50.6^\circ\ \Omega} = 4.66\angle -50.6^\circ\text{ A}$$

*(c)* The stator power factor of this motor is

$$\text{PF} = \cos(-50.6^\circ) = 0.635\text{ lagging}$$

*(d)* The input power to this motor is

$$P_{\text{in}} = VI\cos\theta$$
$$= (110\text{ V})(4.66\text{ A})(0.635) = 325\text{ W}$$

*(e)* The forward-wave air-gap power is

$$P_{\text{AG},F} = I_1^2(0.5\,R_F) \tag{10–8}$$
$$= (4.66\text{ A})^2(12.7\ \Omega) = 275.8\text{ W}$$

and the reverse-wave air-gap power is

$$P_{\text{AG},B} = I_1^2(0.5\,R_B) \tag{10–9}$$
$$= (4.66\text{ A})2(0.755\text{ V}) = 16.4\text{ W}$$

Therefore, the total air-gap power of this motor is

$$P_{\text{AG}} = P_{\text{AG},F} - P_{\text{AG},B} \tag{10–10}$$
$$= 275.8\text{ W} - 16.4\text{ W} = 259.4\text{ W}$$

*(f)* The power converted from electrical to mechanical form is

$$P_{\text{conv}} = (1 - s)\,P_{\text{AG}} \tag{10–17}$$
$$= (1 - 0.05)(259.4\text{ W}) = 246\text{ W}$$

*(g)* The induced torque in the motor is given by

$$\tau_{\text{ind}} = \frac{P_{\text{AG}}}{\omega_{\text{sync}}} \tag{10–11}$$

$$= \frac{259.4\text{ W}}{(1200\text{ r/min})(1\text{ min/60 s})(2\pi\text{ rad/r})} = 2.06\text{ N}\cdot\text{m}$$

*(h)* The output power is given by

$$P_{\text{out}} = P_{\text{conv}} - P_{\text{rot}} = P_{\text{conv}} - P_{\text{core}} - P_{\text{mech}} - P_{\text{stray}}$$
$$= 246\text{ W} - 35\text{ W} - 16\text{ W} = 195\text{ W}$$

*(i)* The load torque of the motor is given by

$$\tau_{\text{load}} = \frac{P_{\text{out}}}{\omega_m}$$

$$= \frac{195\text{ W}}{(1140\text{ r/min})(1\text{ min/60 s})(2\pi\text{ rad/r})} = 1.63\text{ N} \cdot \text{m}$$

(*j*) Finally, the efficiency of the motor at these conditions is

$$\eta = \frac{P_{\text{out}}}{P_{\text{in}}} \times 100\% = \frac{195\text{ W}}{325\text{ W}} \times 100\% = 60\%$$

## 10.6 OTHER TYPES OF MOTORS

Two other types of motors—reluctance motors and hysteresis motors—are used in certain special-purpose applications. These motors differ in rotor construction from the ones previously described, but use the same stator design. Like induction motors, they can be built with either single- or three-phase stators. A third type of special-purpose motor is the stepper motor. A stepper motor requires a polyphase stator, but it does not require a three-phase power supply. The final special-purpose motor discussed is the brushless dc motor, which as the name suggests runs on a dc power supply.

### Reluctance Motors

A *reluctance motor* is a motor which depends on reluctance torque for its operation. Reluctance torque is the torque induced in an iron object (such as a pin) in the presence of an external magnetic field, which causes the object to line up with the external magnetic field. This torque occurs because the external field induces an internal magnetic field in the iron of the object, and a torque appears between the two fields, twisting the object around to line up with the external field. In order for a reluctance torque to be produced in an object, it must be elongated along axes at angles corresponding to the angles between adjacent poles of the external magnetic field.

A simple schematic of a two-pole reluctance motor is shown in Figure 10–31. It can be shown that the torque applied to the rotor of this motor is proportional to $\sin 2\delta$, where $\delta$ is the electrical angle between the rotor and the stator magnetic fields. Therefore, the reluctance torque of a motor is maximum when the angle between the rotor and the stator magnetic fields is 45°.

A simple reluctance motor of the sort shown in Figure 10–31 is a *synchronous motor*, since the rotor will be locked into the stator magnetic fields as long as the pullout torque of the motor is not exceeded. Like a normal synchronous motor, it has no starting torque and will not start by itself.

A *self-starting reluctance motor* that will operate at synchronous speed until its maximum reluctance torque is exceeded can be built by modifying the rotor of an induction motor as shown in Figure 10–32. In this figure, the rotor has salient poles for steady-state operation as a reluctance motor and also has cage or amortisseur windings for starting. The stator of such a motor may be either of single- or three-phase construction. The torque–speed characteristic of this motor, which is sometimes called a *synchronous induction motor*, is shown in Figure 10–33.

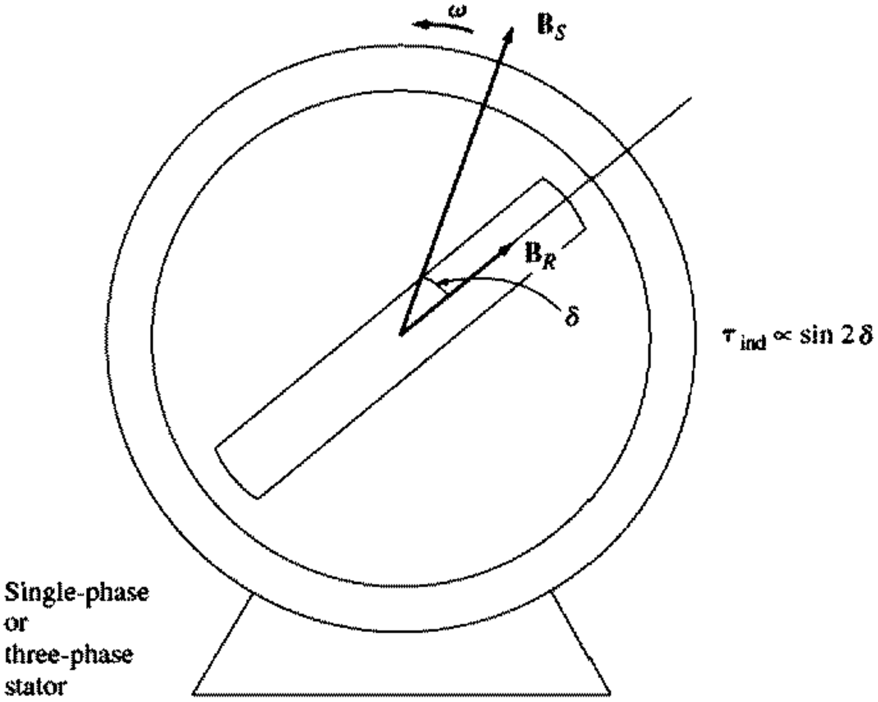


**FIGURE 10–31**
The basic concept of a reluctance motor.

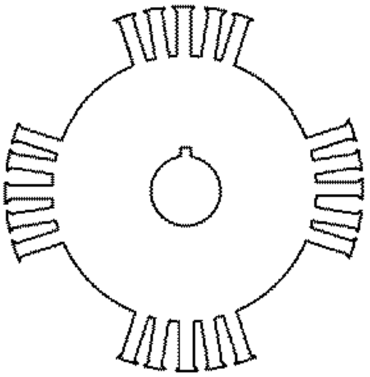


**FIGURE 10–32**
The rotor design of a "synchronous induction" or self-starting reluctance motor.

An interesting variation on the idea of the reluctance motor is the Syncrospeed motor, which is manufactured in the United States by MagneTek, Inc. The rotor of this motor is shown in Figure 10–34. It uses "flux guides" to increase the coupling between adjacent pole faces and therefore to increase the maximum-reluctance torque of the motor. With these flux guides, the maximum-reluctance torque is increased to about 150 percent of the rated torque, as compared to just over 100 percent of the rated torque for a conventional reluctance motor.

## Hysteresis Motors

Another special-purpose motor employs the phenomenon of hysteresis to produce a mechanical torque. The rotor of a hysteresis motor is a smooth cylinder of magnetic material with no teeth, protrusions, or windings. The stator of the motor can be either single- or three-phase; but if it is single-phase, a permanent capacitor

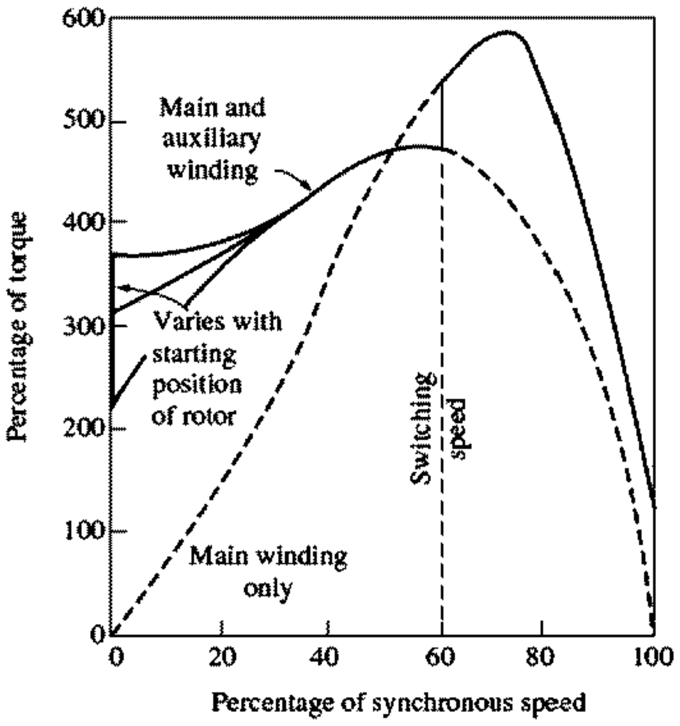
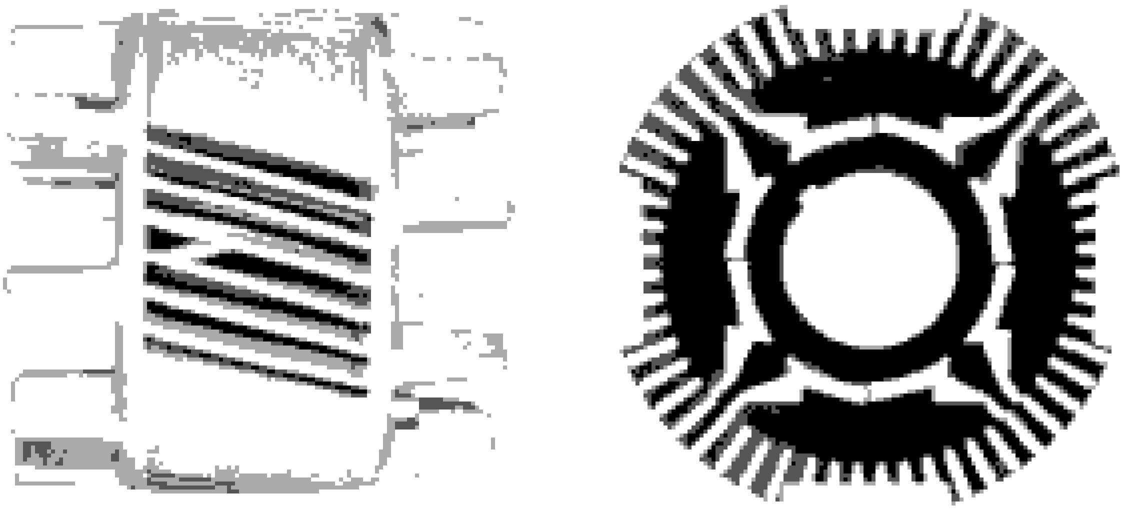


**FIGURE 10–33**
The torque–speed characteristic of a single-phase self-starting reluctance motor.



(a)

(b)

**FIGURE 10–34**
(a) The aluminum casting of a Synchrospeed motor rotor. (b) A rotor lamination from the motor. Notice the flux guides connecting the adjacent poles. These guides increase the reluctance torque of the motor. (*Courtesy of MagneTek, Inc.*)

should be used with an auxiliary winding to provide as smooth a magnetic field as possible, since this greatly reduces the losses of the motor.

Figure 10–35 shows the basic operation of a hysteresis motor. When a three-phase (or single-phase with auxiliary winding) current is applied to the stator of the motor, a rotating magnetic field appears within the machine. This rotating magnetic field magnetizes the metal of the rotor and induces poles within it.

When the motor is operating below synchronous speed, there are two sources of torque within it. Most of the torque is produced by hysteresis. When the

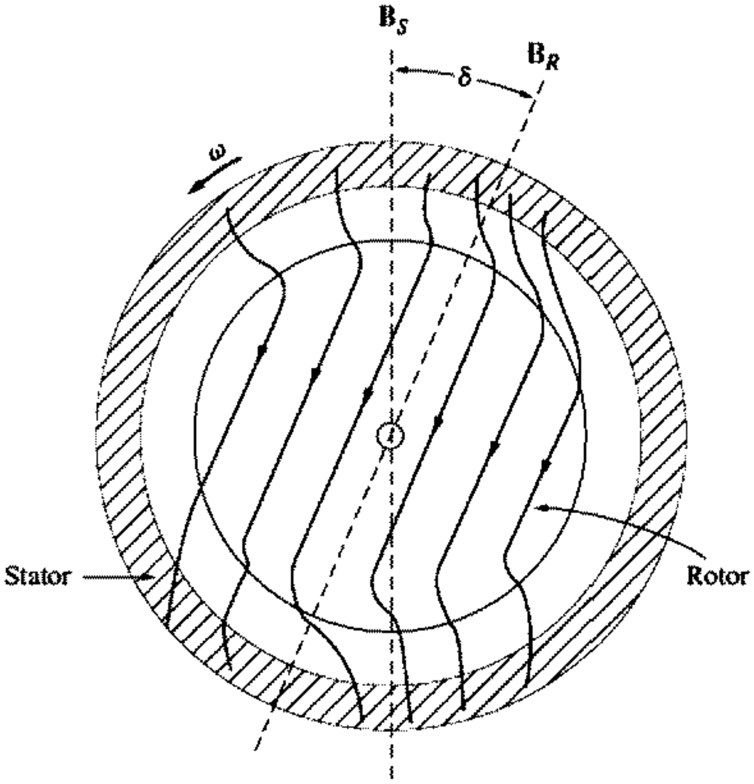


**FIGURE 10–35**
The construction of a hysteresis motor. The main component of torque in this motor is proportional to the angle between the rotor and stator magnetic fields.

magnetic field of the stator sweeps around the surface of the rotor, the rotor flux cannot follow it exactly, because the metal of the rotor has a large hysteresis loss. The greater the intrinsic hysteresis loss of the rotor material, the greater the angle by which the rotor magnetic field lags the stator magnetic field. Since the rotor and stator magnetic fields are at different angles, a finite torque will be produced in the motor. In addition, the stator magnetic field will produce eddy currents in the rotor, and these eddy currents produce a magnetic field of their own, further increasing the torque on the rotor. The greater the relative motion between the rotor and the stator magnetic field, the greater the eddy currents and eddy-current torques.

When the motor reaches synchronous speed, the stator flux ceases to sweep across the rotor, and the rotor acts like a permanent magnet. The induced torque in the motor is then proportional to the angle between the rotor and the stator magnetic field, up to a maximum angle set by the hysteresis in the rotor.

The torque–speed characteristic of a hysteresis motor is shown in Figure 10–36. Since the amount of hysteresis within a particular rotor is a function of only the stator flux density and the material from which it is made, the hysteresis torque of the motor is approximately constant for any speed from zero to $n_{\text{sync}}$. The eddy-current torque is roughly proportional to the slip of the motor. These two facts taken together account for the shape of the hysteresis motor's torque–speed characteristic.

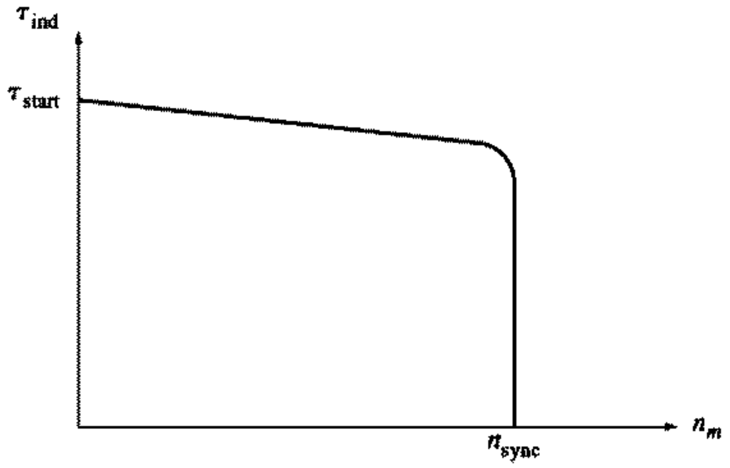


**FIGURE 10–36**
The torque–speed characteristic of a motor.

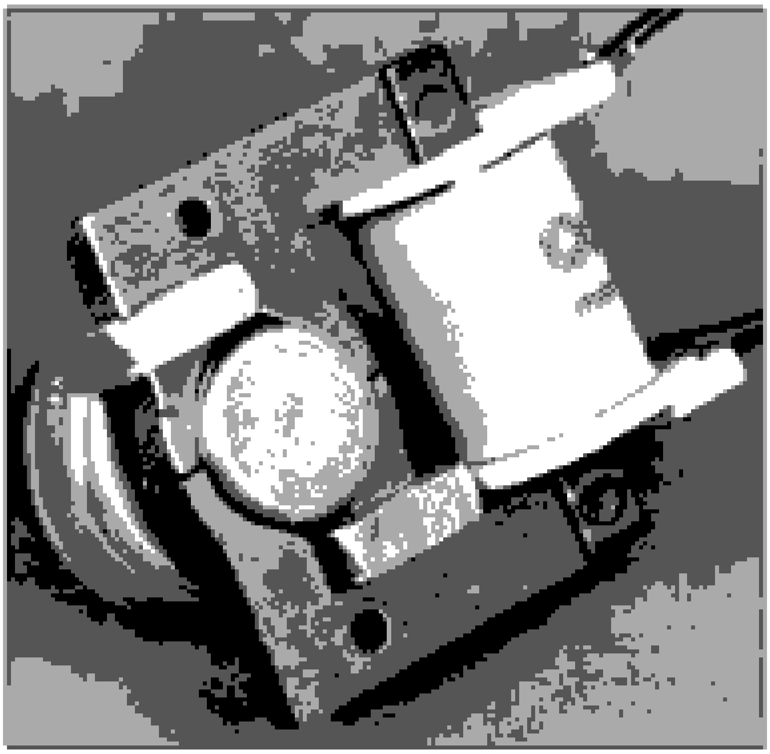


**FIGURE 10–37**
A small hysteresis motor with a shaded-pole stator, suitable for running an electric clock. Note the shaded stator poles. (*Stephen J. Chapman*)

Since the torque of a hysteresis motor at any subsynchronous speed is greater than its maximum synchronous torque, a hysteresis motor can accelerate any load that it can carry during normal operation.

A very small hysteresis motor can be built with shaded-pole stator construction to create a tiny self-starting low-power synchronous motor. Such a motor is shown in Figure 10–37. It is commonly used as the driving mechanism in electric clocks. An electric clock is therefore synchronized to the line frequency of the power system, and the resulting clock is just as accurate (or as inaccurate) as the frequency of the power system to which it is tied.

## Stepper Motors

A *stepper motor* is a special type of synchronous motor which is designed to rotate a specific number of degrees for every electric pulse received by its control unit. Typical steps are 7.5 or 15° per pulse. These motors are used in many control systems, since the position of a shaft or other piece of machinery can be controlled precisely with them.

A simple stepper motor and its associated control unit are shown in Figure 10–38. To understand the operation of the stepper motor, examine Figure 10–39. This figure shows a two-pole three-phase stator with a permanent-magnet rotor. If a dc voltage is applied to phase $a$ of the stator and no voltage is applied to phases $b$ and $c$, then a torque will be induced in the rotor which causes it to line up with the stator magnetic field $\mathbf{B}_S$, as shown in Figure 10–39b.

Now assume that phase $a$ is turned off and that a negative dc voltage is applied to phase $c$. The new stator magnetic field is rotated 60° with respect to the previous magnetic field, and the rotor of the motor follows it around. By continuing this pattern, it is possible to construct a table showing the rotor position as a function of the voltage applied to the stator of the motor. If the voltage produced by the control unit changes with each input pulse in the order shown in Table 10–1, then the stepper motor will advance by 60° with each input pulse.

It is easy to build a stepper motor with finer step size by increasing the number of poles on the motor. From Equation (4–31) the number of mechanical degrees corresponding to a given number of electrical degrees is

$$\theta_m = \frac{2}{P}\,\theta_e \tag{10–18}$$

Since each step in Table 10–1 corresponds to 60 electrical degrees, the number of mechanical degrees moved per step decreases with increasing numbers of poles. For example, if the stepper motor has eight poles, then the mechanical angle of the motor's shaft will change by 15° per step.

The speed of a stepper motor can be related to the number of pulses into its control unit per unit time by using Equation (10–18). Equation (10–18) gives the mechanical angle of a stepper motor as a function of the electrical angle. If both sides of this equation are differentiated with respect to time, then we have a relationship between the electrical and mechanical rotational speeds of the motor:

$$\omega_m = \frac{2}{P}\,\omega_e \tag{10–19a}$$

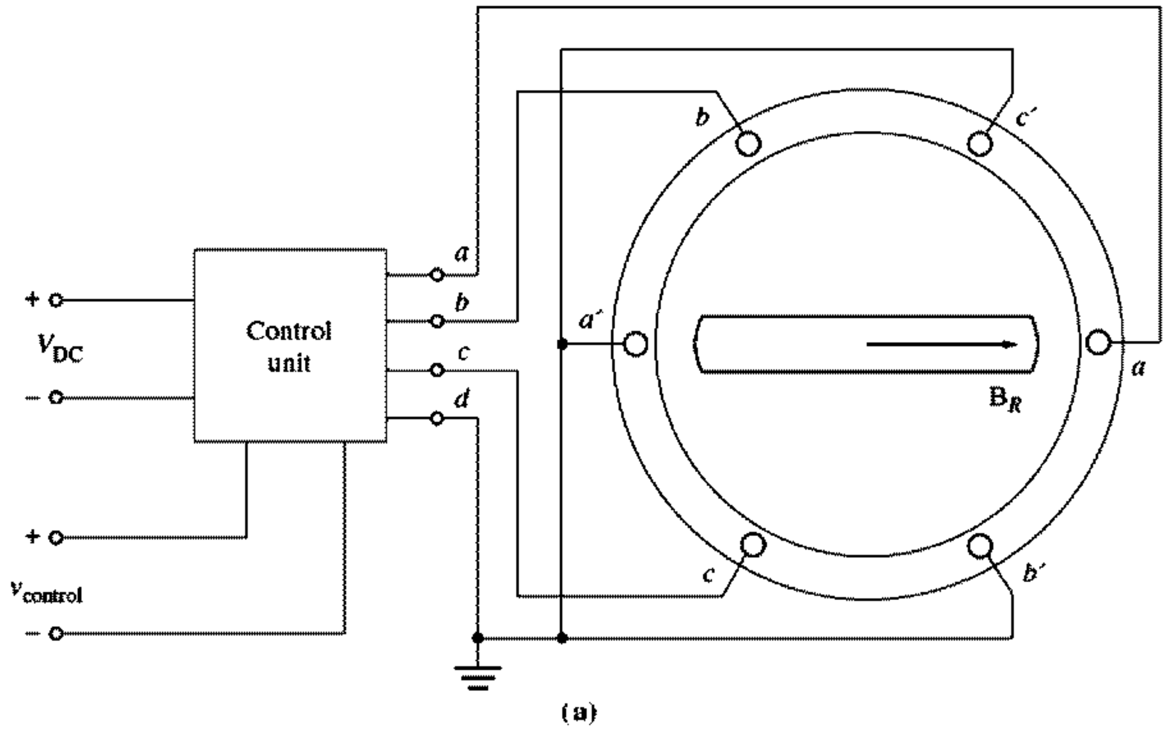
or

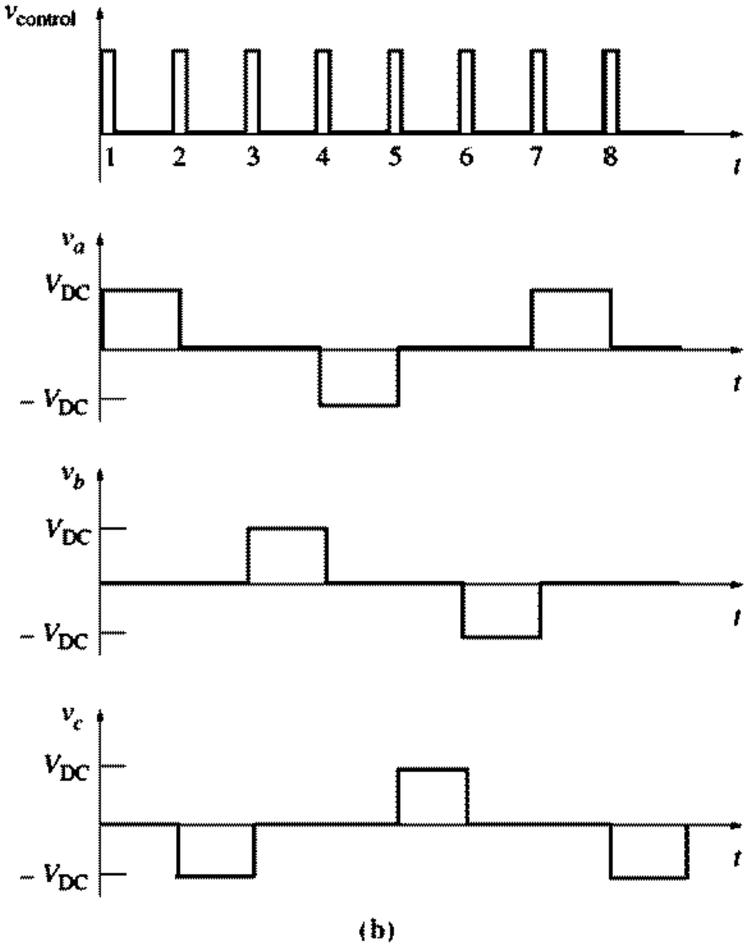
$$n_m = \frac{2}{P}\,n_e \tag{10–19b}$$

Since there are six input pulses per electrical revolution, the relationship between the speed of the motor in revolutions per minute and the number of pulses per minute becomes

$$\boxed{n_m = \frac{1}{3P}\,n_{\text{pulses}}} \tag{10–20}$$

where $n_{\text{pulses}}$ is the number of pulses per minute.





| Pulse number | Phase voltages, V | | |
|---|---|---|---|
| | $v_a$ | $v_b$ | $v_c$ |
| 1 | $V_{DC}$ | 0 | 0 |
| 2 | 0 | 0 | $-V_{DC}$ |
| 3 | 0 | $V_{DC}$ | 0 |
| 4 | $-V_{DC}$ | 0 | 0 |
| 5 | 0 | 0 | $V_{DC}$ |
| 6 | 0 | $-V_{DC}$ | 0 |

(c)

**FIGURE 10–38**
(a) A simple three-phase stepper motor and its associated control unit. The inputs to the control unit consist of a dc power source and a control signal consisting of a train of pulses. (b) A sketch of the output voltage from the control unit as a series of control pulses are input. (c) A table showing the output voltage from the control unit as a function of pulse number.

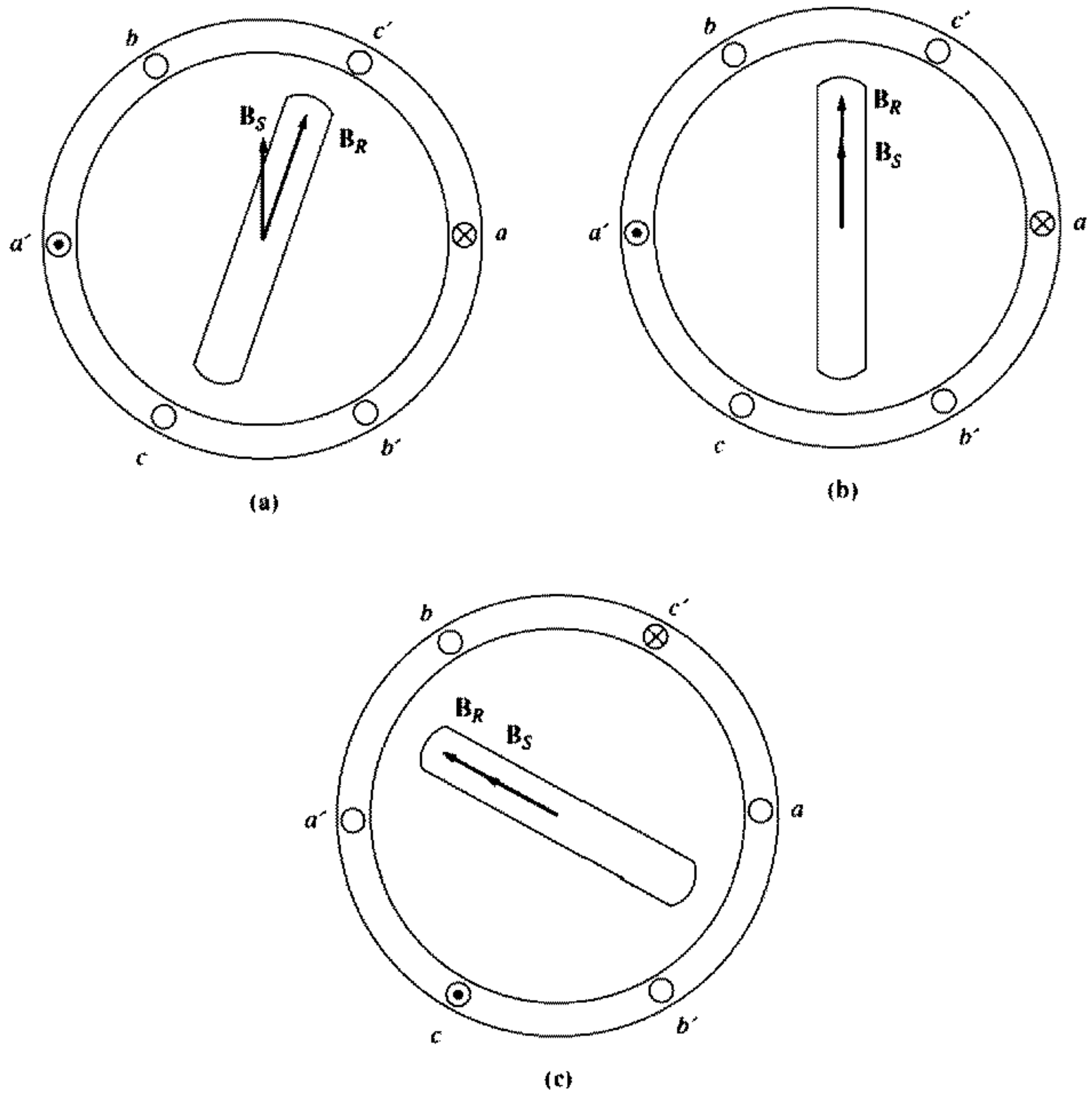


**FIGURE 10–39**
Operation of a stepper motor. (a) A voltage $V$ is applied to phase $a$ of the stator, causing a current to flow in phase $a$ and producing a stator magnetic field $\mathbf{B}_S$. The interaction of $\mathbf{B}_R$ and $\mathbf{B}_S$ produces a counterclockwise torque on the rotor. (b) When the rotor lines up with the stator magnetic field, the net torque falls to zero. (c) A voltage $-V$ is applied to phase $c$ of the stator, causing a current to flow in phase $c$ and producing a stator magnetic field $\mathbf{B}_S$. The interaction of $\mathbf{B}_R$ and $\mathbf{B}_S$ produces a counterclockwise torque on the rotor, causing the rotor to line up with the new position of the magnetic field.

There are two basic types of stepper motors, differing only in rotor construction: *permanent-magnet type* and *reluctance type*. The permanent-magnet type of stepper motor has a permanent-magnet rotor, while the reluctance-type stepper motor has a ferromagnetic rotor which is not a permanent magnet. (The rotor of the reluctance motor described previously in this section is the reluctance type.) In general, the permanent-magnet stepper motor can produce more torque

**TABLE 10–1**
**Rotor position as a function of voltage in a two-pole stepper motor**

| | Phase voltages | | | |
|---|---|---|---|---|
| **Input pulse number** | $a$ | $b$ | $c$ | **Rotor position** |
| 1 | $V$ | 0 | 0 | 0° |
| 2 | 0 | 0 | $-V$ | 60° |
| 3 | 0 | $V$ | 0 | 120° |
| 4 | $-V$ | 0 | 0 | 180° |
| 5 | 0 | 0 | $V$ | 240° |
| 6 | 0 | $-V$ | 0 | 300° |

than the reluctance type, since the permanent-magnet stepper motor has torque from both the permanent rotor magnetic field and reluctance effects.

Reluctance-type stepper motors are often built with a four-phase stator winding instead of the three-phase stator winding described above. A four-phase stator winding reduces the steps between pulses from 60 electrical degrees to 45 electrical degrees. As mentioned earlier, the torque in a reluctance motor varies as sin $2\delta$, so the reluctance torque between steps will be maximum for an angle of 45°. Therefore, a given reluctance-type stepper motor can produce more torque with a four-phase stator winding than with a three-phase stator winding.

Equation (10–20) can be generalized to apply to all stepper motors, regardless of the number of phases on their stator windings. In general, if a stator has $N$ phases, it takes $2N$ pulses per electrical revolution in that motor. Therefore, the relationship between the speed of the motor in revolutions per minute and the number of pulses per minute becomes

$$n_m = \frac{1}{NP} n_{\text{pulses}} \tag{10–21}$$

Stepper motors are very useful in control and positioning systems because the computer doing the controlling can know both the exact *speed* and *position* of the stepper motor without needing feedback information from the shaft of the motor. For example, if a control system sends 1200 pulses per minute to the two-pole stepper motor shown in Figure 10–38, then the speed of the motor will be exactly

$$\begin{aligned} n_m &= \frac{1}{3P} n_{\text{pulses}} \qquad (10–20) \\ &= \frac{1}{3(2\text{ poles})}(1200\text{ pulses/min}) \\ &= 200\text{ r/min} \end{aligned}$$

Furthermore, if the initial position of the shaft is known, then the computer can determine the exact angle of the rotor shaft at any future time by simply counting the total number of pulses which it has sent to the control unit of the stepper motor.

**Example 10–2.** A three-phase permanent-magnet stepper motor required for one particular application must be capable of controlling the position of a shaft in steps of 7.5°, and it must be capable of running at speeds of up to 300 r/min.

*(a)* How many poles must this motor have?

*(b)* At what rate must control pulses be received in the motor's control unit if it is to be driven at 300 r/min?

*Solution*

*(a)* In a three-phase stepper motor, each pulse advances the rotor's position by 60 electrical degrees. This advance must correspond to 7.5 mechanical degrees. Solving Equation (10–18) for $P$ yields

$$P = 2\frac{\theta_e}{\theta_m} = 2\left(\frac{60^\circ}{7.5^\circ}\right) = 16 \text{ poles}$$

*(b)* Solving Equation (10–21) for $n_{\text{pulses}}$ yields

$$\begin{aligned} n_{\text{pulses}} &= NPn_m \\ &= (3 \text{ phases})(16 \text{ poles})(300 \text{ r/min}) \\ &= 240 \text{ pulses/s} \end{aligned}$$

## Brushless DC Motors

Conventional dc motors have traditionally been used in applications where dc power sources are available, such as on aircraft and automobiles. However, small dc motors of these types have a number of disadvantages. The principal disadvantage is excessive sparking and brush wear. Small, fast dc motors are too small to use compensating windings and interpoles, so armature reaction and $L\,di/dt$ effects tend to produce sparking on their commutator brushes. In addition, the high rotational speed of these motors causes increased brush wear and requires regular maintenance every few thousand hours. If the motors must work in a low-pressure environment (such as at high altitudes in an aircraft), brush wear can be so bad that the brushes require replacement after less than an hour of operation!

In some applications, the regular maintenance required by the brushes of these dc motors may be unacceptable. Consider for example a dc motor in an artificial heart—regular maintenance would require opening the patient's chest. In other applications, the sparks at the brushes may create an explosion danger, or unacceptable RF noise. For all of these cases, there is a need for a small, fast dc motor that is highly reliable and has low noise and long life.

Such motors have been developed in the last 25 years by combining a small motor much like a permanent magnetic stepper motor with a rotor position sensor and a solid-state electronic switching circuit. These motors are called ***brushless dc motors*** because they run from a dc power source but do not have commutators and brushes. A sketch of a small brushless dc motor is shown in Figure 10–40, and a photograph of a typical brushless dc motor is shown in Figure 10–41. The rotor is similar to that of a permanent magnet stepper motor, except that it is nonsalient. The stator can have three or more phases (there are four phases in the example shown).

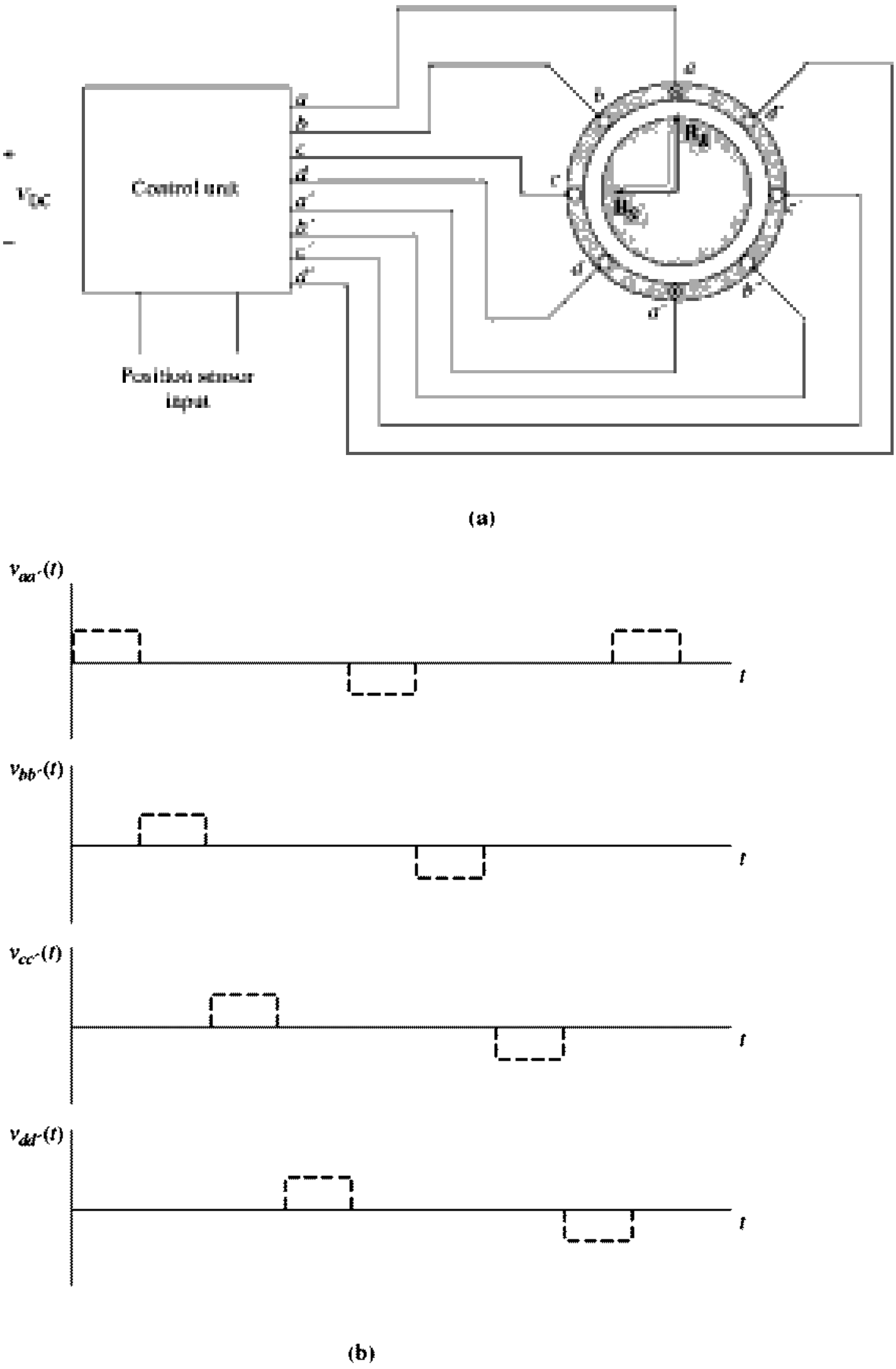


**FIGURE 10–40**
(a) A simple brushless dc motor and its associated control unit. The inputs to the control unit consist of a dc power source and a signal proportional to the current rotor position. (b) The voltages applied to the stator coils.

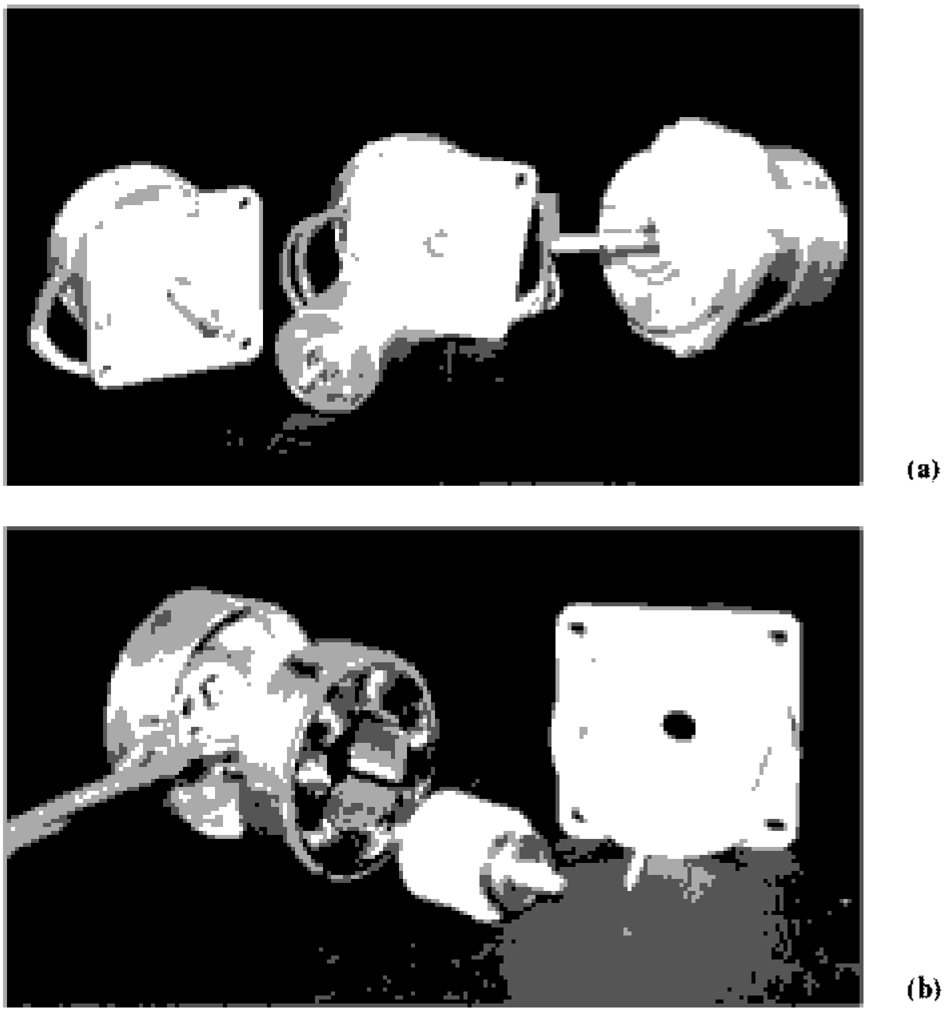


**FIGURE 10–41**
(a) Typical brushless dc motors. (b) Exploded view showing the permanent magnet rotor and a three-phase (6-pole) stator. (*Courtesy of Carson Technologies, Inc.*)

The basic components of a brushless dc motor are

1. A permanent magnet rotor
2. A stator with a three-, four-, or more phase winding
3. A rotor position sensor
4. An electronic circuit to control the phases of the rotor winding

A brushless dc motor functions by energizing one stator coil at a time with a constant dc voltage. When a coil is turned on, it produces a stator magnetic field $\mathbf{B}_S$, and a torque is produced on the rotor given by

$$\tau_{\text{ind}} = k\mathbf{B}_R \times \mathbf{B}_S$$

which tends to align the rotor with the stator magnetic field. At the time shown in Figure 10–40a, the stator magnetic field $\mathbf{B}_S$ points to the left while the permanent magnet rotor magnetic field $\mathbf{B}_R$ points up, producing a counterclockwise torque on the rotor. As a result the rotor will turn to the left.

If coil *a* remained energized all of the time, the rotor would turn until the two magnetic fields are aligned, and then it would stop, just like a stepper motor. The key to the operation of a brushless dc motor is that it includes a *position sensor*, so that the control circuit will know when the rotor is almost aligned with the stator magnetic field. At that time coil *a* will be turned off and coil *b* will be turned on, causing the rotor to again experience a counterclockwise torque, and to continue rotating. This process continues indefinitely with the coils turned on in the order $a, b, c, d, -a, -b, -c, -d$, etc., so that the motor turns continuously.

The electronics of the control circuit can be used to control both the speed and direction of the motor. The net effect of this design is a motor that runs from a dc power source, with full control over both the speed and the direction of rotation.

Brushless dc motors are available only in small sizes, up to 20 W or so, but they have many advantages in the size range over which they are available. Some of the major advantages include:

1. Relatively high efficiency.
2. Long life and high reliability.
3. Little or no maintenance.
4. Very little RF noise compared to a dc motor with brushes.
5. Very high speeds are possible (greater than 50,000 r/min).

The principal disadvantage is that a brushless dc motor is more expensive than a comparable brush dc motor.

## 10.7 SUMMARY

The ac motors described in previous chapters required three-phase power to function. Since most residences and small businesses have only single-phase power sources, these motors cannot be used. A series of motors capable of running from a single-phase power source was described in this chapter.

The first motor described was the universal motor. A universal motor is a series dc motor adapted to run from an ac supply, and its torque–speed characteristic is similar to that of a series dc motor. The universal motor has a very high torque, but its speed regulation is very poor.

Single-phase induction motors have no intrinsic starting torque, but once they are brought up to speed, their torque–speed characteristics are almost as good as those of three-phase motors of comparable size. Starting is accomplished by the addition of an auxiliary winding with a current whose phase angle differs from that of the main winding or by shading portions of the stator poles.

The starting torque of a single-phase induction motor depends on the phase angle between the current in the primary winding and the current in the auxiliary winding, with maximum torque occurring when that angle reaches 90°. Since the split-phase construction provides only a small phase difference between the main and auxiliary windings, its starting torque is modest. Capacitor-start motors have auxiliary windings with an approximately 90° phase shift, so they have large starting torques. Permanent split-capacitor motors, which have smaller capacitors, have starting torques intermediate between those of the split-phase motor and the capacitor-start motor. Shaded-pole motors have a very small effective phase shift and therefore a small starting torque.

Reluctance motors and hysteresis motors are special-purpose ac motors which can operate at synchronous speed without the rotor field windings required by synchronous motors and which can accelerate up to synchronous speed by themselves. These motors can have either single- or three-phase stators.

Stepper motors are motors used to advance the position of a shaft or other mechanical device by a fixed amount each time a control pulse is received. They are used extensively in control systems for positioning objects.

Brushless dc motors are similar to stepper motors with permanent magnet rotors, except that they include a position sensor. The position sensor is used to switch the energized stator coil whenever the rotor is almost aligned with it, keeping the rotor rotating a speed set by the control electronics. Brushless dc motors are more expensive than ordinary dc motors, but require low maintenance and have high reliability, long life, and low RF noise. They are available only in small sizes (20 W and down).

## QUESTIONS

**10–1.** What changes are necessary in a series dc motor to adapt it for operation from an ac power source?

**10–2.** Why is the torque–speed characteristic of a universal motor on an ac source different from the torque–speed characteristic of the same motor on a dc source?

**10–3.** Why is a single-phase induction motor unable to start itself without special auxiliary windings?

**10–4.** How is induced torque developed in a single-phase induction motor (*a*) according to the double revolving-field theory and (*b*) according to the cross-field theory?

**10–5.** How does an auxiliary winding provide a starting torque for single-phase induction motors?

**10–6.** How is the current phase shift accomplished in the auxiliary winding of a split-phase induction motor?

**10–7.** How is the current phase shift accomplished in the auxiliary winding of a capacitor-start induction motor?

**10–8.** How does the starting torque of a permanent split-capacitor motor compare to that of a capacitor-start motor of the same size?

**10–9.** How can the direction of rotation of a split-phase or capacitor-start induction motor be reversed?

**10–10.** How is starting torque produced in a shaded-pole motor?

**10–11.** How does a reluctance motor start?

**10–12.** How can a reluctance motor run at synchronous speed?

**10–13.** What mechanisms produce the starting torque in a hysteresis motor?

**10–14.** What mechanism produces the synchronous torque in a hysteresis motor?

**10–15.** Explain the operation of a stepper motor.

**10–16.** What is the difference between a permanent-magnet type of stepper motor and a reluctance-type stepper motor?

**10–17.** What is the optimal spacing between phases for a reluctance-type stepper motor? Why?

**10–18.** What are the advantages and disadvantages of brushless dc motors compared to ordinary brush dc motors?

## PROBLEMS

**10–1.** A 120-V, ⅓-hp, 60-Hz, four-pole, split-phase induction motor has the following impedances:

$$R_1 = 1.80\ \Omega \qquad X_1 = 2.40\ \Omega \qquad X_M = 60\ \Omega$$
$$R_2 = 2.50\ \Omega \qquad X_2 = 2.40\ \Omega$$

At a slip of 0.05, the motor's rotational losses are 51 W. The rotational losses may be assumed constant over the normal operating range of the motor. If the slip is 0.05, find the following quantities for this motor:

*(a)* Input power
*(b)* Air-gap power
*(c)* $P_{\text{conv}}$
*(d)* $P_{\text{out}}$
*(e)* $\tau_{\text{ind}}$
*(f)* $\tau_{\text{load}}$
*(g)* Overall motor efficiency
*(h)* Stator power factor

**10–2.** Repeat Problem 10–1 for a rotor slip of 0.025.

**10–3.** Suppose that the motor in Problem 10–1 is started and the auxiliary winding fails open while the rotor is accelerating through 400 r/min. How much induced torque will the motor be able to produce on its main winding alone? Assuming that the rotational losses are still 51 W, will this motor continue accelerating or will it slow down again? Prove your answer.

**10–4.** Use MATLAB to calculate and plot the torque–speed characteristic of the motor in Problem 10–1, ignoring the starting winding.

**10–5.** A 220-V, 1.5-hp, 50-Hz, two-pole, capacitor-start induction motor has the following main-winding impedances:

$$R_1 = 1.40\ \Omega \qquad X_1 = 1.90\ \Omega \qquad X_M = 100\ \Omega$$
$$R_2 = 1.50\ \Omega \qquad X_2 = 1.90\ \Omega$$

At a slip of 0.05, the motor's rotational losses are 291 W. The rotational losses may be assumed constant over the normal operating range of the motor. Find the following quantities for this motor at 5 percent slip:

*(a)* Stator current
*(b)* Stator power factor
*(c)* Input power
*(d)* $P_{\text{AG}}$
*(e)* $P_{\text{conv}}$

(*f*) $P_{\text{out}}$
(*g*) $\tau_{\text{ind}}$
(*h*) $\tau_{\text{load}}$
(*i*) Efficiency

**10–6.** Find the induced torque in the motor in Problem 10–5 if it is operating at 5 percent slip and its terminal voltage is (*a*) 190 V, (*b*) 208 V, (*c*) 230 V.

**10–7.** What type of motor would you select to perform each of the following jobs? Why?
(*a*) Vacuum cleaner
(*b*) Refrigerator
(*c*) Air conditioner compressor
(*d*) Air conditioner fan
(*e*) Variable-speed sewing machine
(*f*) Clock
(*g*) Electric drill

**10–8.** For a particular application, a three-phase stepper motor must be capable of stepping in 10° increments. How many poles must it have?

**10–9.** How many pulses per second must be supplied to the control unit of the motor in Problem 10–8 to achieve a rotational speed of 600 r/min?

**10–10.** Construct a table showing step size versus number of poles for three-phase and four-phase stepper motors.

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