CHAPTER 2

TRANSFORMERS

A *transformer* is a device that changes ac electric power at one voltage level to ac electric power at another voltage level through the action of a magnetic field. It consists of two or more coils of wire wrapped around a common ferromagnetic core. These coils are (usually) not directly connected. The only connection between the coils is the common magnetic flux present within the core.

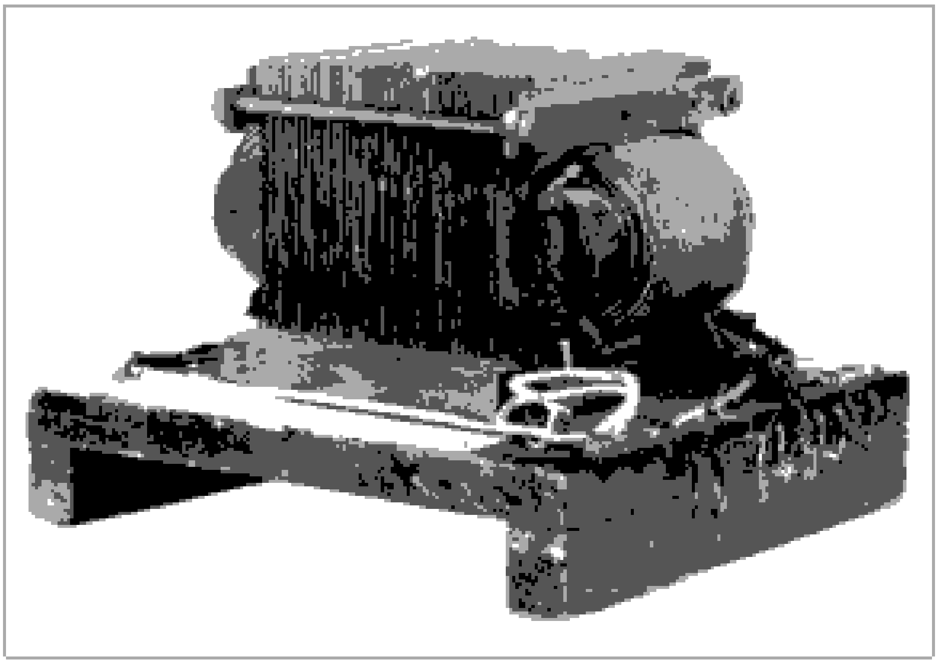


FIGURE 2–1
The first practical modern transformer, built by William Stanley in 1885. Note that the core is made up of individual sheets of metal (laminations). (*Courtesy of General Electric Company.*)

One of the transformer windings is connected to a source of ac electric power, and the second (and perhaps third) transformer winding supplies electric power to loads. The transformer winding connected to the power source is called the ***primary winding*** or ***input winding***, and the winding connected to the loads is called the ***secondary winding*** or ***output winding***. If there is a third winding on the transformer, it is called the ***tertiary winding***.

2.1 WHY TRANSFORMERS ARE IMPORTANT TO MODERN LIFE

The first power distribution system in the United States was a 120-V dc system invented by Thomas A. Edison to supply power for incandescent light bulbs. Edison's first central power station went into operation in New York City in September 1882. Unfortunately, his power system generated and transmitted power at such low voltages that very large currents were necessary to supply significant amounts of power. These high currents caused huge voltage drops and power losses in the transmission lines, severely restricting the service area of a generating station. In the 1880s, central power stations were located every few city blocks to overcome this problem. The fact that power could not be transmitted far with low-voltage dc power systems meant that generating stations had to be small and localized and so were relatively inefficient.

The invention of the transformer and the concurrent development of ac power sources eliminated forever these restrictions on the range and power level of power systems. A transformer ideally changes one ac voltage level to another voltage level without affecting the actual power supplied. If a transformer steps up the voltage level of a circuit, it must decrease the current to keep the power into the device equal to the power out of it. Therefore, ac electric power can be generated at one central location, its voltage stepped up for transmission over long distances at very low losses, and its voltage stepped down again for final use. Since the transmission losses in the lines of a power system are proportional to the square of the current in the lines, raising the transmission voltage and reducing the resulting transmission currents by a factor of 10 with transformers reduces power transmission losses by a factor of 100. Without the transformer, it would simply not be possible to use electric power in many of the ways it is used today.

In a modern power system, electric power is generated at voltages of 12 to 25 kV. Transformers step up the voltage to between 110 kV and nearly 1000 kV for transmission over long distances at very low losses. Transformers then step down the voltage to the 12- to 34.5-kV range for local distribution and finally permit the power to be used safely in homes, offices, and factories at voltages as low as 120 V.

2.2 TYPES AND CONSTRUCTION OF TRANSFORMERS

The principal purpose of a transformer is to convert ac power at one voltage level to ac power of the same frequency at another voltage level. Transformers are also

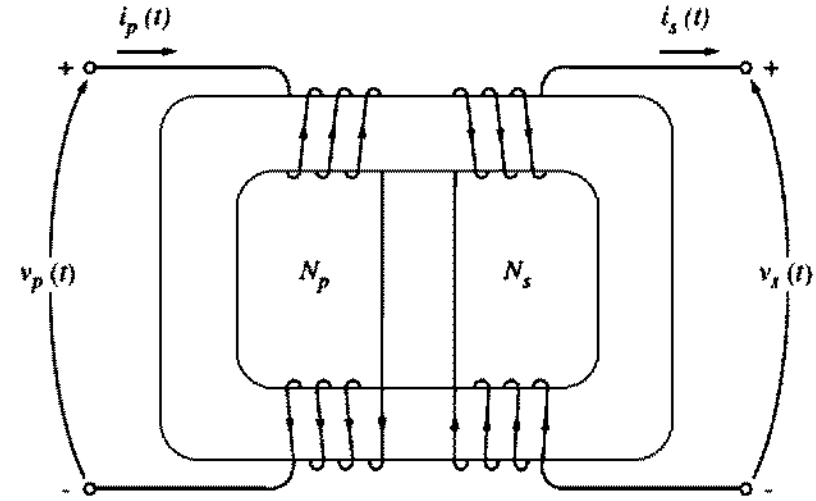


FIGURE 2–2
Core-form transformer construction.

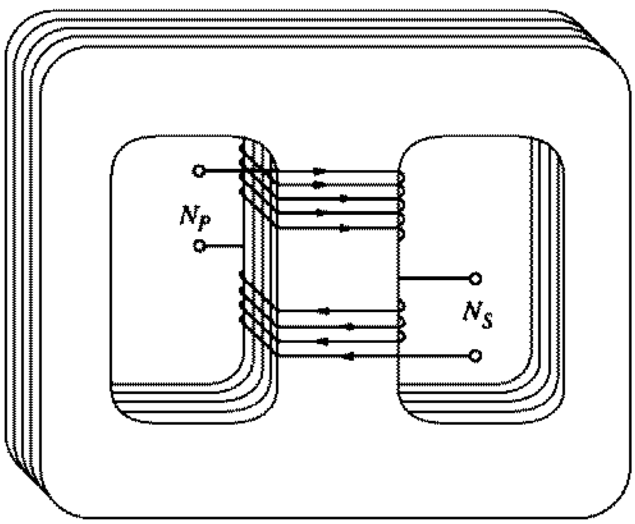
used for a variety of other purposes (e.g., voltage sampling, current sampling, and impedance transformation), but this chapter is primarily devoted to the power transformer.

Power transformers are constructed on one of two types of cores. One type of construction consists of a simple rectangular laminated piece of steel with the transformer windings wrapped around two sides of the rectangle. This type of construction is known as *core form* and is illustrated in Figure 2–2. The other type consists of a three-legged laminated core with the windings wrapped around the center leg. This type of construction is known as *shell form* and is illustrated in Figure 2–3. In either case, the core is constructed of thin laminations electrically isolated from each other in order to minimize eddy currents.

The primary and secondary windings in a physical transformer are wrapped one on top of the other with the low-voltage winding innermost. Such an arrangement serves two purposes:

1. It simplifies the problem of insulating the high-voltage winding from the core.
2. It results in much less leakage flux than would be the case if the two windings were separated by a distance on the core.

Power transformers are given a variety of different names, depending on their use in power systems. A transformer connected to the output of a generator and used to step its voltage up to transmission levels (110+ kV) is sometimes called a *unit transformer*. The transformer at the other end of the transmission line, which steps the voltage down from transmission levels to distribution levels (from 2.3 to 34.5 kV), is called a *substation transformer*. Finally, the transformer that takes the distribution voltage and steps it down to the final voltage at which the power is actually used (110, 208, 220 V, etc.) is called a *distribution transformer*. All these devices are essentially the same—the only difference among them is their intended use.



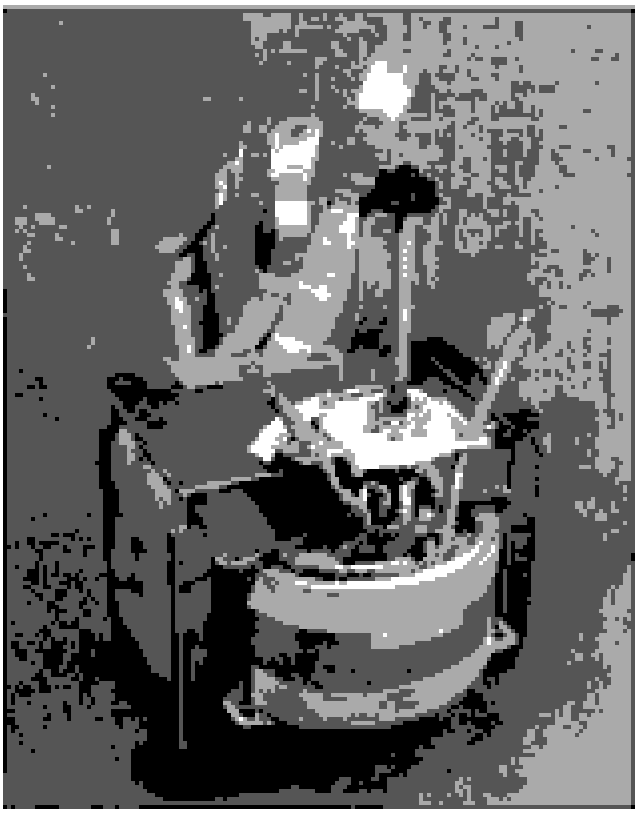


FIGURE 2–3
(a) Shell-form transformer construction. (b) A typical shell-form transformer. *(Courtesy of General Electric Company.)*

In addition to the various power transformers, two special-purpose transformers are used with electric machinery and power systems. The first of these special transformers is a device specially designed to sample a high voltage and produce a low secondary voltage directly proportional to it. Such a transformer is called a *potential transformer.* A power transformer also produces a secondary voltage directly proportional to its primary voltage; the difference between a potential transformer and a power transformer is that the potential transformer is designed to handle only a very small current. The second type of special transformer is a device designed to provide a secondary current much smaller than but directly proportional to its primary current. This device is called a *current transformer.* Both special-purpose transformers are discussed in a later section of this chapter.

2.3 THE IDEAL TRANSFORMER

An *ideal transformer* is a lossless device with an input winding and an output winding. The relationships between the input voltage and the output voltage, and between the input current and the output current, are given by two simple equations. Figure 2–4 shows an ideal transformer.

The transformer shown in Figure 2–4 has N_P turns of wire on its primary side and N_S turns of wire on its secondary side. The relationship between the volt-

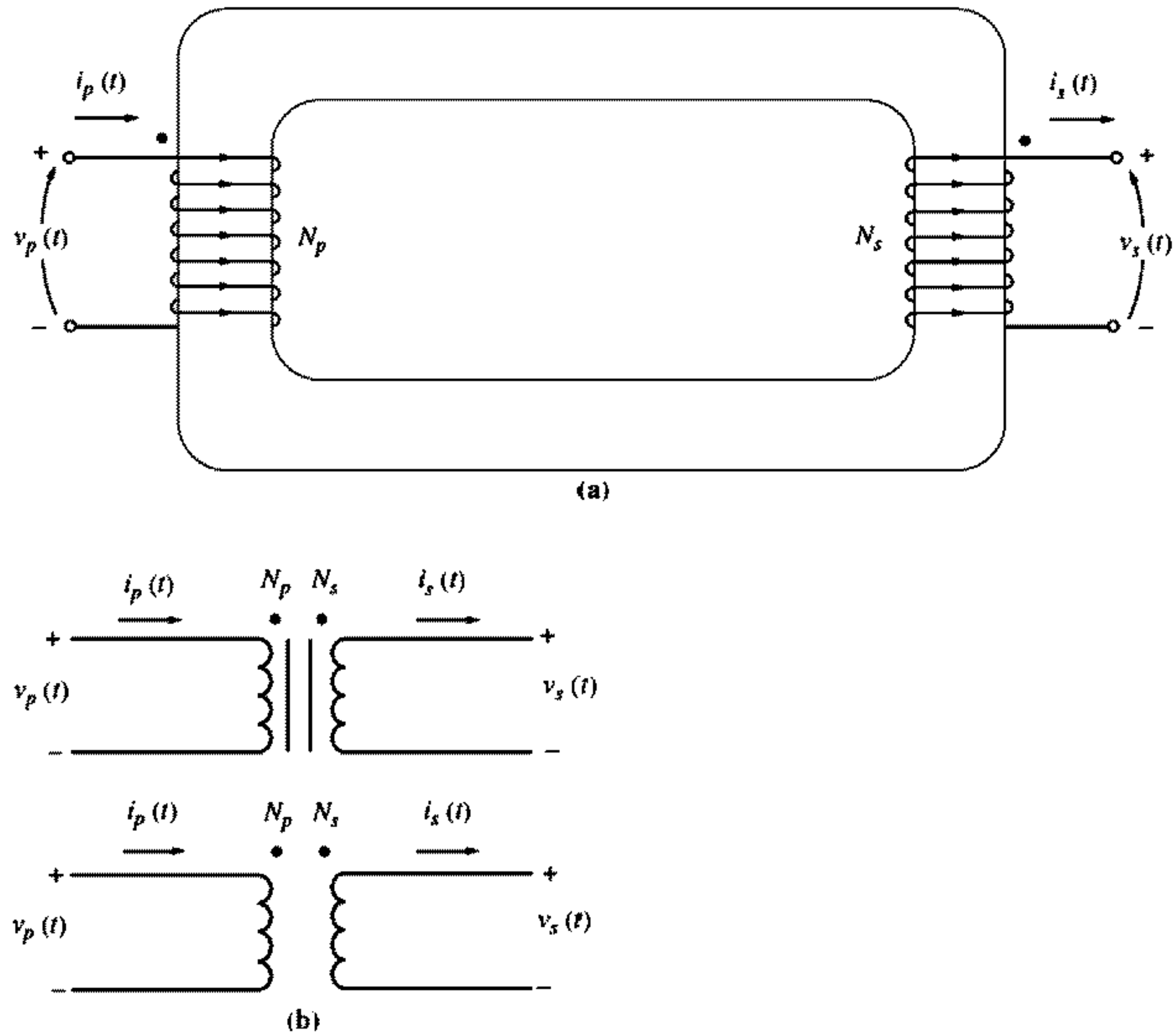


FIGURE 2–4
(a) Sketch of an ideal transformer. (b) Schematic symbols of a transformer.

age $v_P(t)$ applied to the primary side of the transformer and the voltage $v_S(t)$ produced on the secondary side is

$$\boxed{\frac{v_P(t)}{v_S(t)} = \frac{N_P}{N_S} = a} \tag{2–1}$$

where a is defined to be the *turns ratio* of the transformer:

$$a = \frac{N_P}{N_S} \tag{2–2}$$

The relationship between the current $i_P(t)$ flowing into the primary side of the transformer and the current $i_S(t)$ flowing out of the secondary side of the transformer is

$$\boxed{N_P i_P(t) = N_S i_S(t)} \tag{2–3a}$$

or

$$\boxed{\frac{i_P(t)}{i_S(t)} = \frac{1}{a}} \tag{2–3b}$$

In terms of phasor quantities, these equations are

$$\boxed{\frac{\mathbf{V}_P}{\mathbf{V}_S} = a} \tag{2–4}$$

and

$$\boxed{\frac{\mathbf{I}_P}{\mathbf{I}_S} = \frac{1}{a}} \tag{2–5}$$

Notice that the phase angle of $\mathbf{V}_P$ is the same as the angle of $\mathbf{V}_S$ and the phase angle of $\mathbf{I}_P$ is the same as the phase angle of $\mathbf{I}_S$. The turns ratio of the ideal transformer affects the *magnitudes* of the voltages and currents, but not their *angles.*

Equations (2–1) to (2–5) describe the relationships between the magnitudes and angles of the voltages and currents on the primary and secondary sides of the transformer, but they leave one question unanswered: Given that the primary circuit's voltage is positive at a specific end of the coil, what would the *polarity* of the secondary circuit's voltage be? In real transformers, it would be possible to tell the secondary's polarity only if the transformer were opened and its windings examined. To avoid this necessity, transformers utilize the *dot convention.* The dots appearing at one end of each winding in Figure 2–4 tell the polarity of the voltage and current on the secondary side of the transformer. The relationship is as follows:

1. If the primary *voltage* is positive at the dotted end of the winding with respect to the undotted end, then the secondary voltage will be positive at the dotted end also. Voltage polarities are the same with respect to the dots on each side of the core.
2. If the primary *current* of the transformer flows *into* the dotted end of the primary winding, the secondary current will flow *out* of the dotted end of the secondary winding.

The physical meaning of the dot convention and the reason polarities work out this way will be explained in Section 2.4, which deals with the real transformer.

Power in an Ideal Transformer

The power supplied to the transformer by the primary circuit is given by the equation

$$\boxed{P_{\text{in}} = V_P I_P \cos \theta_P} \tag{2–6}$$

where θ_P is the angle between the primary voltage and the primary current. The power supplied by the transformer secondary circuit to its loads is given by the equation

$$\boxed{P_{\text{out}} = V_S I_S \cos \theta_S} \tag{2–7}$$

where θ_S is the angle between the secondary voltage and the secondary current. Since voltage and current angles are unaffected by an ideal transformer, $\theta_P - \theta_S = \theta$. The primary and secondary windings of an ideal transformer have the *same power factor.*

How does the power going into the primary circuit of the ideal transformer compare to the power coming out of the other side? It is possible to find out through a simple application of the voltage and current equations [Equations (2–4) and (2–5)]. The power out of a transformer is

$$P_{\text{out}} = V_S I_S \cos\theta \tag{2–8}$$

Applying the turns-ratio equations gives $V_S = V_P/a$ and $I_S = aI_P$, so

$$P_{\text{out}} = \frac{V_P}{a}(aI_P)\cos\theta$$

$$\boxed{P_{\text{out}} = V_P I_P \cos\theta = P_{\text{in}}} \tag{2–9}$$

Thus, *the output power of an ideal transformer is equal to its input power.*

The same relationship applies to reactive power Q and apparent power S:

$$\boxed{Q_{\text{in}} = V_P I_P \sin\theta = V_S I_S \sin\theta = Q_{\text{out}}} \tag{2–10}$$

and

$$\boxed{S_{\text{in}} = V_P I_P = V_S I_S = S_{\text{out}}} \tag{2–11}$$

Impedance Transformation through a Transformer

The *impedance* of a device or an element is defined as the ratio of the phasor voltage across it to the phasor current flowing through it:

$$Z_L = \frac{\mathbf{V}_L}{\mathbf{I}_L} \tag{2–12}$$

One of the interesting properties of a transformer is that, since it changes voltage and current levels, it changes the *ratio* between voltage and current and hence the apparent impedance of an element. To understand this idea, refer to Figure 2–5. If the secondary current is called $\mathbf{I}_S$ and the secondary voltage $\mathbf{V}_S$, then the impedance of the load is given by

$$Z_L = \frac{\mathbf{V}_S}{\mathbf{I}_S} \tag{2–13}$$

The apparent impedance of the primary circuit of the transformer is

$$Z_L' = \frac{\mathbf{V}_P}{\mathbf{I}_P} \tag{2–14}$$

Since the primary voltage can be expressed as

$$\mathbf{V}_P = a\mathbf{V}_S$$

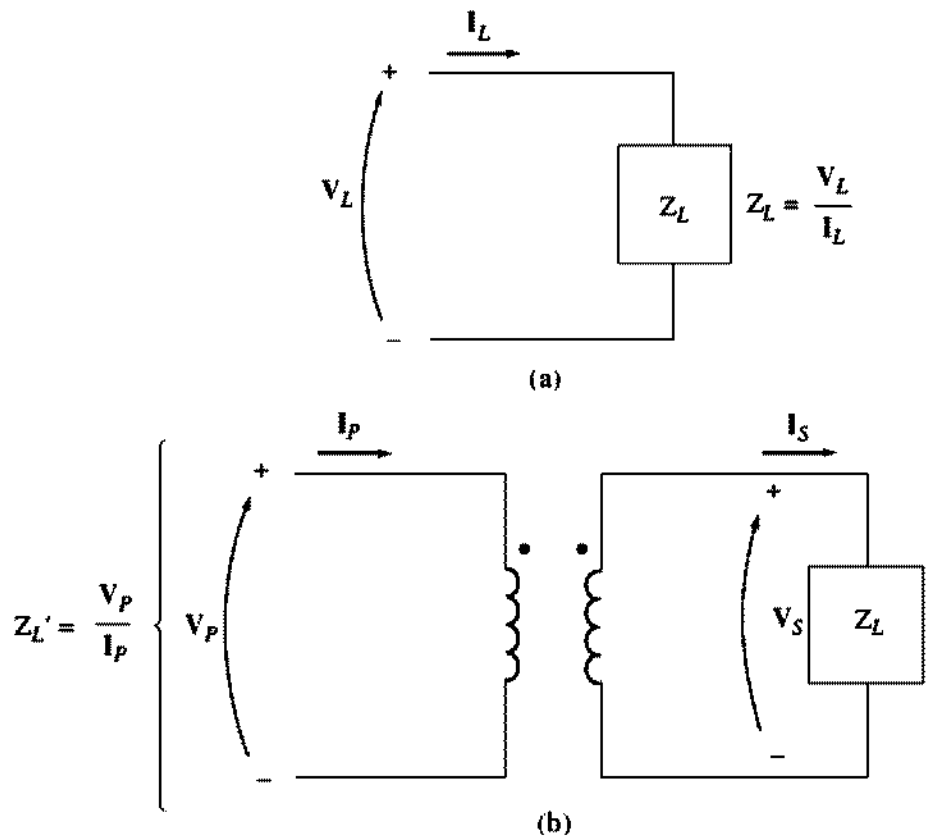


FIGURE 2–5
(a) Definition of impedance. (b) Impedance scaling through a transformer.

and the primary current can be expressed as

$$\mathbf{I}_P = \frac{\mathbf{I}_S}{a}$$

the apparent impedance of the primary is

$$Z_L' = \frac{\mathbf{V}_P}{\mathbf{I}_P} = \frac{a\mathbf{V}_S}{\mathbf{I}_S/a} = a^2\frac{\mathbf{V}_S}{\mathbf{I}_S}$$

$$\boxed{Z_L' = a^2 Z_L} \tag{2–15}$$

With a transformer, it is possible to match the magnitude of a load impedance to a source impedance simply by picking the proper turns ratio.

Analysis of Circuits Containing Ideal Transformers

If a circuit contains an ideal transformer, then the easiest way to analyze the circuit for its voltages and currents is to replace the portion of the circuit on one side of the transformer by an equivalent circuit with the same terminal characteristics. After the equivalent circuit has been substituted for one side, then the new circuit (without a transformer present) can be solved for its voltages and currents. In the portion of the circuit that was not replaced, the solutions obtained will be the cor-

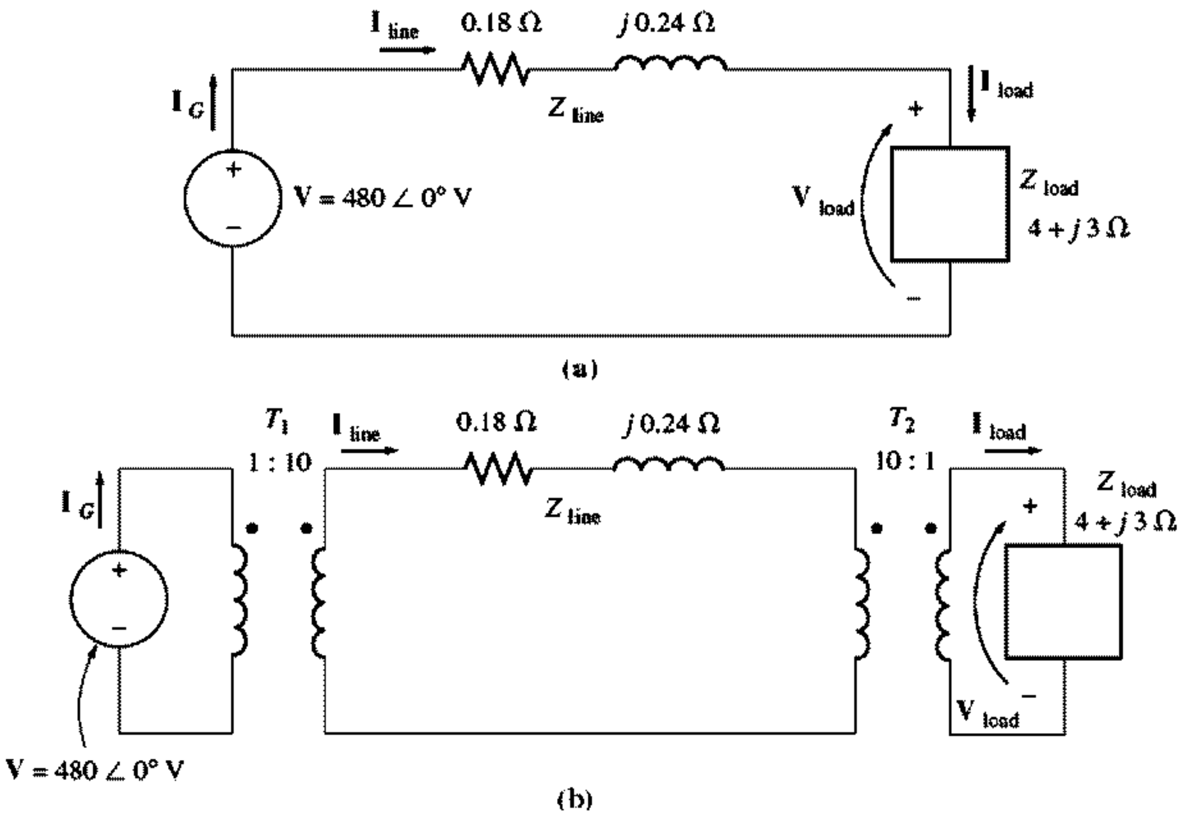


FIGURE 2–6
The power system of Example 2–1 (a) without and (b) with transformers at the ends of the transmission line.

rect values of voltage and current for the original circuit. Then the turns ratio of the transformer can be used to determine the voltages and currents on the other side of the transformer. The process of replacing one side of a transformer by its equivalent at the other side's voltage level is known as *referring* the first side of the transformer to the second side.

How is the equivalent circuit formed? Its shape is exactly the same as the shape of the original circuit. The values of voltages on the side being replaced are scaled by Equation (2–4), and the values of the impedances are scaled by Equation (2–15). The polarities of voltage sources in the equivalent circuit will be reversed from their direction in the original circuit if the dots on one side of the transformer windings are reversed compared to the dots on the other side of the transformer windings.

The solution for circuits containing ideal transformers is illustrated in the following example.

Example 2–1. A single-phase power system consists of a 480-V 60-Hz generator supplying a load $Z_{load} = 4 + j3\ \Omega$ through a transmission line of impedance $Z_{line} = 0.18 + j0.24\ \Omega$. Answer the following questions about this system.

(*a*) If the power system is exactly as described above (Figure 2–6a), what will the voltage at the load be? What will the transmission line losses be?

(*b*) Suppose a 1:10 step-up transformer is placed at the generator end of the transmission line and a 10:1 step-down transformer is placed at the load end of the line (Figure 2–6b). What will the load voltage be now? What will the transmission line losses be now?

Solution

(*a*) Figure 2–6a shows the power system without transformers. Here $\mathbf{I}_G = \mathbf{I}_{\text{line}} = \mathbf{I}_{\text{load}}$. The line current in this system is given by

$$\begin{aligned}\mathbf{I}_{\text{line}} &= \frac{\mathbf{V}}{\mathbf{Z}_{\text{line}} + \mathbf{Z}_{\text{load}}} \\ &= \frac{480\angle 0^\circ \text{ V}}{(0.18\ \Omega + j0.24\ \Omega) + (4\ \Omega + j3\ \Omega)} \\ &= \frac{480\angle 0^\circ}{4.18 + j3.24} = \frac{480\angle 0^\circ}{5.29\angle 37.8^\circ} \\ &= 90.8\angle -37.8^\circ \text{ A}\end{aligned}$$

Therefore the load voltage is

$$\begin{aligned}\mathbf{V}_{\text{load}} &= \mathbf{I}_{\text{line}}\mathbf{Z}_{\text{load}} \\ &= (90.8\angle -37.8^\circ \text{ A})(4\ \Omega + j3\ \Omega) \\ &= (90.8\angle -37.8^\circ \text{ A})(5\angle 36.9^\circ\ \Omega) \\ &= 454\angle -0.9^\circ \text{ V}\end{aligned}$$

and the line losses are

$$\begin{aligned}P_{\text{loss}} &= (I_{\text{line}})^2 R_{\text{line}} \\ &= (90.8 \text{ A})^2 (0.18\ \Omega) = 1484 \text{ W}\end{aligned}$$

(*b*) Figure 2–6b shows the power system with the transformers. To analyze this system, it is necessary to convert it to a common voltage level. This is done in two steps:

1. Eliminate transformer T_2 by referring the load over to the transmission line's voltage level.
2. Eliminate transformer T_1 by referring the transmission line's elements and the equivalent load at the transmission line's voltage over to the source side.

The value of the load's impedance when reflected to the transmission system's voltage is

$$\begin{aligned}\mathbf{Z}'_{\text{load}} &= a^2\mathbf{Z}_{\text{load}} \\ &= \left(\frac{10}{1}\right)^2 (4\ \Omega + j3\ \Omega) \\ &= 400\ \Omega + j300\ \Omega\end{aligned}$$

The total impedance at the transmission line level is now

$$\begin{aligned}\mathbf{Z}_{\text{eq}} &= \mathbf{Z}_{\text{line}} + \mathbf{Z}'_{\text{load}} \\ &= 400.18 + j300.24\ \Omega = 500.3\angle 36.88^\circ\ \Omega\end{aligned}$$

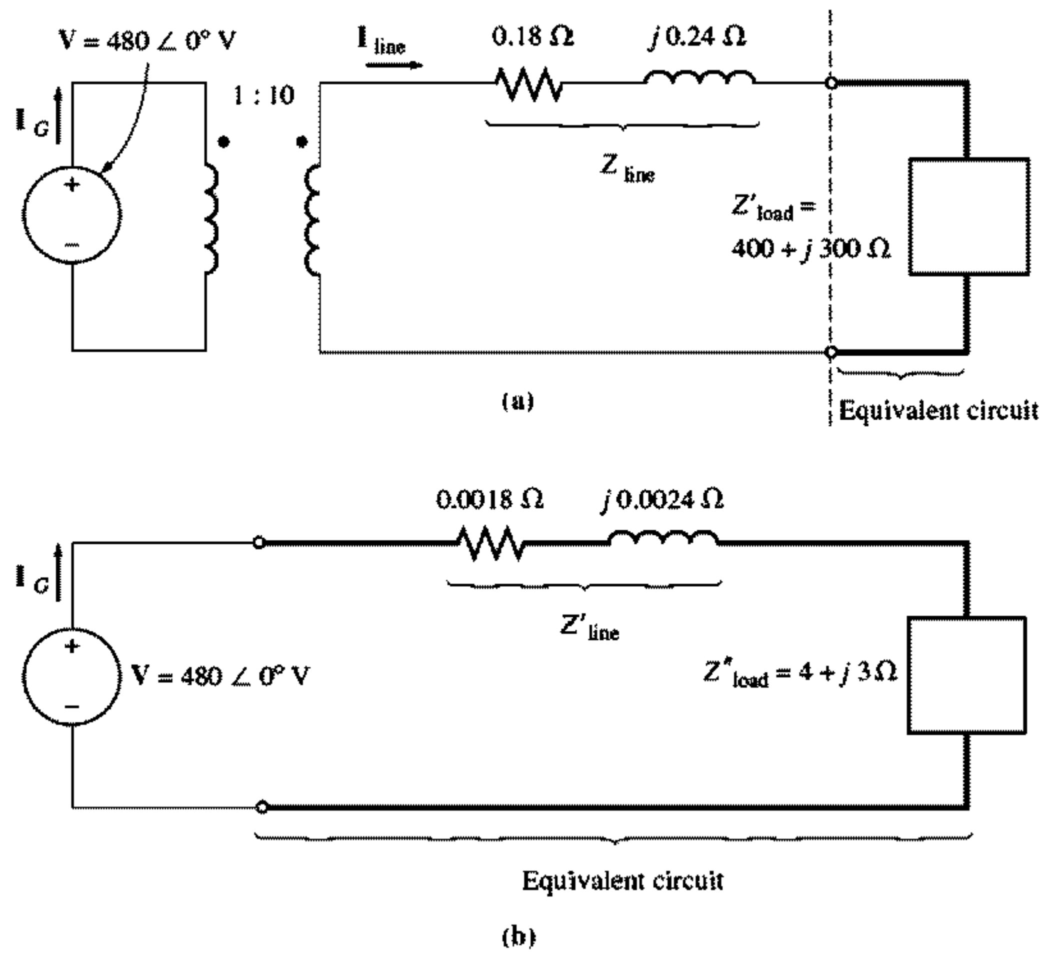


FIGURE 2–7
(a) System with the load referred to the transmission system voltage level. (b) System with the load and transmission line referred to the generator's voltage level.

This equivalent circuit is shown in Figure 2–7a. The total impedance at the transmission line level ($Z_{line} + Z'_{load}$) is now reflected across T_1 to the source's voltage level:

$$\begin{aligned}
\mathbf{Z}'_{eq} &= a^2\,\mathbf{Z}_{eq} \\
&= a^2(\mathbf{Z}_{line} + \mathbf{Z}'_{load}) \\
&= \left(\frac{1}{10}\right)^2 (0.18\ \Omega + j0.24\ \Omega + 400\ \Omega + j300\ \Omega) \\
&= (0.0018\ \Omega + j0.0024\ \Omega + 4\ \Omega + j3\ \Omega) \\
&= 5.003\ \angle 36.88°\ \Omega
\end{aligned}$$

Notice that $\mathbf{Z}''_{load} = 4 + j3\ \Omega$ and $\mathbf{Z}'_{line} = 0.0018 + j0.0024\ \Omega$. The resulting equivalent circuit is shown in Figure 2–7*b*. The generator's current is

$$\mathbf{I}_G = \frac{480\angle 0°\ \text{V}}{5.003\angle 36.88°\ \Omega} = 95.94\angle -36.88°\ \text{A}$$

Knowing the current $\mathbf{I}_G$, we can now work back and find $\mathbf{I}_{line}$ and $\mathbf{I}_{load}$. Working back through T_1, we get

$$N_{P1}\mathbf{I}_G = N_{S1}\mathbf{I}_{\text{line}}$$

$$\mathbf{I}_{\text{line}} = \frac{N_{P1}}{N_{S1}}\mathbf{I}_G$$

$$= \frac{1}{10}(95.94\angle -36.88^\circ\ \text{A}) = 9.594\angle -36.88^\circ\ \text{A}$$

Working back through T_2 gives

$$N_{P2}\mathbf{I}_{\text{line}} = N_{S2}\mathbf{I}_{\text{load}}$$

$$\mathbf{I}_{\text{load}} = \frac{N_{P2}}{N_{S2}}\mathbf{I}_{\text{line}}$$

$$= \frac{10}{1}(9.594\angle -36.88^\circ\ \text{A}) = 95.94\angle -36.88^\circ\ \text{A}$$

It is now possible to answer the questions originally asked. The load voltage is given by

$$\mathbf{V}_{\text{load}} = \mathbf{I}_{\text{load}}Z_{\text{load}}$$

$$= (95.94\angle -36.88^\circ\ \text{A})(5\angle 36.87^\circ\ \Omega)$$

$$= 479.7\angle -0.01^\circ\ \text{V}$$

and the line losses are given by

$$P_{\text{loss}} = (I_{\text{line}})^2R_{\text{line}}$$

$$= (9.594\ \text{A})^2\,(0.18\ \Omega) = 16.7\ \text{W}$$

Notice that raising the transmission voltage of the power system reduced transmission losses by a factor of nearly 90! Also, the voltage at the load dropped much less in the system with transformers compared to the system without transformers. This simple example dramatically illustrates the advantages of using higher-voltage transmission lines as well as the extreme importance of transformers in modern power systems.

2.4 THEORY OF OPERATION OF REAL SINGLE-PHASE TRANSFORMERS

The ideal transformers described in Section 2.3 can of course never actually be made. What can be produced are real transformers—two or more coils of wire physically wrapped around a ferromagnetic core. The characteristics of a real transformer approximate the characteristics of an ideal transformer, but only to a degree. This section deals with the behavior of real transformers.

To understand the operation of a real transformer, refer to Figure 2–8. Figure 2–8 shows a transformer consisting of two coils of wire wrapped around a transformer core. The primary of the transformer is connected to an ac power source, and the secondary winding is open-circuited. The hysteresis curve of the transformer is shown in Figure 2–9.

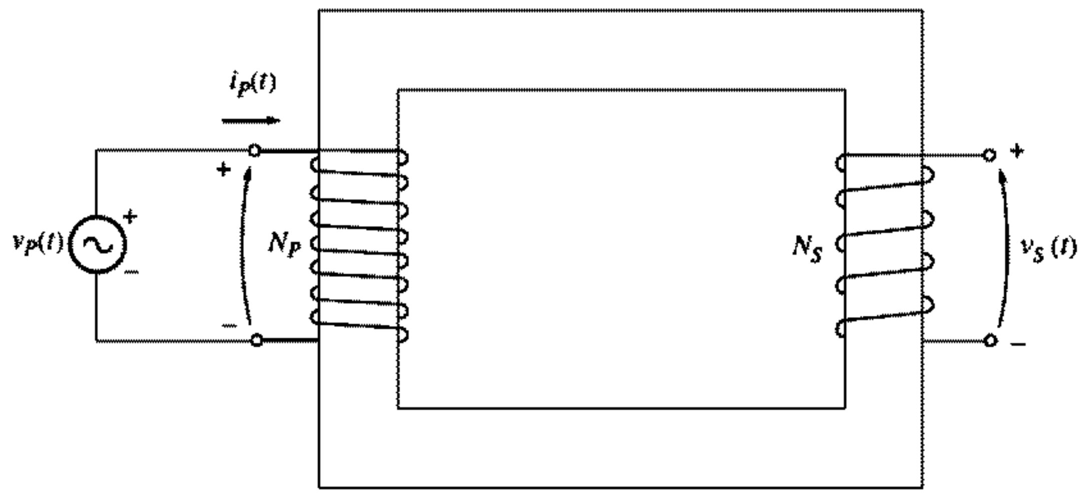


FIGURE 2–8
Sketch of a real transformer with no load attached to its secondary.

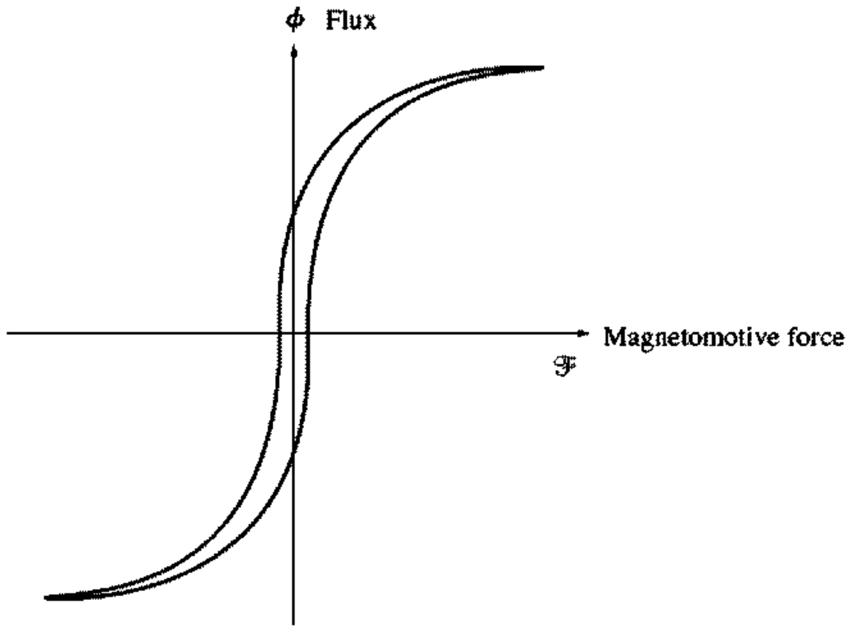


FIGURE 2–9
The hysteresis curve of the transformer.

The basis of transformer operation can be derived from Faraday's law:

$$e_{\text{ind}} = \frac{d\lambda}{dt} \tag{1–41}$$

where λ is the flux linkage in the coil across which the voltage is being induced. The flux linkage λ is the sum of the flux passing through each turn in the coil added over all the turns of the coil:

$$\lambda = \sum_{i=1}^{N} \phi_i \tag{1–42}$$

The total flux linkage through a coil is not just $N\phi$, where N is the number of turns in the coil, because the flux passing through each turn of a coil is slightly different from the flux in the other turns, depending on the position of the turn within the coil.

However, it is possible to define an *average* flux per turn in a coil. If the total flux linkage in all the turns of the coils is λ and if there are N turns, then the *average flux per turn* is given by

$$\bar{\phi} = \frac{\lambda}{N} \tag{2–16}$$

and Faraday's law can be written as

$$e_{\text{ind}} = N\frac{d\bar{\phi}}{dt} \tag{2–17}$$

The Voltage Ratio across a Transformer

If the voltage of the source in Figure 2–8 is $v_P(t)$, then that voltage is placed directly across the coils of the primary winding of the transformer. How will the transformer react to this applied voltage? Faraday's law explains what will happen. When Equation (2–17) is solved for the average flux present in the primary winding of the transformer, the result is

$$\bar{\phi} = \frac{1}{N_P}\int v_P(t)\, dt \tag{2–18}$$

This equation states that the average flux in the winding is proportional to the integral of the voltage applied to the winding, and the constant of proportionality is the reciprocal of the number of turns in the primary winding $1/N_P$.

This flux is present in the *primary coil* of the transformer. What effect does it have on the secondary coil of the transformer? The effect depends on how much of the flux reaches the secondary coil. Not all the flux produced in the primary coil also passes through the secondary coil—some of the flux lines leave the iron core and pass through the air instead (see Figure 2–10). The portion of the flux that goes through one of the transformer coils but not the other one is called *leakage flux*. The flux in the primary coil of the transformer can thus be divided into two components: a *mutual flux*, which remains in the core and links both windings, and a small *leakage flux*, which passes through the primary winding but returns through the air, bypassing the secondary winding:

$$\boxed{\bar{\phi}_P = \phi_M + \phi_{\text{LP}}} \tag{2–19}$$

where $\bar{\phi}_P$ = total average primary flux

ϕ_M = flux component linking both primary and secondary coils

ϕ_{LP} = primary leakage flux

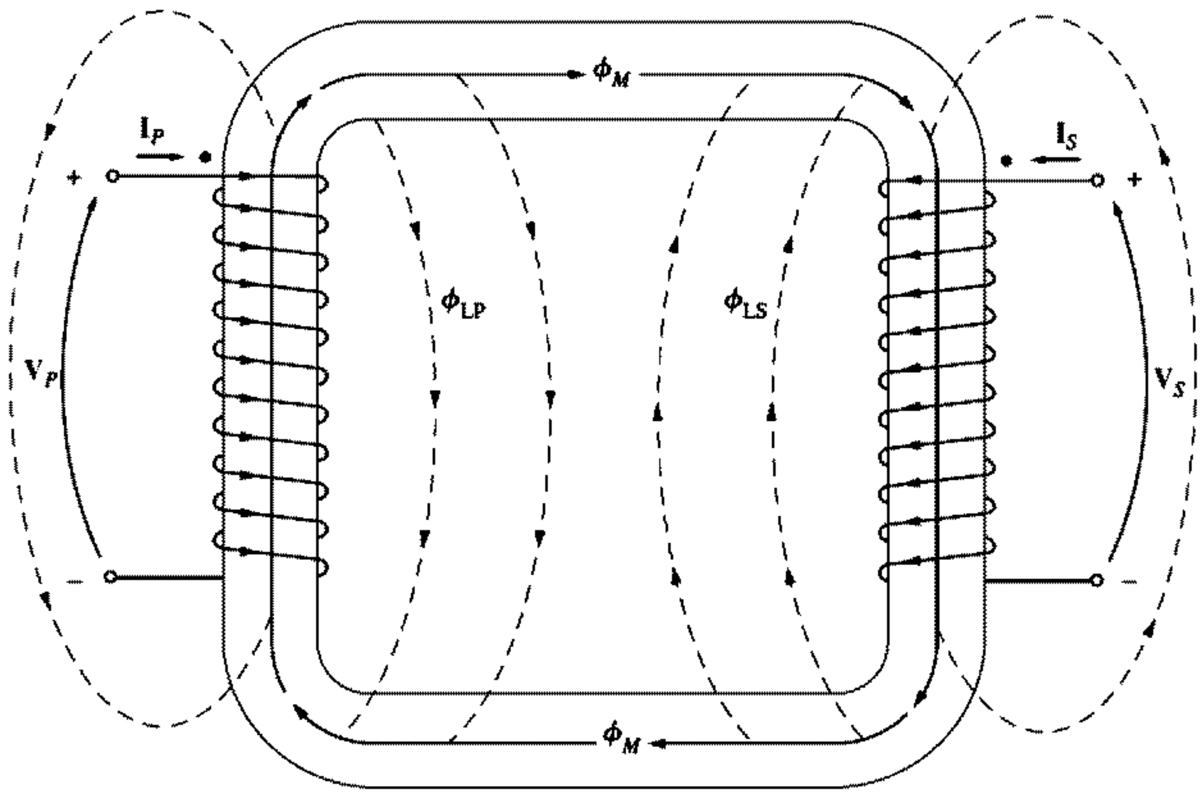


FIGURE 2–10
Mutual and leakage fluxes in a transformer core.

There is a similar division of flux in the secondary winding between mutual flux and leakage flux which passes through the secondary winding but returns through the air, bypassing the primary winding:

$$\boxed{\bar{\phi}_S = \phi_M + \phi_{LS}} \tag{2–20}$$

where $\bar{\phi}_S$ = total average secondary flux
ϕ_M = flux component linking both primary and secondary coils
ϕ_{LS} = secondary leakage flux

With the division of the average primary flux into mutual and leakage components, Faraday's law for the primary circuit can be reexpressed as

$$v_P(t) = N_P \frac{d\bar{\phi}_P}{dt}$$

$$= N_P \frac{d\phi_M}{dt} + N_P \frac{d\phi_{LP}}{dt} \tag{2–21}$$

The first term of this expression can be called $e_P(t)$, and the second term can be called $e_{LP}(t)$. If this is done, then Equation (2–21) can be rewritten as

$$v_P(t) = e_P(t) + e_{LP}(t) \tag{2–22}$$

The voltage on the secondary coil of the transformer can also be expressed in terms of Faraday's law as

$$v_S(t) = N_S \frac{d\overline{\phi}_S}{dt}$$

$$= N_S \frac{d\phi_M}{dt} + N_S \frac{d\phi_{LS}}{dt} \tag{2-23}$$

$$= e_S(t) + e_{LS}(t) \tag{2-24}$$

The primary voltage *due to the mutual flux* is given by

$$e_P(t) = N_P \frac{d\phi_M}{dt} \tag{2-25}$$

and the secondary voltage *due to the mutual flux* is given by

$$e_S(t) = N_S \frac{d\phi_M}{dt} \tag{2-26}$$

Notice from these two relationships that

$$\frac{e_P(t)}{N_P} = \frac{d\phi_M}{dt} = \frac{e_S(t)}{N_S}$$

Therefore,

$$\boxed{\frac{e_P(t)}{e_S(t)} = \frac{N_P}{N_S} = a} \tag{2-27}$$

This equation means that *the ratio of the primary voltage caused by the mutual flux to the secondary voltage caused by the mutual flux is equal to the turns ratio of the transformer.* Since in a well-designed transformer $\phi_M >> \phi_{LP}$ and $\phi_M >> \phi_{LS}$, the ratio of the total voltage on the primary of a transformer to the total voltage on the secondary of a transformer is approximately

$$\frac{v_P(t)}{v_S(t)} = \frac{N_P}{N_S} = a \tag{2-28}$$

The smaller the leakage fluxes of the transformer are, the closer the total transformer voltage ratio approximates that of the ideal transformer discussed in Section 2.3.

The Magnetization Current in a Real Transformer

When an ac power source is connected to a transformer as shown in Figure 2–8, a current flows in its primary circuit, *even when the secondary circuit is open-circuited.* This current is the current required to produce flux in a real ferromagnetic core, as explained in Chapter 1. It consists of two components:

1. The *magnetization current* i_M, which is the current required to produce the flux in the transformer core
2. The *core-loss current* i_{h+e}, which is the current required to make up for hysteresis and eddy current losses

Figure 2–11 shows the magnetization curve of a typical transformer core. If the flux in the transformer core is known, then the magnitude of the magnetization current can be found directly from Figure 2–11.

Ignoring for the moment the effects of leakage flux, we see that the average flux in the core is given by

$$\bar{\phi} = \frac{1}{N_P}\int v_P(t)\,dt \tag{2–18}$$

If the primary voltage is given by the expression $v_P(t) = V_M \cos \omega t$ V, then the resulting flux must be

$$\begin{aligned}\bar{\phi} &= \frac{1}{N_P}\int V_M \cos \omega t \, dt \\ &= \frac{V_M}{\omega N_P}\sin \omega t \quad \text{Wb}\end{aligned} \tag{2–29}$$

If the values of current required to produce a given flux (Figure 2–11a) are compared to the flux in the core at different times, it is possible to construct a sketch of the magnetization current in the winding on the core. Such a sketch is shown in Figure 2–11b. Notice the following points about the magnetization current:

1. The magnetization current in the transformer is not sinusoidal. The higher-frequency components in the magnetization current are due to magnetic saturation in the transformer core.
2. Once the peak flux reaches the saturation point in the core, a small increase in peak flux requires a very large increase in the peak magnetization current.
3. The fundamental component of the magnetization current lags the voltage applied to the core by 90°.
4. The higher-frequency components in the magnetization current can be quite large compared to the fundamental component. In general, the further a transformer core is driven into saturation, the larger the harmonic components will become.

The other component of the no-load current in the transformer is the current required to supply power to make up the hysteresis and eddy current losses in the core. This is the core-loss current. Assume that the flux in the core is sinusoidal. Since the eddy currents in the core are proportional to $d\phi/dt$, the eddy currents are largest when the flux in the core is passing through 0 Wb. Therefore, the core-loss current is greatest as the flux passes through zero. The total current required to make up for core losses is shown in Figure 2–12.

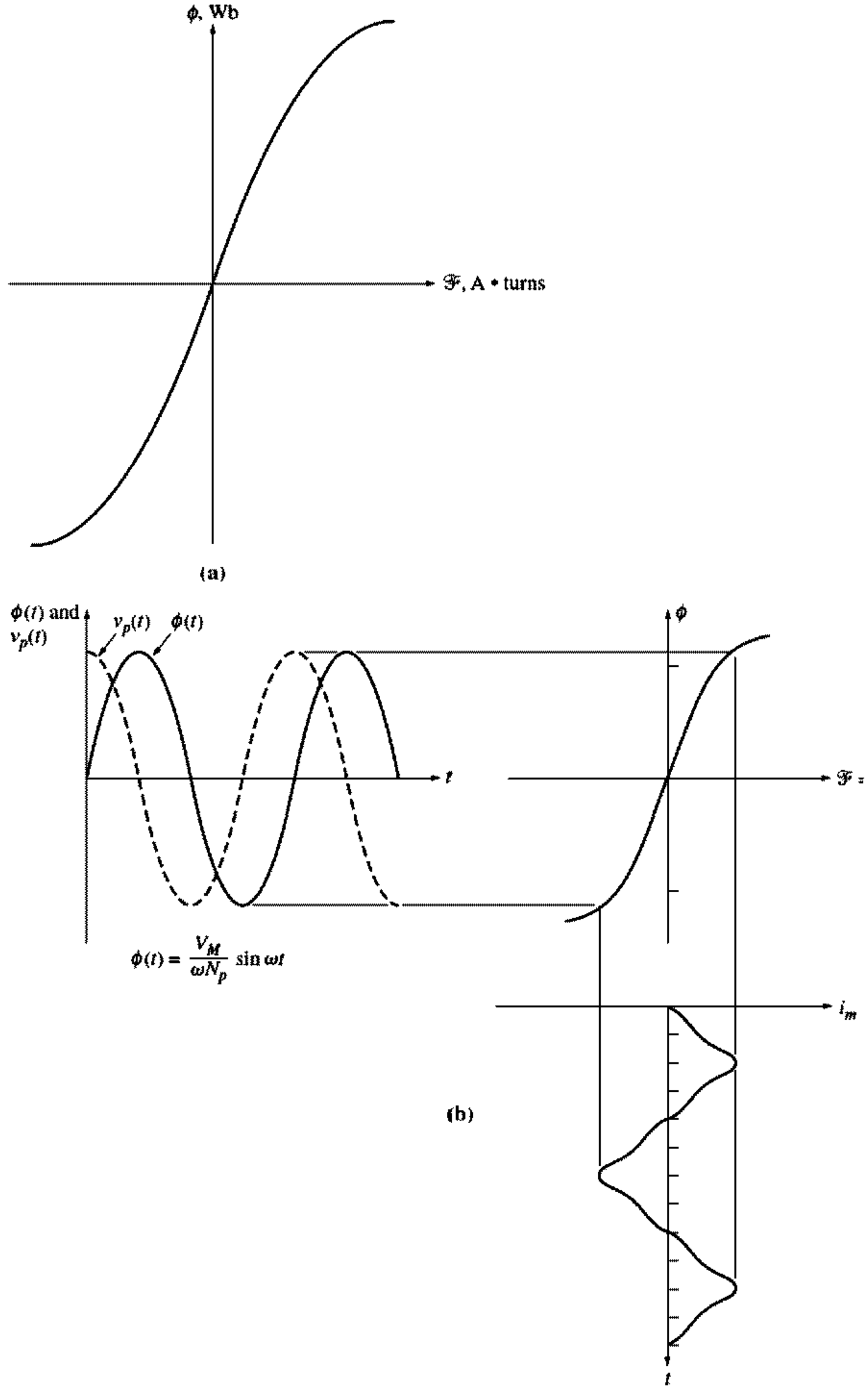


FIGURE 2–11
(a) The magnetization curve of the transformer core. (b) The magnetization current caused by the flux in the transformer core.

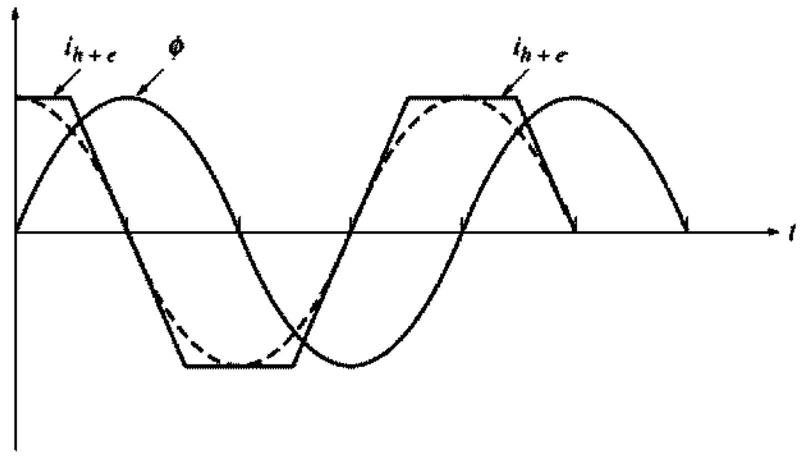


FIGURE 2–12
The core-loss current in a transformer.

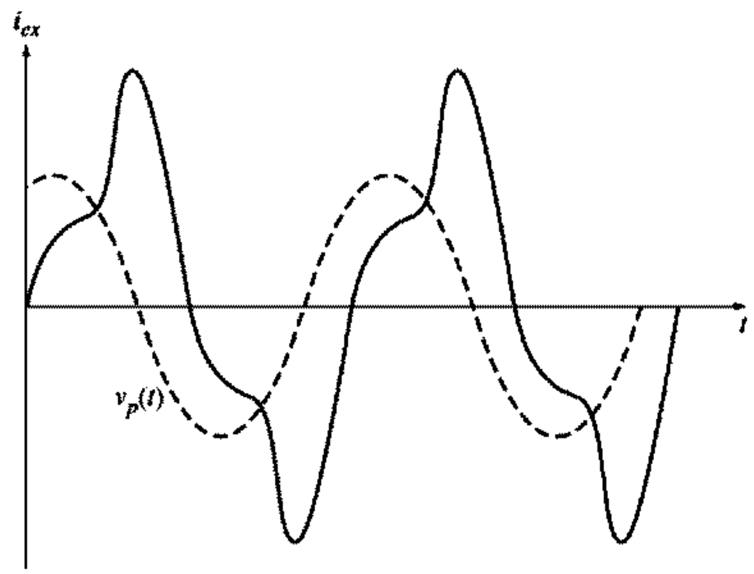


FIGURE 2–13
The total excitation current in a transformer.

Notice the following points about the core-loss current:

1. The core-loss current is nonlinear because of the nonlinear effects of hysteresis.
2. The fundamental component of the core-loss current is in phase with the voltage applied to the core.

The total no-load current in the core is called the *excitation current* of the transformer. It is just the sum of the magnetization current and the core-loss current in the core:

$$i_{ex} = i_m + i_{h+e} \tag{2–30}$$

The total excitation current in a typical transformer core is shown in Figure 2–13.

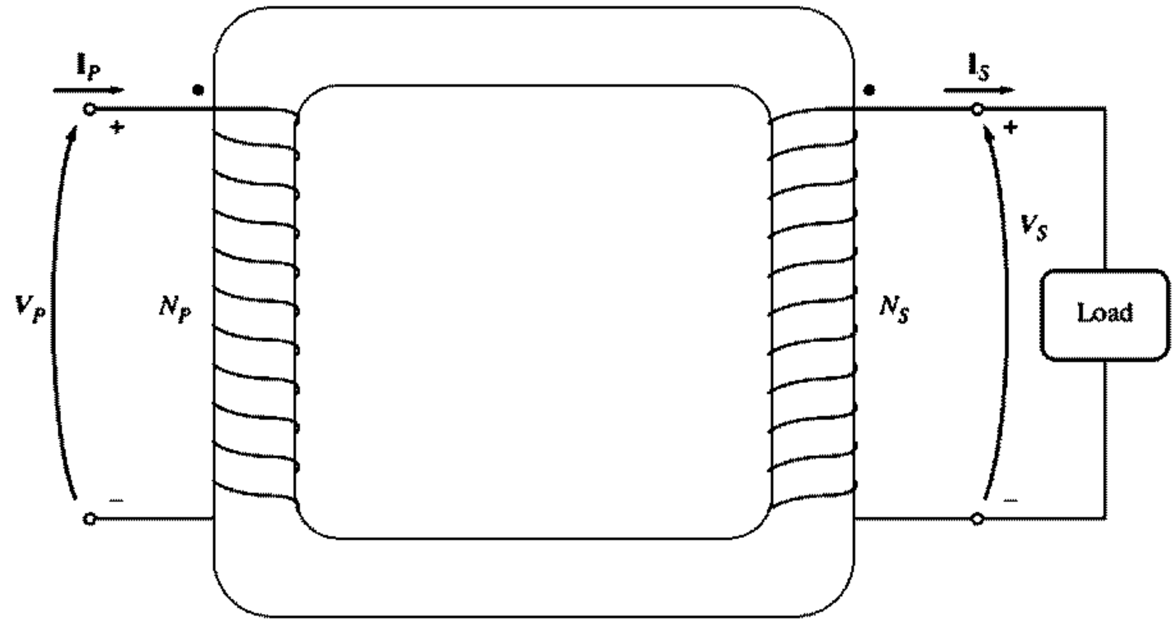


FIGURE 2–14
A real transformer with a load connected to its secondary.

The Current Ratio on a Transformer and the Dot Convention

Now suppose that a load is connected to the secondary of the transformer. The resulting circuit is shown in Figure 2–14. Notice the dots on the windings of the transformer. As in the ideal transformer previously described, the dots help determine the polarity of the voltages and currents in the core without having physically to examine its windings. The physical significance of the dot convention is that *a current flowing into the dotted end of a winding produces a positive magnetomotive force* $\mathcal{F}$, while a current flowing into the undotted end of a winding produces a negative magnetomotive force. Therefore, two currents flowing into the dotted ends of their respective windings produce magnetomotive forces that add. If one current flows into a dotted end of a winding and one flows out of a dotted end, then the magnetomotive forces will subtract from each other.

In the situation shown in Figure 2–14, the primary current produces a positive magnetomotive force $\mathcal{F}_P = N_P i_P$, and the secondary current produces a negative magnetomotive force $\mathcal{F}_S = -N_S i_S$. Therefore, the net magnetomotive force on the core must be

$$\mathcal{F}_{\text{net}} = N_P i_P - N_S i_S \tag{2–31}$$

This net magnetomotive force must produce the net flux in the core, so the net magnetomotive force must be equal to

$$\boxed{\mathcal{F}_{\text{net}} = N_P i_P - N_S i_S = \phi \mathcal{R}} \tag{2–32}$$

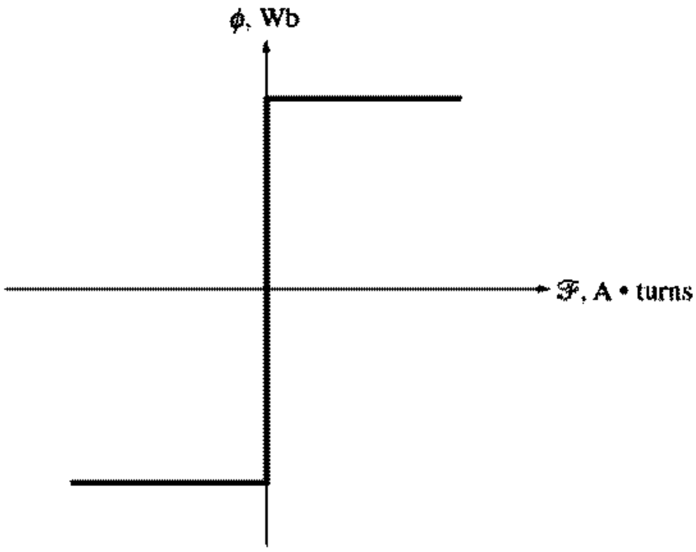


FIGURE 2–15
The magnetization curve of an ideal transformer.

where $\mathcal{R}$ is the reluctance of the transformer core. Because the reluctance of a well-designed transformer core is very small (nearly zero) until the core is saturated, the relationship between the primary and secondary currents is approximately

$$\mathcal{F}_{\text{net}} = N_P i_P - N_S i_S \approx 0 \tag{2–33}$$

as long as the core is unsaturated. Therefore,

$$\boxed{N_P i_P \approx N_S i_S} \tag{2–34}$$

or

$$\boxed{\frac{i_P}{i_S} \approx \frac{N_S}{N_P} = \frac{1}{a}} \tag{2–35}$$

It is the fact that the magnetomotive force in the core is nearly zero which gives the dot convention the meaning in Section 2.3. In order for the magnetomotive force to be nearly zero, current must flow *into one dotted end* and *out of the other dotted end*. The voltages must be built up in the same way with respect to the dots on each winding in order to drive the currents in the direction required. (The polarity of the voltages can also be determined by Lenz' law if the construction of the transformer coils is visible.)

What assumptions are required to convert a real transformer into the ideal transformer described previously? They are as follows:

1. The core must have no hysteresis or eddy currents.
2. The magnetization curve must have the shape shown in Figure 2–15. Notice that for an unsaturated core the net magnetomotive force $\mathcal{F}_{\text{net}} = 0$, implying that $N_P i_P = N_S i_S$.
3. The leakage flux in the core must be zero, implying that all the flux in the core couples both windings.
4. The resistance of the transformer windings must be zero.

While these conditions are never exactly met, well-designed power transformers can come quite close.

2.5 THE EQUIVALENT CIRCUIT OF A TRANSFORMER

The losses that occur in real transformers have to be accounted for in any accurate model of transformer behavior. The major items to be considered in the construction of such a model are

1. *Copper (I^2R) losses.* Copper losses are the resistive heating losses *in the primary and secondary windings* of the transformer. They are proportional to the square of the current in the windings.
2. *Eddy current losses.* Eddy current losses are resistive heating losses *in the core* of the transformer. They are proportional to the square of the voltage applied to the transformer.
3. *Hysteresis losses.* Hysteresis losses are associated with the rearrangement of the magnetic domains in the core during each half-cycle, as explained in Chapter 1. They are a complex, nonlinear function of the voltage applied to the transformer.
4. *Leakage flux.* The fluxes ϕ_{LP} and ϕ_{LS} which escape the core and pass through only one of the transformer windings are leakage fluxes. These escaped fluxes produce a *self-inductance* in the primary and secondary coils, and the effects of this inductance must be accounted for.

The Exact Equivalent Circuit of a Real Transformer

It is possible to construct an equivalent circuit that takes into account all the major imperfections in real transformers. Each major imperfection is considered in turn, and its effect is included in the transformer model.

The easiest effect to model is the copper losses. Copper losses are resistive losses in the primary and secondary windings of the transformer core. They are modeled by placing a resistor R_P in the primary circuit of the transformer and a resistor R_S in the secondary circuit.

As explained in Section 2.4, the leakage flux in the primary windings ϕ_{LP} produces a voltage ϕ_{LP} given by

$$e_{LP}(t) = N_P \frac{d\phi_{LP}}{dt} \qquad (2\text{–}36a)$$

and the leakage flux in the secondary windings ϕ_{LS} produces a voltage e_{LS} given by

$$e_{LS}(t) = N_S \frac{d\phi_{LS}}{dt} \qquad (2\text{–}36b)$$

Since much of the leakage flux path is through air, and since air has a constant reluctance much higher than the core reluctance, the flux ϕ_{LP} is directly proportional to the primary circuit current i_P and the flux ϕ_{LS} is directly proportional to the secondary current i_S:

$$\phi_{LP} = (\mathcal{P}N_P)i_P \tag{2–37a}$$

$$\phi_{LS} = (\mathcal{P}N_S)i_S \tag{2–37b}$$

where $\mathcal{P}$ = permeance of flux path
N_P = number of turns on primary coil
N_S = number of turns on secondary coil

Substitute Equations (2–37) into Equations (2–36). The result is

$$e_{LP}(t) = N_P \frac{d}{dt}(\mathcal{P}N_P)i_P = N_P^2\mathcal{P}\frac{di_P}{dt} \tag{2–38a}$$

$$e_{LS}(t) = N_S \frac{d}{dt}(\mathcal{P}N_S)i_S = N_S^2\mathcal{P}\frac{di_S}{dt} \tag{2–38b}$$

The constants in these equations can be lumped together. Then

$$\boxed{e_{LP}(t) = L_P\frac{di_P}{dt}} \tag{2–39a}$$

$$\boxed{e_{LS}(t) = L_S\frac{di_S}{dt}} \tag{2–39b}$$

where $L_P = N_P^2\mathcal{P}$ is the self-inductance of the primary coil and $L_S = N_S^2\mathcal{P}$ is the self-inductance of the secondary coil. Therefore, the leakage flux will be modeled by primary and secondary inductors.

How can the core excitation effects be modeled? The magnetization current i_m is a current proportional (in the unsaturated region) to the voltage applied to the core and *lagging the applied voltage by* 90°, so it can be modeled by a reactance X_M connected across the primary voltage source. The core-loss current i_{h+e} is a current proportional to the voltage applied to the core that is *in phase with the applied voltage*, so it can be modeled by a resistance R_C connected across the primary voltage source. (Remember that both these currents are really nonlinear, so the inductance X_M and the resistance R_C are, at best, approximations of the real excitation effects.)

The resulting equivalent circuit is shown in Figure 2–16. Notice that the elements forming the excitation branch are placed inside the primary resistance R_P and the primary inductance L_P. This is because the voltage actually applied to the core is really equal to the input voltage less the internal voltage drops of the winding.

Although Figure 2–16 is an accurate model of a transformer, it is not a very useful one. To analyze practical circuits containing transformers, it is normally necessary to convert the entire circuit to an equivalent circuit at a single voltage level. (Such a conversion was done in Example 2–1.) Therefore, the equivalent

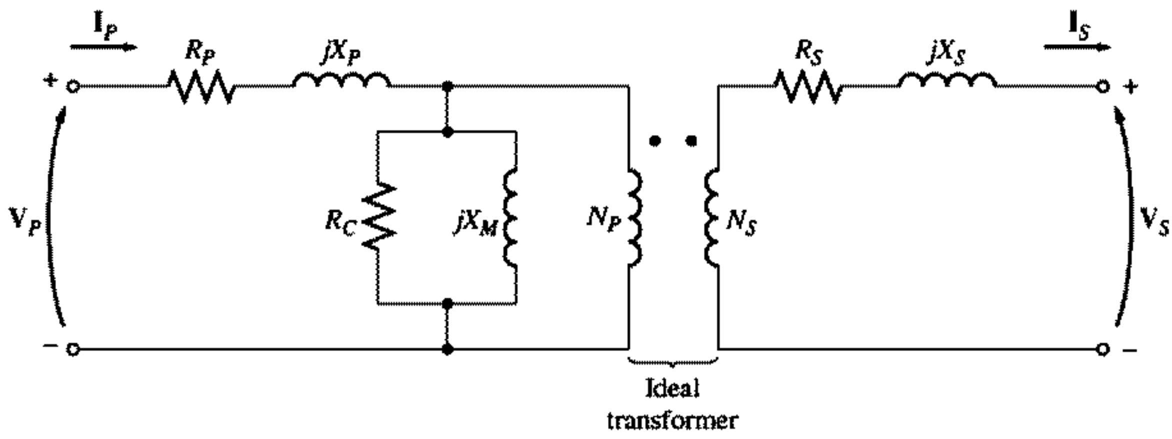


FIGURE 2–16
The model of a real transformer.

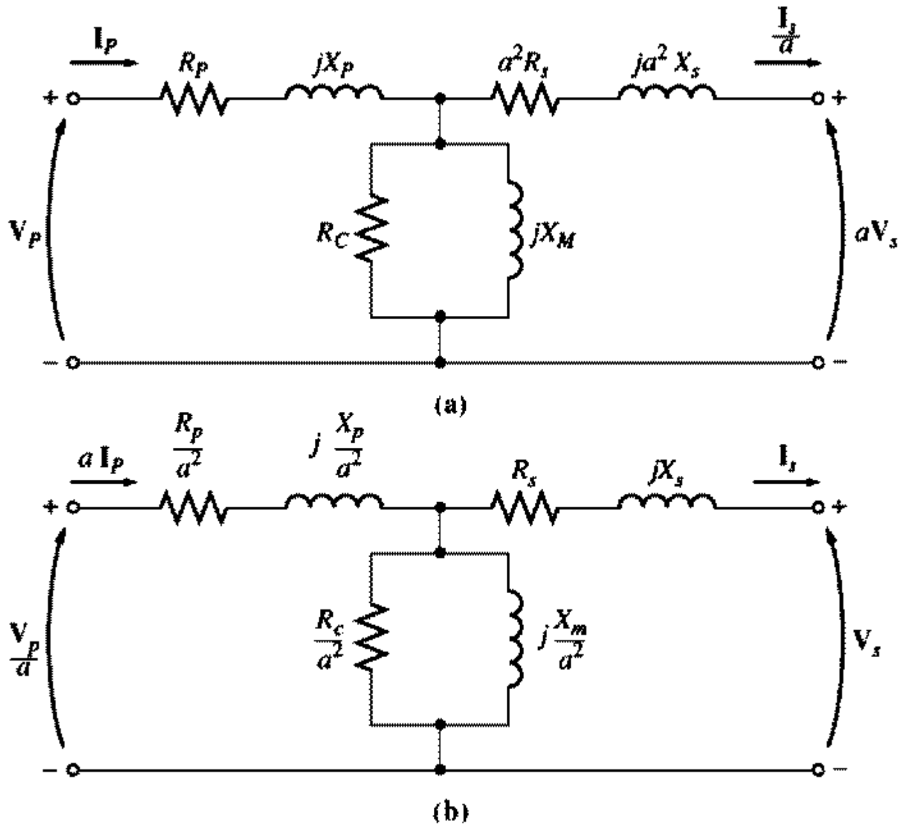


FIGURE 2–17
(a) The transformer model referred to its primary voltage level. (b) The transformer model referred to its secondary voltage level.

circuit must be referred either to its primary side or to its secondary side in problem solutions. Figure 2–17a is the equivalent circuit of the transformer referred to its primary side, and Figure 2–17b is the equivalent circuit referred to its secondary side.

Approximate Equivalent Circuits of a Transformer

The transformer models shown before are often more complex than necessary in order to get good results in practical engineering applications. One of the principal complaints about them is that the excitation branch of the model adds another node to the circuit being analyzed, making the circuit solution more complex than necessary. The excitation branch has a very small current compared to the load current of the transformers. In fact, it is so small that under normal circumstances it causes a completely negligible voltage drop in R_P and X_P. Because this is true, a simplified equivalent circuit can be produced that works almost as well as the original model. The excitation branch is simply moved to the front of the transformer, and the primary and secondary impedances are left in series with each other. These impedances are just added, creating the approximate equivalent circuits in Figure 2–18a and b.

In some applications, the excitation branch may be neglected entirely without causing serious error. In these cases, the equivalent circuit of the transformer reduces to the simple circuits in Figure 2–18c and d.

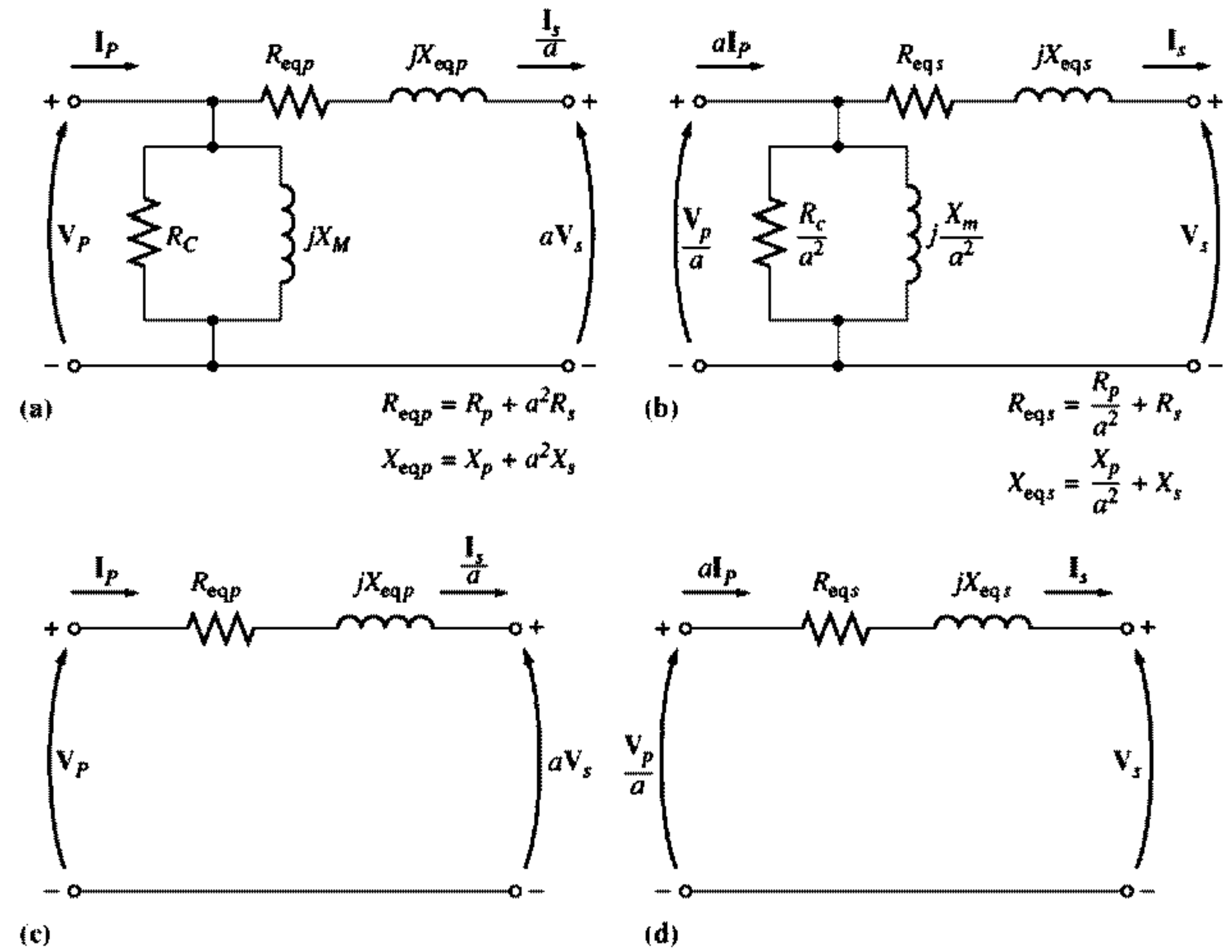


FIGURE 2–18
Approximate transformer models. (a) Referred to the primary side; (b) referred to the secondary side; (c) with no excitation branch, referred to the primary side; (d) with no excitation branch, referred to the secondary side.

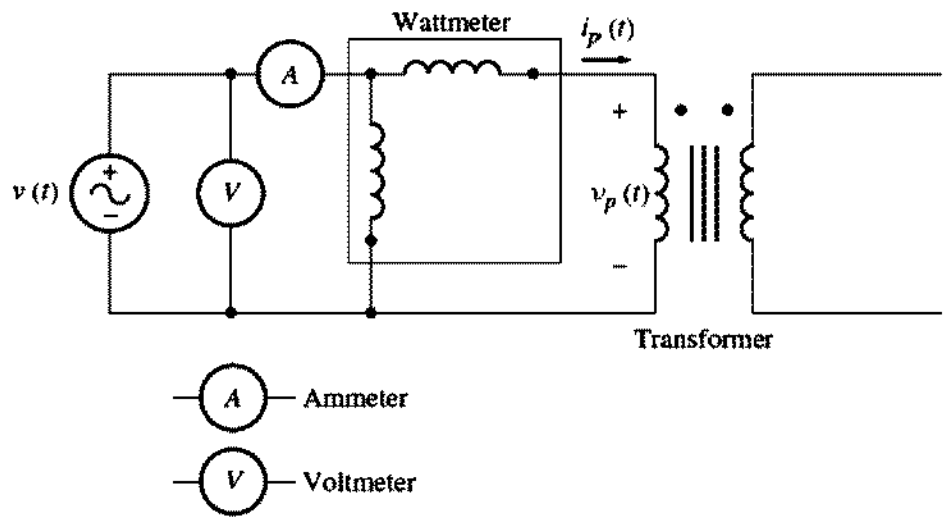


FIGURE 2–19
Connection for transformer open-circuit test.

Determining the Values of Components in the Transformer Model

It is possible to experimentally determine the values of the inductances and resistances in the transformer model. An adequate approximation of these values can be obtained with only two tests, the open-circuit test and the short-circuit test.

In the *open-circuit test,* a transformer's secondary winding is open-circuited, and its primary winding is connected to a full-rated line voltage. Look at the equivalent circuit in Figure 2–17. Under the conditions described, all the input current must be flowing through the excitation branch of the transformer. The series elements R_P and X_P are too small in comparison to R_C and X_M to cause a significant voltage drop, so essentially all the input voltage is dropped across the excitation branch.

The open-circuit test connections are shown in Figure 2–19. Full line voltage is applied to the primary of the transformer, and the input voltage, input current, and input power to the transformer are measured. From this information, it is possible to determine the power factor of the input current and therefore both the *magnitude* and the *angle* of the excitation impedance.

The easiest way to calculate the values of R_C and X_M is to look first at the *admittance* of the excitation branch. The conductance of the core-loss resistor is given by

$$G_C = \frac{1}{R_C} \tag{2–40}$$

and the susceptance of the magnetizing inductor is given by

$$B_M = \frac{1}{X_M} \tag{2–41}$$

Since these two elements are in parallel, their admittances add, and the total excitation admittance is

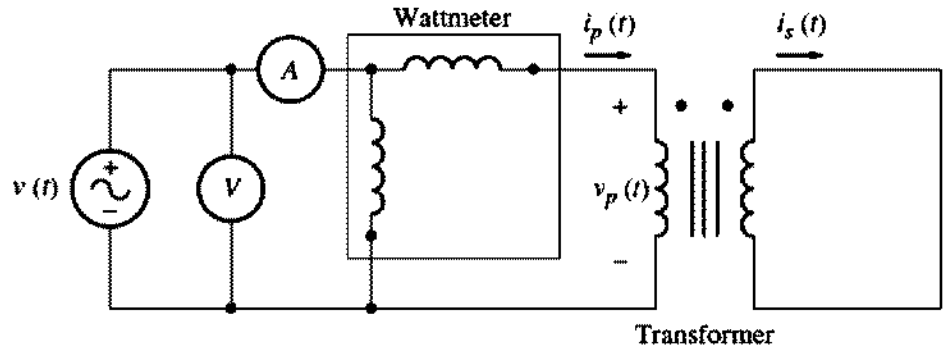


FIGURE 2–20
Connection for transformer short-circuit test.

$$Y_E = G_C - jB_M \tag{2–42}$$

$$= \frac{1}{R_C} - j\frac{1}{X_M} \tag{2–43}$$

The *magnitude* of the excitation admittance (referred to the primary circuit) can be found from the open-circuit test voltage and current:

$$|Y_E| = \frac{I_{OC}}{V_{OC}} \tag{2–44}$$

The *angle* of the admittance can be found from a knowledge of the circuit power factor. The open-circuit power factor (PF) is given by

$$\text{PF} = \cos\theta = \frac{P_{OC}}{V_{OC}I_{OC}} \tag{2–45}$$

and the power-factor angle θ is given by

$$\theta = \cos^{-1}\frac{P_{OC}}{V_{OC}I_{OC}} \tag{2–46}$$

The power factor is always lagging for a real transformer, so the angle of the current always lags the angle of the voltage by θ degrees. Therefore, the admittance Y_E is

$$Y_E = \frac{I_{OC}}{V_{OC}}\angle-\theta$$

$$= \frac{I_{OC}}{V_{OC}}\angle-\cos^{-1}\text{PF} \tag{2–47}$$

By comparing Equations (2–43) and (2–47), it is possible to determine the values of R_C and X_M directly from the open-circuit test data.

In the *short-circuit test,* the secondary terminals of the transformer are short-circuited, and the primary terminals are connected to a fairly low-voltage source, as shown in Figure 2–20. The input voltage is adjusted until the current in the short-circuited windings is equal to its rated value. (Be sure to keep the primary voltage at a safe level. It would not be a good idea to burn out the transformer's windings while trying to test it.) The input voltage, current, and power are again measured.

Since the input voltage is so low during the short-circuit test, negligible current flows through the excitation branch. If the excitation current is ignored, then all the voltage drop in the transformer can be attributed to the series elements in the circuit. The magnitude of the series impedances referred to the primary side of the transformer is

$$|Z_{SE}| = \frac{V_{SC}}{I_{SC}} \tag{2–48}$$

The power factor of the current is given by

$$PF = \cos\theta = \frac{P_{SC}}{V_{SC} I_{SC}} \tag{2–49}$$

and is lagging. The current angle is thus negative, and the overall impedance angle θ is positive:

$$\theta = \cos^{-1} \frac{P_{SC}}{V_{SC} I_{SC}} \tag{2–50}$$

Therefore,

$$Z_{SE} = \frac{V_{SC}\angle 0^\circ}{I_{SC}\angle -\theta^\circ} = \frac{V_{SC}}{I_{SC}}\angle \theta^\circ \tag{2–51}$$

The series impedance Z_{SE} is equal to

$$\begin{aligned} Z_{SE} &= R_{eq} + jX_{eq} \\ &= (R_P + a^2 R_S) + j(X_P + a^2 X_S) \end{aligned} \tag{2–52}$$

It is possible to determine the total series impedance referred to the primary side by using this technique, but there is no easy way to split the series impedance into primary and secondary components. Fortunately, such separation is not necessary to solve normal problems.

These same tests may also be performed on the *secondary* side of the transformer if it is more convenient to do so because of voltage levels or other reasons. If the tests are performed on the secondary side, the results will naturally yield the equivalent circuit impedances referred to the secondary side of the transformer instead of to the primary side.

Example 2–2. The equivalent circuit impedances of a 20-kVA, 8000/240-V, 60-Hz transformer are to be determined. The open-circuit test and the short-circuit test were performed on the primary side of the transformer, and the following data were taken:

Open-circuit test (on primary)	**Short-circuit test (on primary)**
$V_{OC} = 8000$ V	$V_{SC} = 489$ V
$I_{OC} = 0.214$ A	$I_{SC} = 2.5$ A
$V_{OC} = 400$ W	$P_{SC} = 240$ W

Find the impedances of the approximate equivalent circuit referred to the primary side, and sketch that circuit.

Solution

The power factor during the *open-circuit* test is

$$\mathrm{PF} = \cos\theta = \frac{P_{OC}}{V_{OC} I_{OC}} \tag{2-45}$$

$$= \cos\theta = \frac{400 \text{ W}}{(8000 \text{ V})(0.214 \text{ A})}$$

$$= 0.234 \text{ lagging}$$

The excitation admittance is given by

$$Y_E = \frac{I_{OC}}{V_{OC}} \angle -\cos^{-1} \mathrm{PF} \tag{2-47}$$

$$= \frac{0.214 \text{ A}}{8000 \text{ V}} \angle -\cos^{-1} 0.234$$

$$= 0.0000268 \angle -76.5^\circ \ \Omega$$

$$= 0.0000063 - j0.0000261 = \frac{1}{R_C} - j\frac{1}{X_M}$$

Therefore,

$$R_C = \frac{1}{0.0000063} = 159 \text{ k}\Omega$$

$$X_M = \frac{1}{0.0000261} = 38.4 \text{ k}\Omega$$

The power factor during the *short-circuit* test is

$$\mathrm{PF} = \cos\theta = \frac{P_{SC}}{V_{SC} I_{SC}} \tag{2-49}$$

$$= \cos\theta = \frac{240 \text{ W}}{(489 \text{ V})(2.5 \text{ A})} = 0.196 \text{ lagging}$$

The series impedance is given by

$$Z_{SE} = \frac{V_{SC}}{I_{SC}} \angle -\cos^{-1} \mathrm{PF}$$

$$= \frac{489 \text{ V}}{2.5 \text{ A}} \angle 78.7^\circ$$

$$= 195.6 \angle 78.7^\circ = 38.4 + j192 \ \Omega$$

Therefore, the equivalent resistance and reactance are

$$R_{eq} = 38.4 \ \Omega \qquad X_{eq} = 192 \ \Omega$$

The resulting simplified equivalent circuit is shown in Figure 2–21.

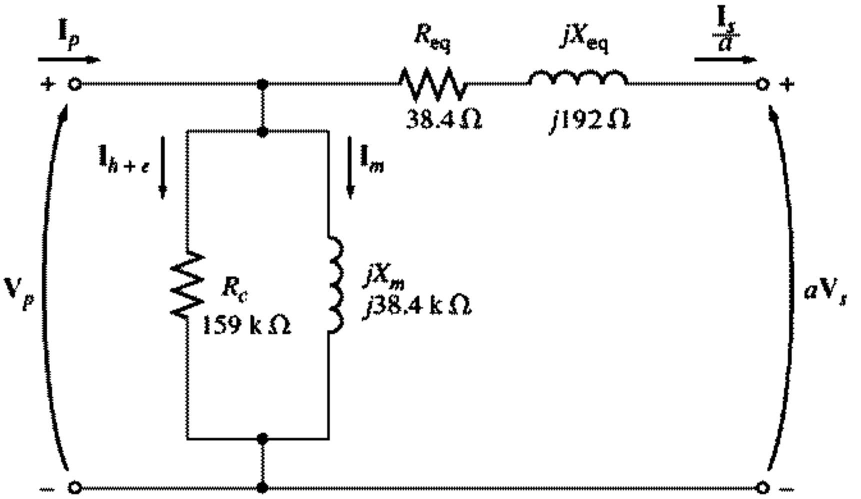


FIGURE 2–21
The equivalent circuit of Example 2–2.

2.6 THE PER-UNIT SYSTEM OF MEASUREMENTS

As the relatively simple Example 2–1 showed, solving circuits containing transformers can be quite a tedious operation because of the need to refer all the different voltage levels on different sides of the transformers in the system to a common level. Only after this step has been taken can the system be solved for its voltages and currents.

There is another approach to solving circuits containing transformers which eliminates the need for explicit voltage-level conversions at every transformer in the system. Instead, the required conversions are handled automatically by the method itself, without ever requiring the user to worry about impedance transformations. Because such impedance transformations can be avoided, circuits containing many transformers can be solved easily with less chance of error. This method of calculation is known as the *per-unit (pu) system* of measurements.

There is yet another advantage to the per-unit system that is quite significant for electric machinery and transformers. As the size of a machine or transformer varies, its internal impedances vary widely. Thus, a primary circuit reactance of 0.1 Ω might be an atrociously high number for one transformer and a ridiculously low number for another—it all depends on the device's voltage and power ratings. However, it turns out that in a per-unit system related to the device's ratings, *machine and transformer impedances fall within fairly narrow ranges* for each type and construction of device. This fact can serve as a useful check in problem solutions.

In the per-unit system, the voltages, currents, powers, impedances, and other electrical quantities are not measured in their usual SI units (volts, amperes, watts, ohms, etc.). Instead, *each electrical quantity is measured as a decimal fraction* of some base level. Any quantity can be expressed on a per-unit basis by the equation

$$\text{Quantity per unit} = \frac{\text{Actual value}}{\text{base value of quantity}} \tag{2–53}$$

where "actual value" is a value in volts, amperes, ohms, etc.

It is customary to select two base quantities to define a given per-unit system. The ones usually selected are voltage and power (or apparent power). Once these base quantities have been selected, all the other base values are related to them by the usual electrical laws. In a single-phase system, these relationships are

$$P_{\text{base}},\ Q_{\text{base}},\ \text{or}\ S_{\text{base}} = V_{\text{base}} I_{\text{base}} \tag{2–54}$$

$$Z_{\text{base}} = \frac{V_{\text{base}}}{I_{\text{base}}} \tag{2–55}$$

$$Y_{\text{base}} = \frac{I_{\text{base}}}{V_{\text{base}}} \tag{2–56}$$

and

$$Z_{\text{base}} = \frac{(V_{\text{base}})^2}{S_{\text{base}}} \tag{2–57}$$

Once the base values of S (or P) and V have been selected, all other base values can be computed easily from Equations (2–54) to (2–57).

In a power system, a base apparent power and voltage are selected *at a specific point in the system*. A transformer has no effect on the base apparent power of the system, since the apparent power into a transformer equals the apparent power out of the transformer [Equation (2–11)]. On the other hand, voltage changes when it goes through a transformer, so the value of V_{base} changes at every transformer in the system according to its turns ratio. Because the *base quantities* change in passing through a transformer, the process of referring quantities to a common voltage level is automatically taken care of during per-unit conversion.

Example 2–3. A simple power system is shown in Figure 2–22. This system contains a 480-V generator connected to an ideal 1:10 step-up transformer, a transmission line, an ideal 20:1 step-down transformer, and a load. The impedance of the transmission line is $20 + j60\ \Omega$, and the impedance of the load is $10\angle 30°\Omega$. The base values for this system are chosen to be 480 V and 10 kVA at the generator.

(a) Find the base voltage, current, impedance, and apparent power at every point in the power system.
(b) Convert this system to its per-unit equivalent circuit.
(c) Find the power supplied to the load in this system.
(d) Find the power lost in the transmission line.

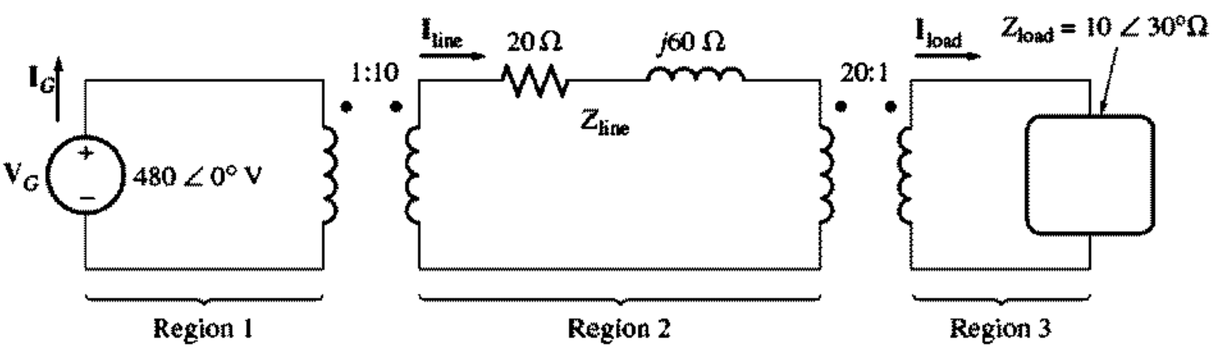


FIGURE 2–22
The power system of Example 2–3.

Solution

(a) *In the generator region*, $V_{base} = 480$ V and $S_{base} = 10$ kVA, so

$$I_{base\,1} = \frac{S_{base}}{V_{base\,1}} = \frac{10{,}000 \text{ VA}}{480 \text{ V}} = 20.83 \text{ A}$$

$$Z_{base\,1} = \frac{V_{base\,1}}{I_{base\,1}} = \frac{480 \text{ V}}{20.83 \text{ A}} = 23.04\ \Omega$$

The turns ratio of transformer T_1 is $a = 1/10 = 0.1$, so the base voltage *in the transmission line region* is

$$V_{base\,2} = \frac{V_{base\,1}}{a} = \frac{480 \text{ V}}{0.1} = 4800 \text{ V}$$

The other base quantities are

$$S_{base\,2} = 10 \text{ kVA}$$

$$I_{base\,2} = \frac{10{,}000 \text{ VA}}{4800 \text{ V}} = 2.083 \text{ A}$$

$$Z_{base\,2} = \frac{4800 \text{ V}}{2.083 \text{ A}} = 2304\ \Omega$$

The turns ratio of transformer T_2 is $a = 20/1 = 20$, so the base voltage *in the load region* is

$$V_{base\,3} = \frac{V_{base\,2}}{a} = \frac{4800 \text{ V}}{20} = 240 \text{ V}$$

The other base quantities are

$$S_{base\,3} = 10 \text{ kVA}$$

$$I_{base\,3} = \frac{10{,}000 \text{ VA}}{240 \text{ V}} = 41.67 \text{ A}$$

$$Z_{base\,3} = \frac{240 \text{ V}}{41.67 \text{ A}} = 5.76\ \Omega$$

(b) To convert a power system to a per-unit system, each component must be divided by its base value in its region of the system. The *generator's* per-unit voltage is its actual value divided by its base value:

$$V_{G,pu} = \frac{480\angle 0° \text{ V}}{480 \text{ V}} = 1.0\angle 0° \text{ pu}$$

The *transmission line's* per-unit impedance is its actual value divided by its base value:

$$Z_{line,pu} = \frac{20 + j60\ \Omega}{2304\ \Omega} = 0.0087 + j0.0260 \text{ pu}$$

The *load's* per-unit impedance is also given by actual value divided by base value:

$$Z_{load,pu} = \frac{10\angle 30°\ \Omega}{5.76\ \Omega} = 1.736\angle 30° \text{ pu}$$

The per-unit equivalent circuit of the power system is shown in Figure 2–23.

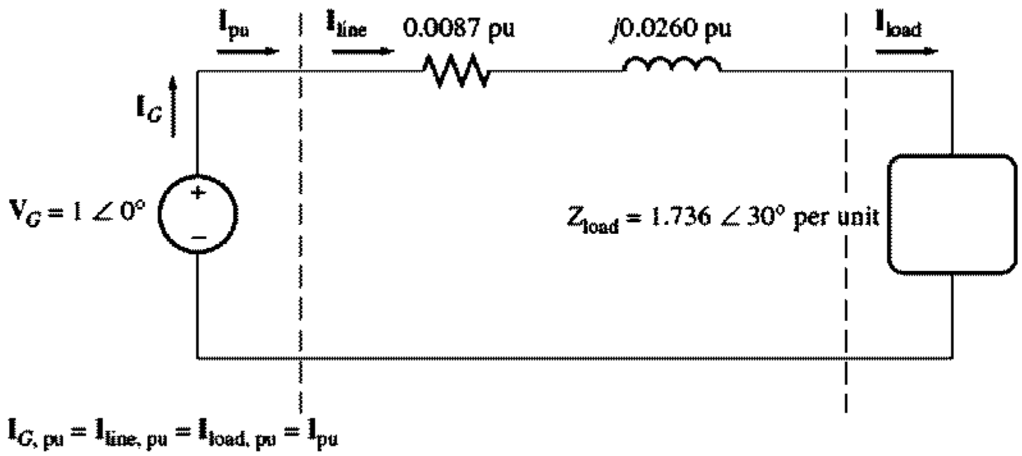


FIGURE 2–23
The per-unit equivalent circuit for Example 2–3.

(*c*) The current flowing in this per-unit power system is

$$\begin{aligned} \mathbf{I}_{pu} &= \frac{\mathbf{V}_{pu}}{Z_{tot,pu}} \\ &= \frac{1\angle 0°}{(0.0087 + j0.0260) + (1.736\angle 30°)} \\ &= \frac{1\angle 0°}{(0.0087 + j0.0260) + (1.503 + j0.868)} \\ &= \frac{1\angle 0°}{1.512 + j0.894} = \frac{1\angle 0°}{1.757\angle 30.6°} \\ &= 0.569\angle -30.6° \text{ pu} \end{aligned}$$

Therefore, the per-unit power of the load is

$$P_{load,pu} = I_{pu}^2 R_{pu} = (0.569)^2(1.503) = 0.487$$

and the actual power supplied to the load is

$$\begin{aligned} P_{load} &= P_{load,pu} S_{base} = (0.487)(10{,}000 \text{ VA}) \\ &= 4870 \text{ W} \end{aligned}$$

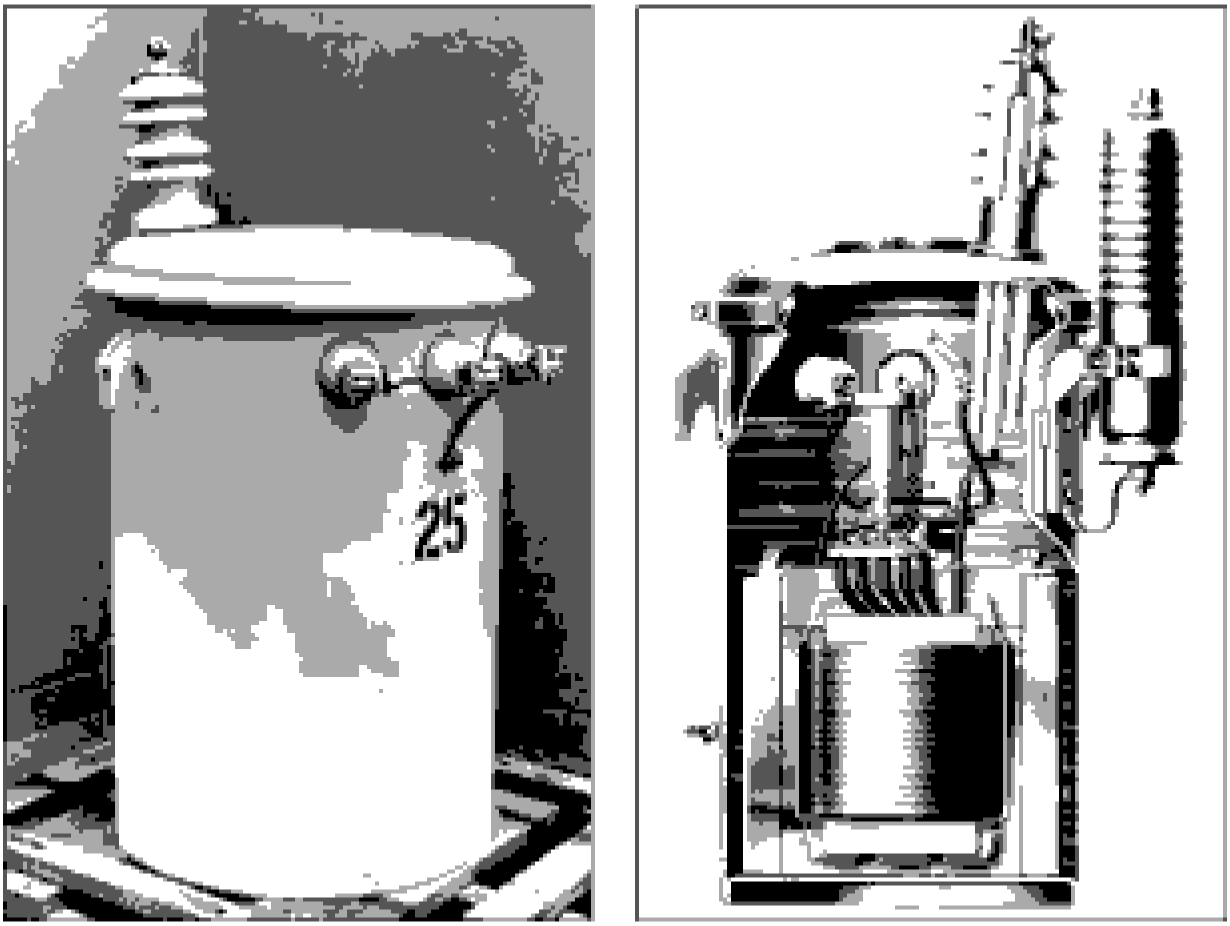
(*d*) The per-unit power lost in the transmission line is

$$P_{line,pu} = I_{pu}^2 R_{line,pu} = (0.569)^2(0.0087) = 0.00282$$

and the actual power lost in the transmission line is

$$\begin{aligned} P_{line} &= P_{line,pu} S_{base} = (0.00282)(10{,}000 \text{ VA}) \\ &= 28.2 \text{ W} \end{aligned}$$

When only one device (transformer or motor) is being analyzed, its own ratings are usually used as the base for the per-unit system. If a per-unit system based on the transformer's own ratings is used, a power or distribution transformer's characteristics will not vary much over a wide range of voltage and power ratings. For example, the series resistance of a transformer is usually about 0.01 per unit,



(a) (b)

FIGURE 2–24
(a) A typical 13.2-kV to 120/240-V distribution transformer. (*Courtesy of General Electric Company.*) (b) A cutaway view of the distribution transformer showing the shell-form transformer inside it. (*Courtesy of General Electric Company.*)

and the series reactance is usually between 0.02 and 0.10 per unit. In general, the larger the transformer, the smaller the series impedances. The magnetizing reactance is usually between about 10 and 40 per unit, while the core-loss resistance is usually between about 50 and 200 per unit. Because per-unit values provide a convenient and meaningful way to compare transformer characteristics when they are of different sizes, transformer impedances are normally given in per-unit or as a percentage on the transformer's nameplate (see Figure 2–46, later in this chapter).

The same idea applies to synchronous and induction machines as well: Their per-unit impedances fall within relatively narrow ranges over quite large size ranges.

If more than one machine and one transformer are included in a single power system, the system base voltage and power may be chosen arbitrarily, but the *entire system must have the same base*. One common procedure is to choose the system base quantities to be equal to the base of the largest component in the system. Per-unit values given to another base can be converted to the new base by converting them to their actual values (volts, amperes, ohms, etc.) as an in-between step. Alternatively, they can be converted directly by the equations

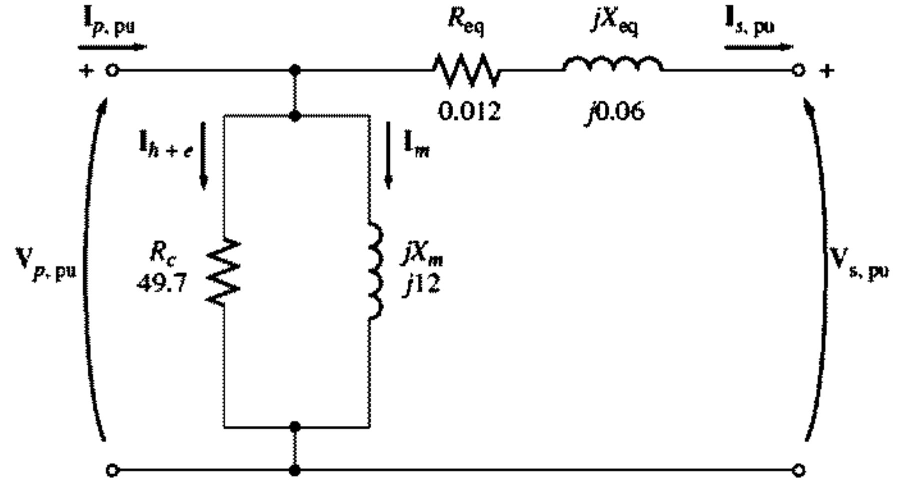


FIGURE 2–25
The per-unit equivalent circuit of Example 2–4.

$$(P, Q, S)_{\text{pu on base 2}} = (P, Q, S)_{\text{pu on base 1}} \frac{S_{\text{base 1}}}{S_{\text{base 2}}} \tag{2–58}$$

$$V_{\text{pu on base 2}} = V_{\text{pu on base 1}} \frac{V_{\text{base 1}}}{V_{\text{base 2}}} \tag{2–59}$$

$$(R, X, Z)_{\text{pu on base 2}} = (R, X, Z)_{\text{pu on base 1}} \frac{(V_{\text{base 1}})^2 (S_{\text{base 2}})}{(V_{\text{base 2}})^2 (S_{\text{base 1}})} \tag{2–60}$$

Example 2–4. Sketch the approximate per-unit equivalent circuit for the transformer in Example 2–2. Use the transformer's ratings as the system base.

Solution

The transformer in Example 2–2 is rated at 20 kVA, 8000/240 V. The approximate equivalent circuit (Figure 2–21) developed in the example was referred to the high-voltage side of the transformer, so to convert it to per-unit, the primary circuit base impedance must be found. On the primary,

$$V_{\text{base 1}} = 8000 \text{ V}$$

$$S_{\text{base 1}} = 20{,}000 \text{ VA}$$

$$Z_{\text{base 1}} = \frac{(V_{\text{base 1}})^2}{S_{\text{base 1}}} = \frac{(8000 \text{ V})^2}{20{,}000 \text{ VA}} = 3200 \ \Omega$$

Therefore,

$$Z_{SE,pu} = \frac{38.4 + j192 \ \Omega}{3200 \ \Omega} = 0.012 + j0.06 \text{ pu}$$

$$R_{C,pu} = \frac{159 \text{ k}\Omega}{3200 \ \Omega} = 49.7 \text{ pu}$$

$$Z_{M,pu} = \frac{38.4 \text{ k}\Omega}{3200 \ \Omega} = 12 \text{ pu}$$

The per-unit approximate equivalent circuit, expressed to the transformer's own base, is shown in Figure 2–25.

2.7 TRANSFORMER VOLTAGE REGULATION AND EFFICIENCY

Because a real transformer has series impedances within it, the output voltage of a transformer varies with the load even if the input voltage remains constant. To conveniently compare transformers in this respect, it is customary to define a quantity called *voltage regulation* (VR). ***Full-load voltage regulation*** is a quantity that compares the output voltage of the transformer at no load with the output voltage at full load. It is defined by the equation

$$\boxed{\mathrm{VR} = \frac{V_{S,\mathrm{nl}} - V_{S,\mathrm{fl}}}{V_{S,\mathrm{fl}}} \times 100\%} \tag{2–61}$$

Since at no load, $V_S = V_P/a$, the voltage regulation can also be expressed as

$$\boxed{\mathrm{VR} = \frac{V_P/a - V_{S,\mathrm{fl}}}{V_{S,\mathrm{fl}}} \times 100\%} \tag{2–62}$$

If the transformer equivalent circuit is in the per-unit system, then voltage regulation can be expressed as

$$\boxed{\mathrm{VR} = \frac{V_{P,\mathrm{pu}} - V_{S,\mathrm{fl,pu}}}{V_{S,\mathrm{fl,pu}}} \times 100\%} \tag{2–63}$$

Usually it is a good practice to have as small a voltage regulation as possible. For an ideal transformer, VR = 0 percent. It is not always a good idea to have a low-voltage regulation, though—sometimes high-impedance and high-voltage regulation transformers are deliberately used to reduce the fault currents in a circuit.

How can the voltage regulation of a transformer be determined?

The Transformer Phasor Diagram

To determine the voltage regulation of a transformer, it is necessary to understand the voltage drops within it. Consider the simplified transformer equivalent circuit in Figure 2–18b. The effects of the excitation branch on transformer voltage regulation can be ignored, so only the series impedances need be considered. The voltage regulation of a transformer depends both on the magnitude of these series impedances and on the phase angle of the current flowing through the transformer. The easiest way to determine the effect of the impedances and the current phase angles on the transformer voltage regulation is to examine a ***phasor diagram***, a sketch of the phasor voltages and currents in the transformer.

In all the following phasor diagrams, the phasor voltage $\mathbf{V}_S$ is assumed to be at an angle of 0°, and all other voltages and currents are compared to that reference. By applying Kirchhoff's voltage law to the equivalent circuit in Figure 2–18b, the primary voltage can be found as

$$\frac{\mathbf{V}_P}{a} = \mathbf{V}_S + R_{eq}\mathbf{I}_S + jX_{eq}\mathbf{I}_S \tag{2–64}$$

A transformer phasor diagram is just a visual representation of this equation.

Figure 2–26 shows a phasor diagram of a transformer operating at a lagging power factor. It is easy to see that $V_P/a > V_S$ for lagging loads, so the voltage regulation of a transformer with lagging loads must be greater than zero.

A phasor diagram at unity power factor is shown in Figure 2–27a. Here again, the voltage at the secondary is lower than the voltage at the primary, so VR > 0. However, this time the voltage regulation is a smaller number than it was with a lagging current. If the secondary current is leading, the secondary voltage can actually be ***higher*** than the referred primary voltage. If this happens, the transformer actually has a *negative* voltage regulation (see Figure 2–27b).

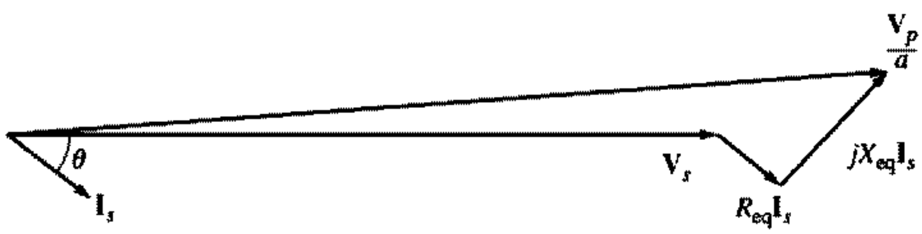


FIGURE 2–26
Phasor diagram of a transformer operating at a lagging power factor.

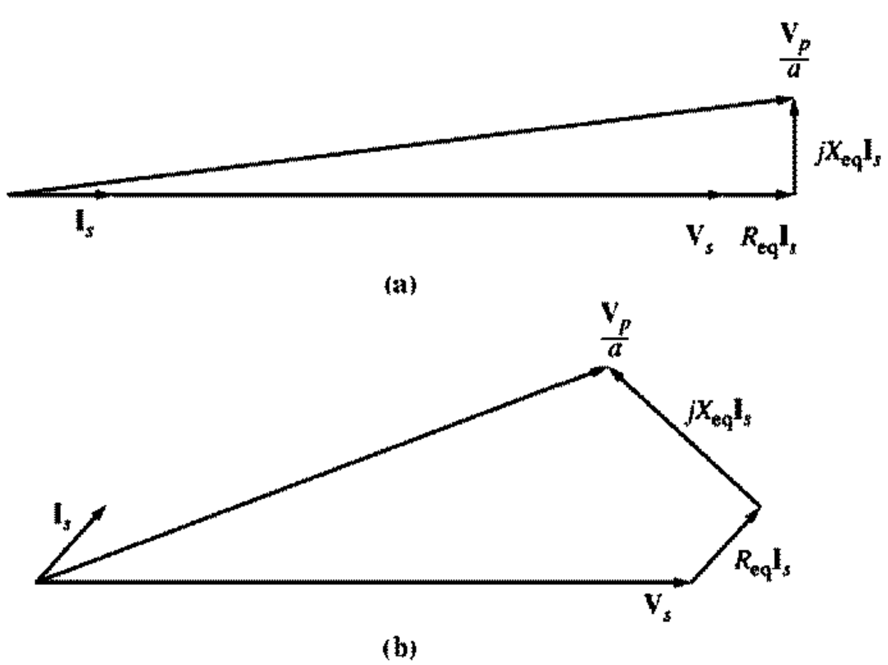


FIGURE 2–27
Phasor diagram of a transformer operating at (a) unity and (b) leading power factor.

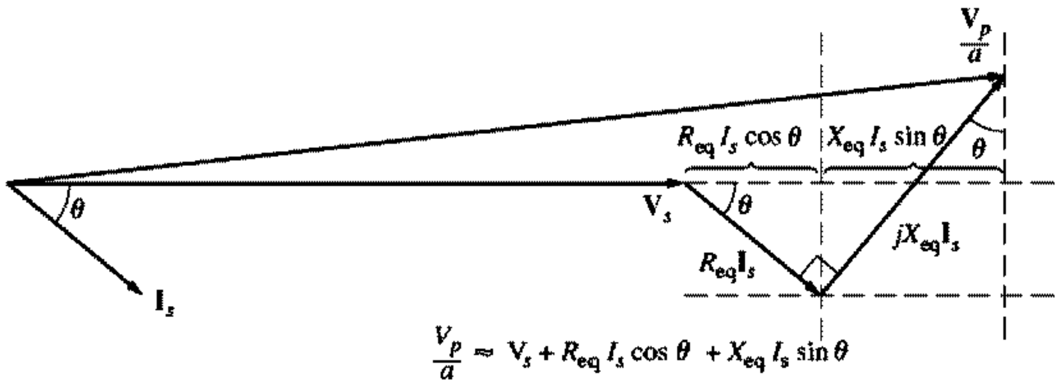


FIGURE 2–28
Derivation of the approximate equation for V_P/a.

Transformer Efficiency

Transformers are also compared and judged on their efficiencies. The efficiency of a device is defined by the equation

$$\eta = \frac{P_{out}}{P_{in}} \times 100\% \tag{2–65}$$

$$\eta = \frac{P_{out}}{P_{out} + P_{loss}} \times 100\% \tag{2–66}$$

These equations apply to motors and generators as well as to transformers.

The transformer equivalent circuits make efficiency calculations easy. There are three types of losses present in transformers:

1. *Copper (I^2R) losses.* These losses are accounted for by the series resistance in the equivalent circuit.
2. *Hysteresis losses.* These losses were explained in Chapter 1 and are accounted for by resistor R_C.
3. *Eddy current losses.* These losses were explained in Chapter 1 and are accounted for by resistor R_C.

To calculate the efficiency of a transformer at a given load, just add the losses from each resistor and apply Equation (2–67). Since the output power is given by

$$P_{out} = V_S I_S \cos\theta_S \tag{2–7}$$

the efficiency of the transformer can be expressed by

$$\eta = \frac{V_S I_S \cos\theta}{P_{Cu} + P_{core} + V_S I_S \cos\theta} \times 100\% \tag{2–67}$$

Example 2–5. A 15-kVA, 2300/230-V transformer is to be tested to determine its excitation branch components, its series impedances, and its voltage regulation. The following test data have been taken from the primary side of the transformer:

Open-circuit test	Short-circuit test
$V_{OC} = 2300$ V	$V_{SC} = 47$ V
$I_{OC} = 0.21$ A	$I_{SC} = 6.0$ A
$P_{OC} = 50$ W	$P_{SC} = 160$ W

The data have been taken by using the connections shown in Figures 2–19 and 2–20.

(a) Find the equivalent circuit of this transformer referred to the high-voltage side.
(b) Find the equivalent circuit of this transformer referred to the low-voltage side.
(c) Calculate the full-load voltage regulation at 0.8 lagging power factor, 1.0 power factor, and at 0.8 leading power factor.
(d) Plot the voltage regulation as load is increased from no load to full load at power factors of 0.8 lagging, 1.0, and 0.8 leading.
(e) What is the efficiency of the transformer at full load with a power factor of 0.8 lagging?

Solution

(*a*) The excitation branch values of the transformer equivalent circuit can be calculated from the *open-circuit test* data, and the series elements can be calculated from the *short-circuit test* data. From the open-circuit test data, the open-circuit impedance angle is

$$\theta_{OC} = \cos^{-1}\frac{P_{OC}}{V_{OC}I_{OC}}$$

$$= \cos^{-1}\frac{50\text{W}}{(2300\text{ V})(0.21\text{ A})} = 84^\circ$$

The excitation admittance is thus

$$Y_E = \frac{I_{OC}}{V_{OC}}\angle -84^\circ$$

$$= \frac{0.21\text{ A}}{2300\text{ V}}\angle -84^\circ$$

$$= 9.13\times 10^{-5}\angle -84^\circ\Omega = 0.0000095 - j0.0000908\ \Omega$$

The elements of the excitation branch referred to the primary are

$$R_C = \frac{1}{0.0000095} = 105\text{ k}\Omega$$

$$X_M = \frac{1}{0.0000908} = 11\text{ k}\Omega$$

From the short-circuit test data, the short-circuit impedance angle is

$$\theta_{SC} = \cos^{-1} \frac{P_{SC}}{V_{SC} I_{SC}}$$

$$= \cos^{-1} \frac{160\text{ W}}{(47\text{ V})(6\text{ A})} = 55.4°$$

The equivalent series impedance is thus

$$Z_{SE} = \frac{V_{SC}}{I_{SC}} \angle \theta_{SC}$$

$$= \frac{47\text{ V}}{6\text{ A}} \angle 55.4°\ \Omega$$

$$= 7.833 \angle 55.4° = 4.45 + j6.45$$

The series elements referred to the primary are

$$R_{eq} = 4.45\ \Omega \qquad X_{eq} = 6.45\ \Omega$$

This equivalent circuit is shown in Figure 2–29a.

(b) To find the equivalent circuit referred to the low-voltage side, it is simply necessary to divide the impedance by a^2. Since $a = N_P/N_S = 10$, the resulting values are

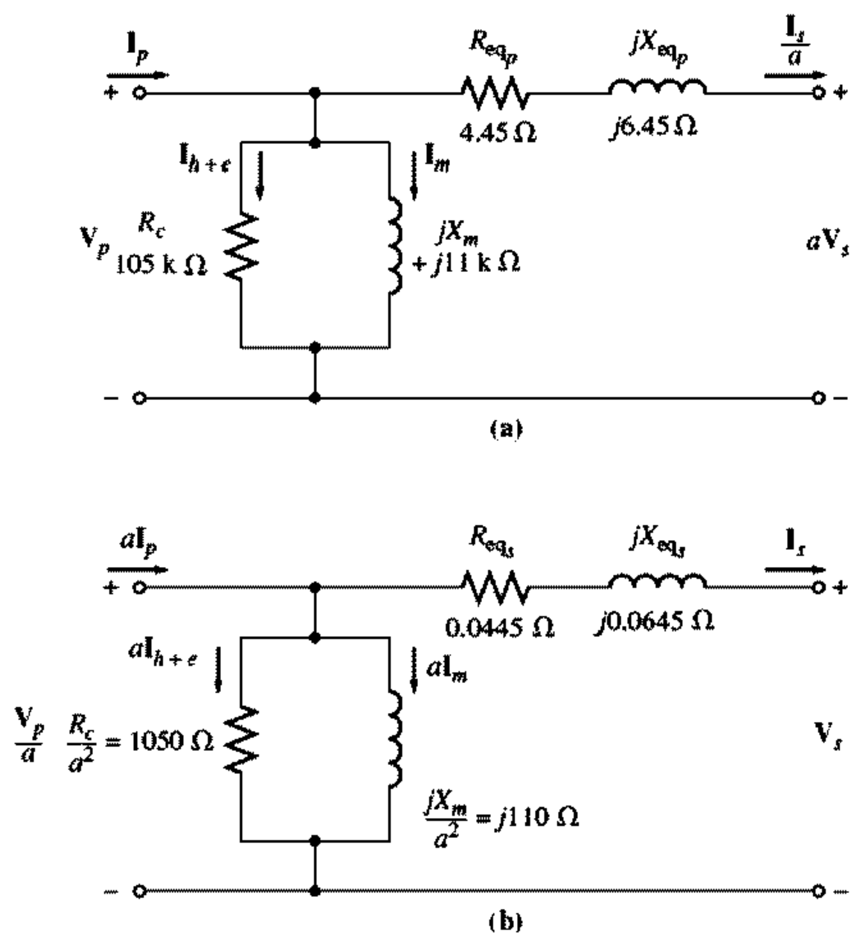


FIGURE 2–29
The transfer equivalent circuit for Example 2–5 referred to (a) its primary side and (b) its secondary side.

$$R_C = 1050\ \Omega \qquad R_{eq} = 0.0445\ \Omega$$
$$X_M = 110\ \Omega \qquad X_{eq} = 0.0645\ \Omega$$

The resulting equivalent circuit is shown in Figure 2–29b.

(c) The full-load current on the secondary side of this transformer is

$$I_{S,\text{rated}} = \frac{S_{\text{rated}}}{V_{S,\text{rated}}} = \frac{15{,}000\text{ VA}}{230\text{ V}} = 65.2\text{ A}$$

To calculate V_P/a, use Equation (2–64):

$$\frac{\mathbf{V}_P}{a} = \mathbf{V}_S + R_{eq}\mathbf{I}_S + jX_{eq}\mathbf{I}_S \tag{2–64}$$

At PF = 0.8 lagging, current $\mathbf{I}_S = 65.2\ \angle{-36.9^\circ}$ A. Therefore,

$$\begin{aligned}\frac{\mathbf{V}_P}{a} &= 230\angle 0^\circ\text{ V} + (0.0445\ \Omega)(65.2\angle{-36.9^\circ}\text{ A}) + j(0.0645\ \Omega)(65.2\angle{-36.9^\circ}\text{ A})\\ &= 230\angle 0^\circ\text{ V} + 2.90\angle{-36.9^\circ}\text{ V} + 4.21\angle 53.1^\circ\text{ V}\\ &= 230 + 2.32 - j1.74 + 2.52 + j3.36\\ &= 234.84 + j1.62 = 234.85\angle 0.40^\circ\text{ V}\end{aligned}$$

The resulting voltage regulation is

$$\text{VR} = \frac{V_P/a - V_{S,\text{fl}}}{V_{S,\text{fl}}} \times 100\% \tag{2–62}$$

$$= \frac{234.85\text{ V} - 230\text{ V}}{230\text{ V}} \times 100\% = 2.1\%$$

At PF = 1.0, current $\mathbf{I}_S = 65.2\ \angle\ 0^\circ$ A. Therefore,

$$\begin{aligned}\frac{\mathbf{V}_P}{a} &= 230\angle 0^\circ\text{ V} + (0.0445\ \Omega)(65.2\angle 0^\circ\text{ A}) + j(0.0645\ \Omega)(65.2\angle 0^\circ\text{ A})\\ &= 230\angle 0^\circ\text{ V} + 2.90\angle 0^\circ\text{ V} + 4.21\angle 90^\circ\text{ V}\\ &= 230 + 2.90 + j4.21\\ &= 232.9 + j4.21 = 232.94\angle 1.04^\circ\text{ V}\end{aligned}$$

The resulting voltage regulation is

$$\text{VR} = \frac{232.94\text{ V} - 230\text{ V}}{230\text{ V}} \times 100\% = 1.28\%$$

At PF = 0.8 leading, current $\mathbf{I}_S = 65.2\ \angle 36.9^\circ$ A. Therefore,

$$\begin{aligned}\frac{\mathbf{V}_P}{a} &= 230\angle 0^\circ\text{ V} + (0.0445\ \Omega)(65.2\angle 36.9^\circ\text{ A}) + j(0.0645\ \Omega)(65.2\angle 36.9^\circ\text{ A})\\ &= 230\angle 0^\circ\text{ V} + 2.90\angle 36.9^\circ\text{ V} + 4.21\angle 126.9^\circ\text{ V}\\ &= 230 + 2.32 + j1.74 - 2.52 + j3.36\\ &= 229.80 + j5.10 = 229.85\angle 1.27^\circ\text{ V}\end{aligned}$$

The resulting voltage regulation is

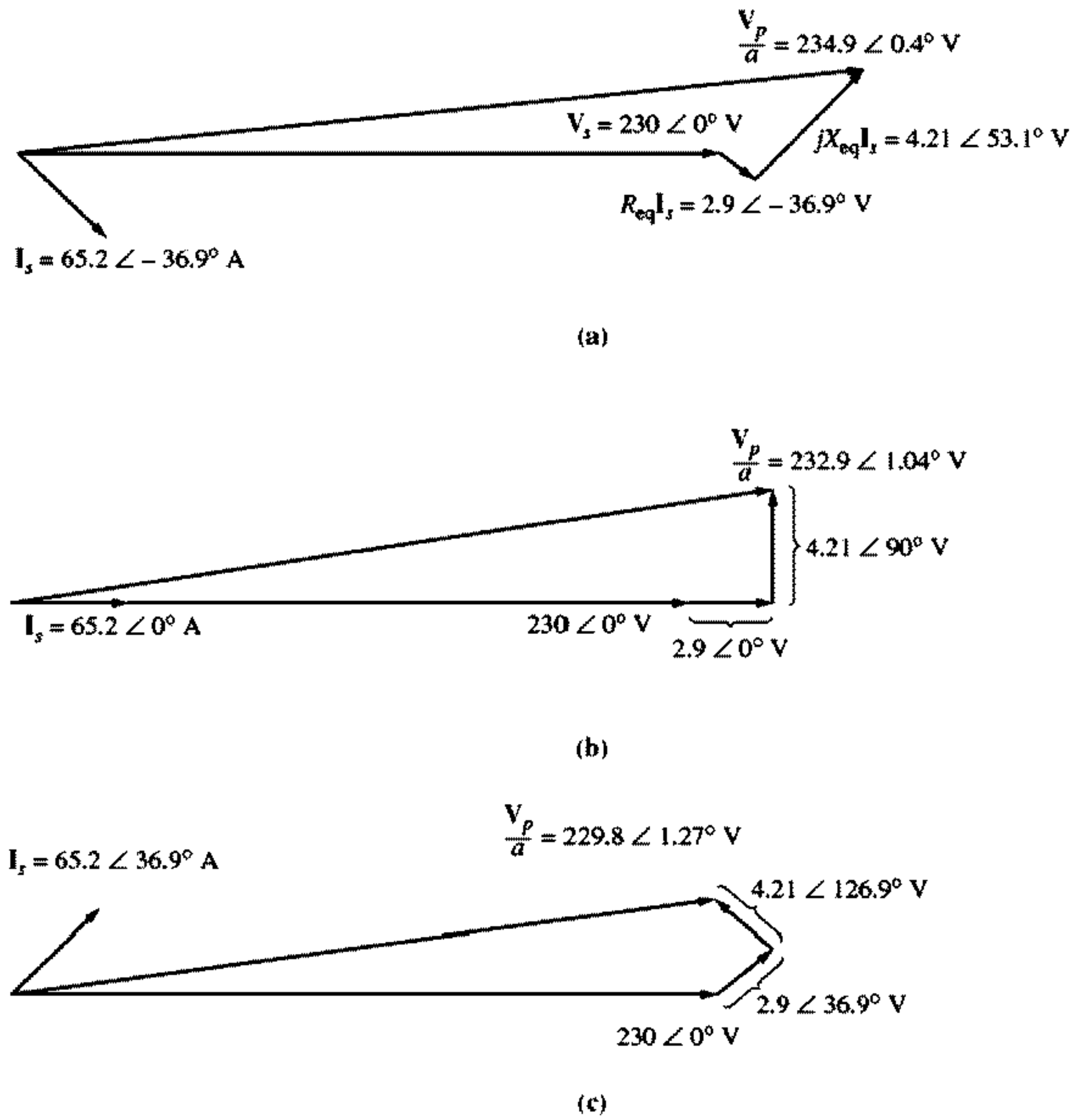


FIGURE 2–30
Transformer phasor diagrams for Example 2–5.

$$VR = \frac{229.85\ V - 230\ V}{230\ V} \times 100\% = -0.062\%$$

Each of these three phasor diagrams is shown in Figure 2–30.

(d) The best way to plot the voltage regulation as a function of load is to repeat the calculations in part *c* for many different loads using MATLAB. A program to do this is shown below.

```
% M-file: trans_vr.m
% M-file to calculate and plot the voltage regulation
% of a transformer as a function of load for power
% factors of 0.8 lagging, 1.0, and 0.8 leading.
VS = 230;                       % Secondary voltage (V)
amps = 0:6.52:65.2;             % Current values (A)
```

```
Req = 0.0445;                  % Equivalent R (ohms)
Xeq = 0.0645;                  % Equivalent X (ohms)

% Calculate the current values for the three
% power factors. The first row of I contains
% the lagging currents, the second row contains
% the unity currents, and the third row contains
% the leading currents.
I(1,:) = amps .* ( 0.8 - j*0.6);      % Lagging
I(2,:) = amps .* ( 1.0        );      % Unity
I(3,:) = amps .* ( 0.8 + j*0.6);      % Leading

% Calculate VP/a.
VPa = VS + Req.*I + j.*Xeq.*I;

% Calculate voltage regulation
VR = (abs(VPa) - VS) ./ VS .* 100;

% Plot the voltage regulation
plot(amps,VR(1,:),'b-');
hold on;
plot(amps,VR(2,:),'k-');
plot(amps,VR(3,:),'r-.');
title ('Voltage Regulation Versus Load');
xlabel ('Load (A)');
ylabel ('Voltage Regulation (%)');
legend('0.8 PF lagging','1.0 PF','0.8 PF leading');
hold off;
```

The plot produced by this program is shown in Figure 2–31.

(e) To find the efficiency of the transformer, first calculate its losses. The copper losses are

$$P_{\mathrm{Cu}} = (I_S)^2 R_{\mathrm{eq}} = (65.2\ \mathrm{A})^2(0.0445\ \Omega) = 189\ \mathrm{W}$$

The core losses are given by

$$P_{\mathrm{core}} = \frac{(V_P/a)^2}{R_C} = \frac{(234.85\ \mathrm{V})^2}{1050\ \Omega} = 52.5\ \mathrm{W}$$

The output power of the transformer at this power factor is

$$\begin{aligned} P_{\mathrm{out}} &= V_S I_S \cos\theta \\ &= (230\ \mathrm{V})(65.2\ \mathrm{A}) \cos 36.9^\circ = 12{,}000\ \mathrm{W} \end{aligned}$$

Therefore, the efficiency of the transformer at this condition is

$$\eta = \frac{V_S I_S \cos\theta}{P_{\mathrm{Cu}} + P_{\mathrm{core}} + V_S I_S \cos\theta} \times 100\% \qquad (2\text{–}68)$$

$$= \frac{12{,}000\ \mathrm{W}}{189\ \mathrm{W} + 52.5\ \mathrm{W} + 12{,}000\ \mathrm{W}} \times 100\%$$

$$= 98.03\%$$

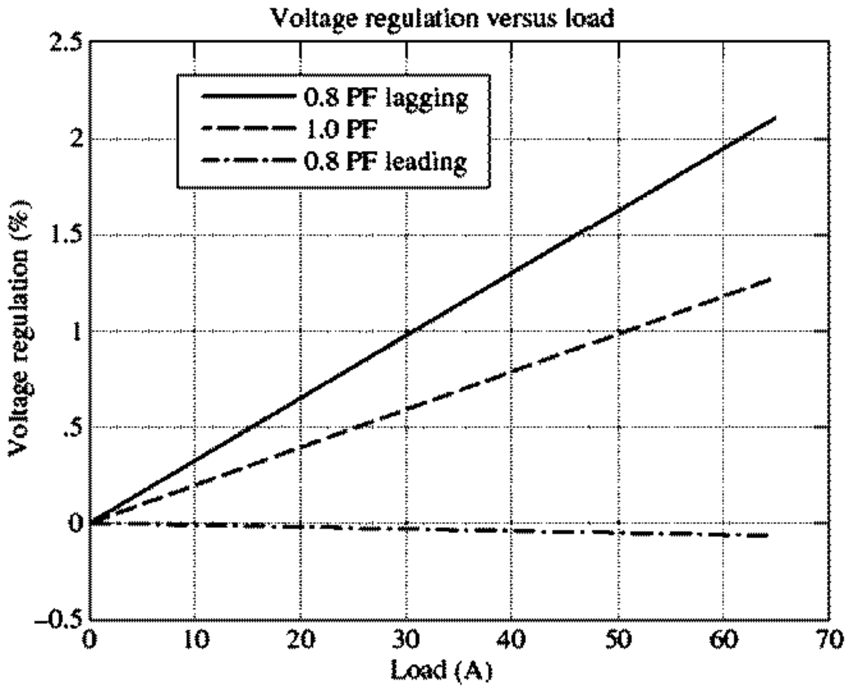


FIGURE 2–31
Plot of voltage regulation versus load for the transformer of Example 2–5.

2.8 TRANSFORMER TAPS AND VOLTAGE REGULATION

In previous sections of this chapter, transformers were described by their turns ratios or by their primary-to-secondary-voltage ratios. Throughout those sections, the turns ratio of a given transformer was treated as though it were completely fixed. In almost all real distribution transformers, this is not quite true. Distribution transformers have a series of *taps* in the windings to permit small changes in the turns ratio of the transformer after it has left the factory. A typical installation might have four taps in addition to the nominal setting with spacings of 2.5 percent of full-load voltage between them. Such an arrangement provides for adjustments up to 5 percent above or below the nominal voltage rating of the transformer.

Example 2–6. A 500-kVA, 13,200/480-V distribution transformer has four 2.5 percent taps on its primary winding. What are the voltage ratios of this transformer at each tap setting?

Solution
The five possible voltage ratings of this transformer are

+5.0% tap	13,860/480 V
+2.5% tap	13,530/480 V
Nominal rating	13,200/480 V
−2.5% tap	12,870/480 V
−5.0% tap	12,540/480 V

The taps on a transformer permit the transformer to be adjusted in the field to accommodate variations in local voltages. However, these taps normally cannot be changed while power is being applied to the transformer. They must be set once and left alone.

Sometimes a transformer is used on a power line whose voltage varies widely with the load. Such voltage variations might be due to a high line impedance between the generators on the power system and that particular load (perhaps it is located far out in the country). Normal loads need to be supplied an essentially constant voltage. How can a power company supply a controlled voltage through high-impedance lines to loads which are constantly changing?

One solution to this problem is to use a special transformer called a *tap changing under load* (TCUL) *transformer* or *voltage regulator.* Basically, a TCUL transformer is a transformer with the ability to change taps while power is connected to it. A voltage regulator is a TCUL transformer with built-in voltage sensing circuitry that automatically changes taps to keep the system voltage constant. Such special transformers are very common in modern power systems.

2.9 THE AUTOTRANSFORMER

On some occasions it is desirable to change voltage levels by only a small amount. For example, it may be necessary to increase a voltage from 110 to 120 V or from 13.2 to 13.8 kV. These small rises may be made necessary by voltage drops that occur in power systems a long way from the generators. In such circumstances, it is wasteful and excessively expensive to wind a transformer with two full windings, each rated at about the same voltage. A special-purpose transformer, called an *autotransformer,* is used instead.

A diagram of a step-up autotransformer is shown in Figure 2–32. In Figure 2–32a, the two coils of the transformer are shown in the conventional manner. In Figure 2–32b, the first winding is shown connected in an additive manner to the second winding. Now, the relationship between the voltage on the first winding and the voltage on the second winding is given by the turns ratio of the transformer. However, *the voltage at the output of the whole transformer is the sum of the voltage on the first winding and the voltage on the second winding.* The first winding here is called the *common winding*, because its voltage appears on both sides of the transformer. The smaller winding is called the *series winding*, because it is connected in series with the common winding.

A diagram of a step-down autotransformer is shown in Figure 2–33. Here the voltage at the input is the sum of the voltages on the series winding and the common winding, while the voltage at the output is just the voltage on the common winding.

Because the transformer coils are physically connected, a different terminology is used for the autotransformer than for other types of transformers. The voltage on the common coil is called the *common voltage* $\mathbf{V}_C$, and the current in that coil is called the *common current* $\mathbf{I}_C$. The voltage on the series coil is called the *series voltage* $\mathbf{V}_{SE}$, and the current in that coil is called the *series current* $\mathbf{I}_{SE}$.

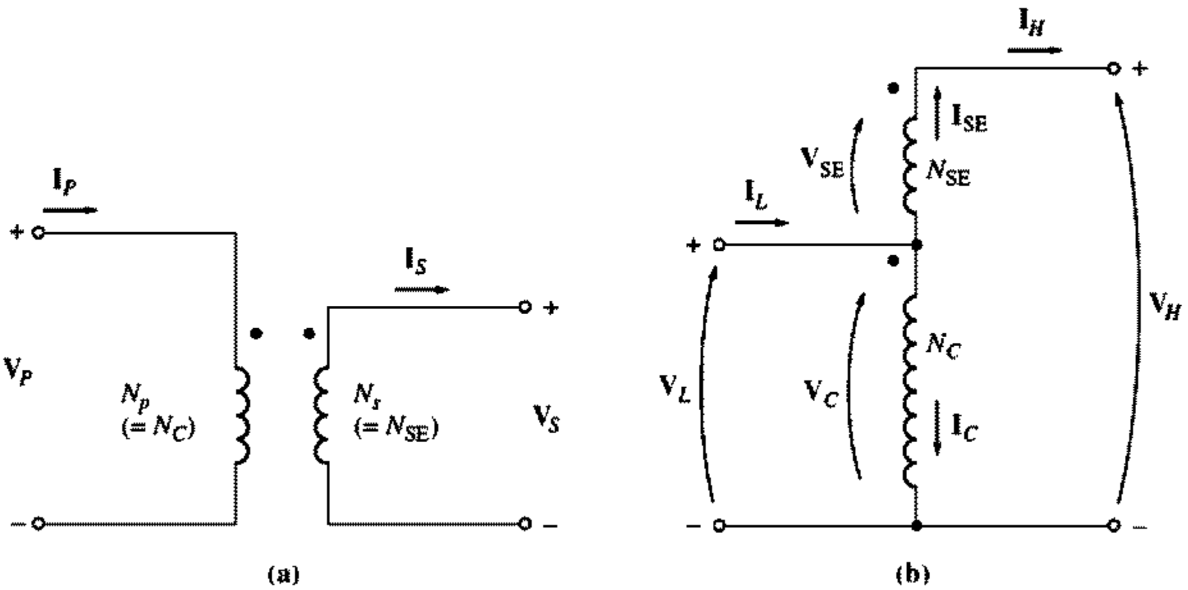


FIGURE 2–32
A transformer with its windings (a) connected in the conventional manner and (b) reconnected as an autotransformer.

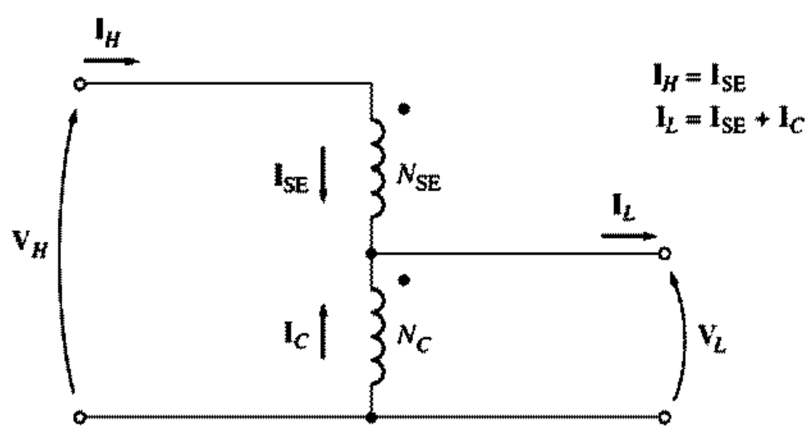


FIGURE 2–33
A step-down autotransformer connection.

The voltage and current on the low-voltage side of the transformer are called $\mathbf{V}_L$ and $\mathbf{I}_L$, respectively, while the corresponding quantities on the high-voltage side of the transformer are called $\mathbf{V}_H$ and $\mathbf{I}_H$. The primary side of the autotransformer (the side with power into it) can be either the high-voltage side or the low-voltage side, depending on whether the autotransformer is acting as a step-down or a step-up transformer. From Figure 2–32*b* the voltages and currents in the coils are related by the equations

$$\frac{\mathbf{V}_C}{\mathbf{V}_{SE}} = \frac{N_C}{N_{SE}} \tag{2–68}$$

$$N_C \mathbf{I}_C = N_{SE} \mathbf{I}_{SE} \tag{2–69}$$

The voltages in the coils are related to the voltages at the terminals by the equations

$$\mathbf{V}_L = \mathbf{V}_C \tag{2–70}$$

$$\mathbf{V}_H = \mathbf{V}_C + \mathbf{V}_{SE} \tag{2–71}$$

and the currents in the coils are related to the currents at the terminals by the equations

$$\mathbf{I}_L = \mathbf{I}_C + \mathbf{I}_{SE} \tag{2–72}$$

$$\mathbf{I}_H = \mathbf{I}_{SE} \tag{2–73}$$

Voltage and Current Relationships in an Autotransformer

What is the voltage relationship between the two sides of an autotransformer? It is quite easy to determine the relationship between $\mathbf{V}_H$ and $\mathbf{V}_L$. The voltage on the high side of the autotransformer is given by

$$\mathbf{V}_H = \mathbf{V}_C + \mathbf{V}_{SE} \tag{2–71}$$

But $\mathbf{V}_C/\mathbf{V}_{SE} = N_C/\mathrm{N}_{SE}$, so

$$\mathbf{V}_H = \mathbf{V}_C + \frac{N_{SE}}{N_C}\mathbf{V}_C \tag{2–74}$$

Finally, noting that $\mathbf{V}_L = V_C$, we get

$$\mathbf{V}_H = \mathbf{V}_L + \frac{N_{SE}}{N_C}\mathbf{V}_L$$

$$= \frac{N_{SE} + N_C}{N_C}\mathbf{V}_L \tag{2–75}$$

or

$$\boxed{\frac{\mathbf{V}_L}{\mathbf{V}_H} = \frac{N_C}{N_{SE} + N_C}} \tag{2–76}$$

The current relationship between the two sides of the transformer can be found by noting that

$$\mathbf{I}_L = \mathbf{I}_C + \mathbf{I}_{SE} \tag{2–72}$$

From Equation (2–69), $\mathbf{I}_C = (N_{SE}/N_C)\mathbf{I}_{SE}$, so

$$\mathbf{I}_L = \frac{N_{SE}}{N_C}\mathbf{I}_{SE} + \mathbf{I}_{SE} \tag{2–77}$$

Finally, noting that $\mathbf{I}_H = \mathbf{I}_{SE}$, we find

$$\mathbf{I}_L = \frac{N_{SE}}{N_C}\mathbf{I}_H + \mathbf{I}_H$$

$$= \frac{N_{SE} + N_C}{N_C} \mathbf{I}_H \tag{2-78}$$

or

$$\boxed{\frac{\mathbf{I}_L}{\mathbf{I}_H} = \frac{N_{SE} + N_C}{N_C}} \tag{2-79}$$

The Apparent Power Rating Advantage of Autotransformers

It is interesting to note that not all the power traveling from the primary to the secondary in the autotransformer goes through the windings. As a result, if a conventional transformer is reconnected as an autotransformer, it can handle much more power than it was originally rated for.

To understand this idea, refer again to Figure 2–32b. Notice that the input apparent power to the autotransformer is given by

$$S_{in} = V_L I_L \tag{2-80}$$

and the output apparent power is given by

$$S_{out} = V_H I_H \tag{2-81}$$

It is easy to show, by using the voltage and current equations [Equations (2–76) and (2–79)], that the input apparent power is again equal to the output apparent power:

$$S_{in} = S_{out} = S_{IO} \tag{2-82}$$

where S_{IO} is defined to be the input and output apparent powers of the transformer. However, *the apparent power in the transformer windings* is

$$S_W = V_C I_C = V_{SE} I_{SE} \tag{2-83}$$

The relationship between the power going into the primary (and out the secondary) of the transformer and the power in the transformer's actual windings can be found as follows:

$$\begin{aligned} S_W &= V_C I_C \\ &= V_L (I_L - I_H) \\ &= V_L I_L - V_L I_H \end{aligned}$$

Using Equation (2–79), we get

$$S_W = V_L I_L - V_L I_L \frac{N_C}{N_{SE} + N_C}$$

$$= V_L I_L \frac{(N_{SE} + N_C) - N_C}{N_{SE} + N_C} \tag{2-84}$$

$$= S_{IO} \frac{N_{SE}}{N_{SE} + N_C} \tag{2-85}$$

Therefore, the ratio of the apparent power in the primary and secondary of the autotransformer to the apparent power actually traveling through its windings is

$$\boxed{\frac{S_{IO}}{S_W} = \frac{N_{SE} + N_C}{N_{SE}}} \tag{2–86}$$

Equation (2–86) describes the *apparent power rating advantage* of an autotransformer over a conventional transformer. Here S_{IO} is the apparent power entering the primary and leaving the secondary of the transformer, while S_W is the apparent power actually traveling through the transformer's windings (the rest passes from primary to secondary without being coupled through the transformer's windings). Note that the smaller the series winding, the greater the advantage.

For example, a 5000-kVA autotransformer connecting a 110-kV system to a 138-kV system would have an N_C/N_{SE} turns ratio of 110:28. Such an autotransformer would actually have windings rated at

$$S_W = S_{IO}\frac{N_{SE}}{N_{SE} + N_C} \tag{2–85}$$

$$= (5000 \text{ kVA})\frac{28}{28 + 110} = 1015 \text{ kVA}$$

The autotransformer would have windings rated at only about 1015 kVA, while a conventional transformer doing the same job would need windings rated at 5000 kVA. The autotransformer could be 5 times smaller than the conventional transformer and also would be much less expensive. For this reason, it is very advantageous to build transformers between two nearly equal voltages as autotransformers.

The following example illustrates autotransformer analysis and the rating advantage of autotransformers.

Example 2–7. A 100-VA 120/12–V transformer is to be connected so as to form a step-up autotransformer (see Figure 2–34). A primary voltage of 120 V is applied to the transformer.

(a) What is the secondary voltage of the transformer?

(b) What is its maximum voltampere rating in this mode of operation?

(c) Calculate the rating advantage of this autotransformer connection over the transformer's rating in conventional 120/12–V operation.

Solution

To accomplish a step-up transformation with a 120-V primary, the ratio of the turns on the common winding N_C to the turns on the series winding N_{SE} in this transformer must be 120:12 (or 10:1).

(a) This transformer is being used as a step-up transformer. The secondary voltage is V_H, and from Equation (2–75),

$$\mathbf{V}_H = \frac{N_{SE} + N_C}{N_C}\mathbf{V}_L \tag{2–75}$$

$$= \frac{12 + 120}{120}\,120 \text{ V} = 132 \text{ V}$$

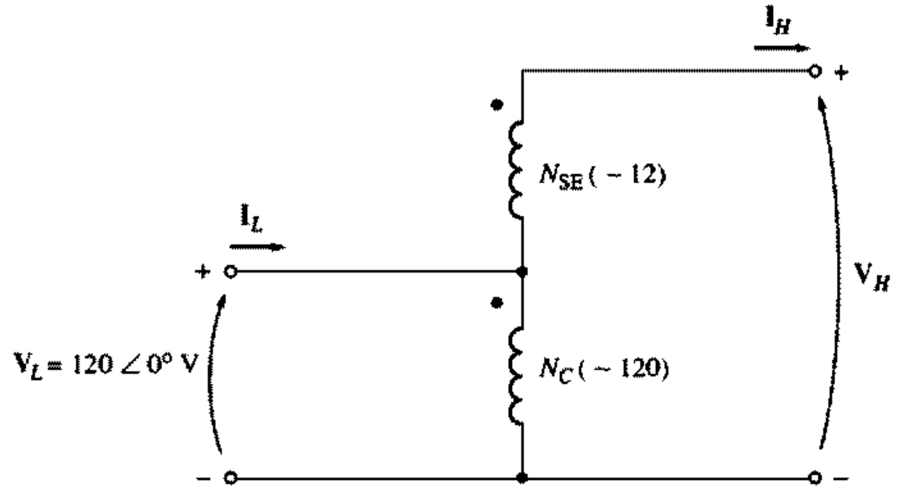


FIGURE 2–34
The autotransformer of Example 2–7.

(*b*) The maximum voltampere rating in either winding of this transformer is 100 VA. How much input or output apparent power can this provide? To find out, examine the series winding. The voltage V_{SE} on the winding is 12 V, and the voltampere rating of the winding is 100 VA. Therefore, the *maximum* series winding current is

$$I_{SE,max} = \frac{S_{max}}{V_{SE}} = \frac{100\text{ VA}}{12\text{V}} = 8.33\text{ A}$$

Since I_{SE} is equal to the secondary current I_S (or I_H) and since the secondary voltage $V_S = V_H = 132$ V, the secondary apparent power is

$$S_{out} = V_S I_S = V_H I_H$$
$$= (132\text{ V})(8.33\text{ A}) = 1100\text{ VA} = S_{in}$$

(*c*) The rating advantage can be calculated from part (*b*) or separately from Equation (2–86). From part *b*,

$$\frac{S_{IO}}{S_W} = \frac{1100\text{ VA}}{100\text{ VA}} = 11$$

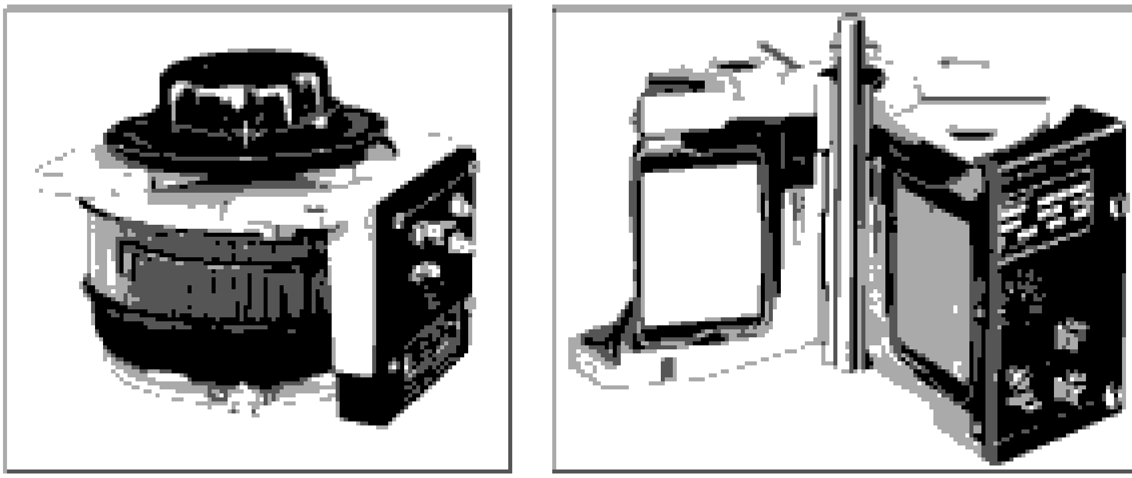
From Equation (2–86),

$$\frac{S_{IO}}{S_W} = \frac{N_{SE} + N_C}{N_{SE}} \tag{2–86}$$

$$= \frac{12 + 120}{12} = \frac{132}{12} = 11$$

By either equation, the apparent power rating is increased by a factor of 11.

It is not normally possible to just reconnect an ordinary transformer as an autotransformer and use it in the manner of Example 2–7, because the insulation on the low-voltage side of the ordinary transformer may not be strong enough to withstand the full output voltage of the autotransformer connection. In transform-



(a) (b)

FIGURE 2–35
(a) A variable-voltage autotransformer. (b) Cutaway view of the autotransformer. (*Courtesy of Superior Electric Company.*)

ers built specifically as autotransformers, the insulation on the smaller coil (the series winding) is made just as strong as the insulation on the larger coil.

It is common practice in power systems to use autotransformers whenever two voltages fairly close to each other in level need to be transformed, because the closer the two voltages are, the greater the autotransformer power advantage becomes. They are also used as variable transformers, where the low-voltage tap moves up and down the winding. This is a very convenient way to get a variable ac voltage. Such a variable autotransformer is shown in Figure 2–35.

The principal disadvantage of autotransformers is that, unlike ordinary transformers, ***there is a direct physical connection between the primary and the secondary circuits,*** so the ***electrical isolation*** of the two sides is lost. If a particular application does not require electrical isolation, then the autotransformer is a convenient and ***inexpensive*** way to tie nearly equal voltages together.

The Internal Impedance of an Autotransformer

Autotransformers have one additional disadvantage compared to conventional transformers. It turns out that, compared to a given transformer connected in the conventional manner, the effective per-unit impedance of an autotransformer is smaller by a factor equal to the reciprocal of the power advantage of the autotransformer connection.

The proof of this statement is left as a problem at the end of the chapter.

The reduced internal impedance of an autotransformer compared to a conventional two-winding transformer can be a serious problem in some applications where the series impedance is needed to limit current flows during power system faults (short circuits). The effect of the smaller internal impedance provided by an autotransformer must be taken into account in practical applications before autotransformers are selected.

Example 2–8. A transformer is rated at 1000 kVA, 12/1.2 kV, 60 Hz when it is operated as a conventional two-winding transformer. Under these conditions, its series resistance and reactance are given as 1 and 8 percent per unit, respectively. This transformer is to be used as a 13.2/12–kV step-down autotransformer in a power distribution system. In the autotransformer connection, (*a*) what is the transformer's rating when used in this manner and (*b*) what is the transformer's series impedance in per-unit?

Solution

(*a*) The N_C/N_{SE} turns ratio must be 12:1.2 or 10:1. The voltage rating of this transformer will be 13.2/12 kV, and the apparent power (voltampere) rating will be

$$S_{IO} = \frac{N_{SE} + N_C}{N_{SE}} S_W$$

$$= \frac{1 + 10}{1} 1000 \text{ kVA} = 11{,}000 \text{ kVA}$$

(*b*) The transformer's impedance in a per-unit system when connected in the conventional manner is

$$Z_{eq} = 0.01 + j0.08 \text{ pu} \qquad \text{separate windings}$$

The apparent power advantage of this autotransformer is 11, so the per-unit impedance of the autotransformer connected as described is

$$Z_{eq} = \frac{0.01 + j0.08}{11}$$

$$= 0.00091 + j0.00727 \text{ pu} \qquad \text{autotransformer}$$

2.10 THREE-PHASE TRANSFORMERS

Almost all the major power generation and distribution systems in the world today are three-phase ac systems. Since three-phase systems play such an important role in modern life, it is necessary to understand how transformers are used in them.

Transformers for three-phase circuits can be constructed in one of two ways. One approach is simply to take three single-phase transformers and connect them in a three-phase bank. An alternative approach is to make a three-phase transformer consisting of three sets of windings wrapped on a common core. These two possible types of transformer construction are shown in Figures 2–36 and 2–37. The construction of a single three-phase transformer is the preferred practice today, since it is lighter, smaller, cheaper, and slightly more efficient. The older construction approach was to use three separate transformers. That approach had the advantage that each unit in the bank could be replaced individually in the event of trouble, but that does not outweigh the advantages of a combined three-phase unit for most applications. However, there are still a great many installations consisting of three single-phase units in service.

A discussion of three-phase circuits is included in Appendix A. Some readers may wish to refer to it before studying the following material.

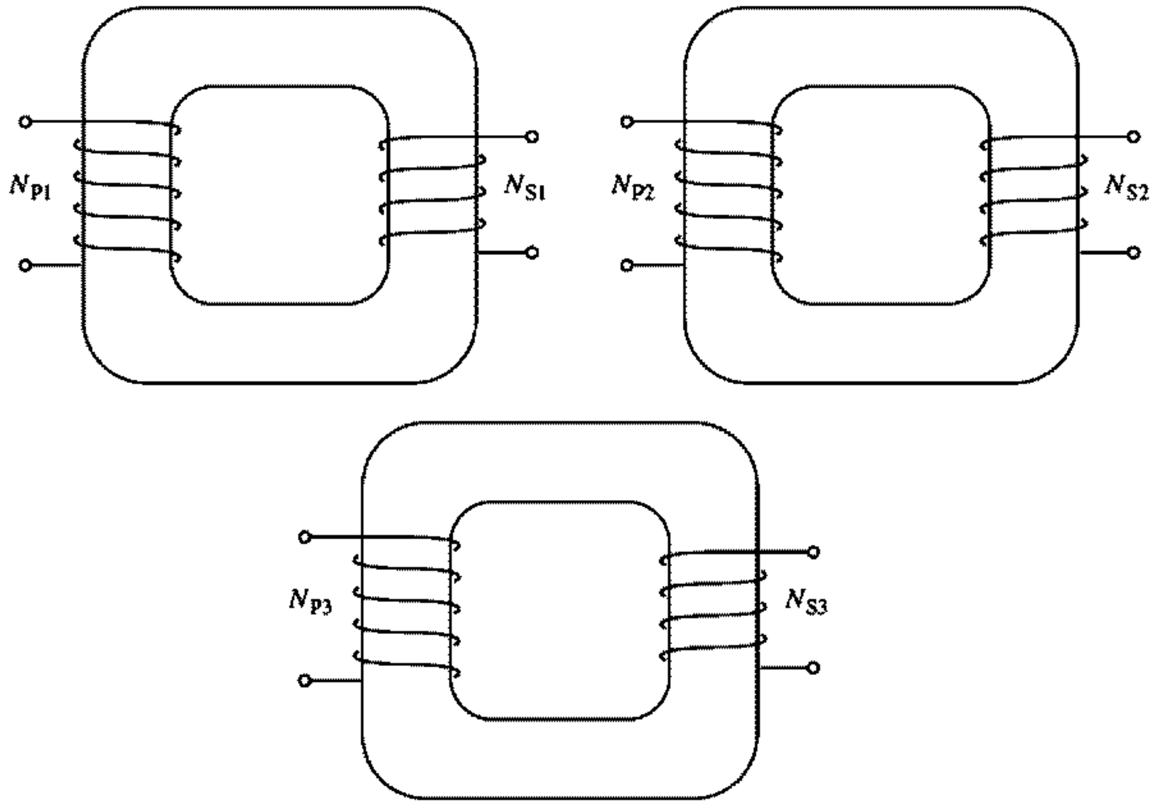


FIGURE 2–36
A three-phase transformer bank composed of independent transformers.

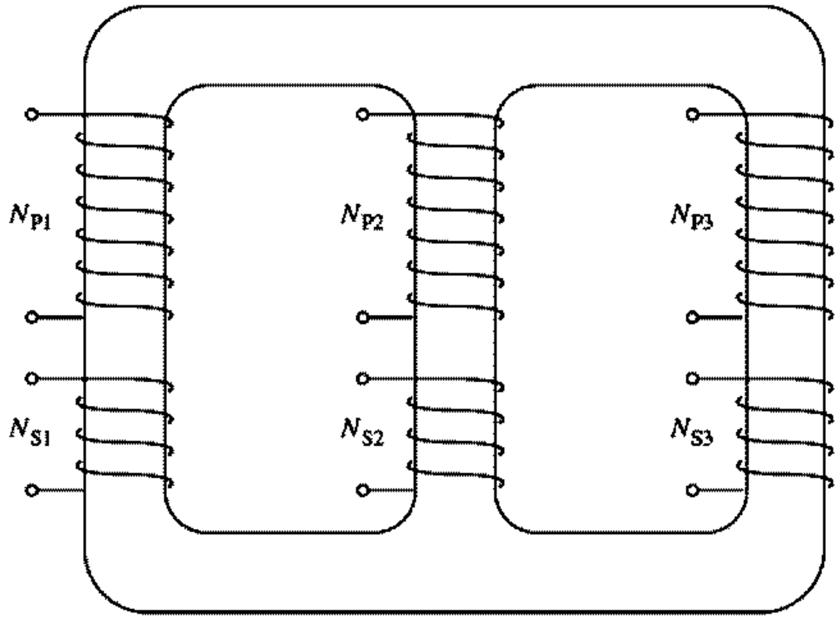


FIGURE 2–37
A three-phase transformer wound on a single three-legged core.

Three-Phase Transformer Connections

A three-phase transformer consists of three transformers, either separate or combined on one core. The primaries and secondaries of any three-phase transformer can be independently connected in either a wye (Y) or a delta (Δ). This gives a total of four possible connections for a three-phase transformer bank:

1. Wye–wye (Y–Y)
2. Wye–delta (Y–Δ)
3. Delta–wye (Δ–Y)
4. Delta–delta (Δ–Δ)

These connections are shown in Figure 2–38.

The key to analyzing any three-phase transformer bank is to look at a single transformer in the bank. *Any single transformer in the bank behaves exactly like the single-phase transformers already studied.* The impedance, voltage regulation, efficiency, and similar calculations for three-phase transformers are done on *a per-phase basis,* using exactly the same techniques already developed for single-phase transformers.

The advantages and disadvantages of each type of three-phase transformer connection are discussed below.

WYE–WYE CONNECTION. The Y–Y connection of three-phase transformers is shown in Figure 2–38a. In a Y–Y connection, the primary voltage on each phase of the transformer is given by $V_{\phi P} = V_{LP} / \sqrt{3}$. The primary-phase voltage is related to the secondary-phase voltage by the turns ratio of the transformer. The phase voltage on the secondary is then related to the line voltage on the secondary by $V_{LS} = \sqrt{3}\, V_{\phi S}$. Therefore, overall the voltage ratio on the transformer is

$$\boxed{\frac{V_{LP}}{V_{LS}} = \frac{\sqrt{3}V_{\phi P}}{\sqrt{3}V_{\phi S}} = a \qquad \text{Y–Y}} \tag{2–87}$$

The Y–Y connection has two very serious problems:

1. If loads on the transformer circuit are unbalanced, then the voltages on the phases of the transformer can become severely unbalanced.
2. Third-harmonic voltages can be large.

If a three-phase set of voltages is applied to a Y–Y transformer, the voltages in any phase will be 120° apart from the voltages in any other phase. However, *the third-harmonic components of each of the three phases will be in phase with each other,* since there are three cycles in the third harmonic for each cycle of the fundamental frequency. There are always some third-harmonic components in a transformer because of the nonlinearity of the core, and these components add up.

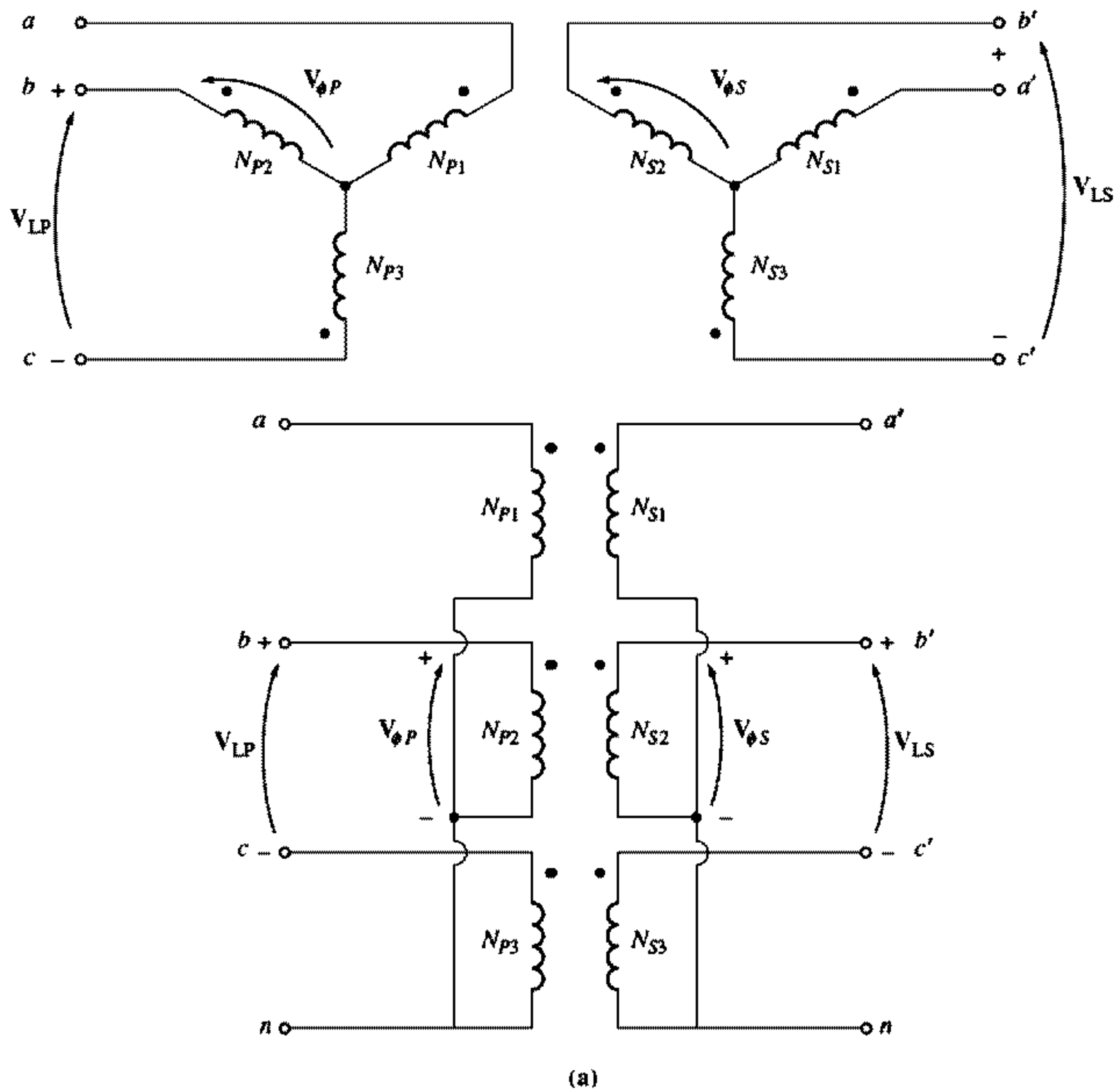


FIGURE 2–38
Three-phase transformer connections and wiring diagrams: (a) Y–Y; (b) Y–Δ; (c) Δ–Y; (d) Δ–Δ.

The result is a very large third-harmonic component of voltage on top of the 50- or 60-Hz fundamental voltage. This third-harmonic voltage can be larger than the fundamental voltage itself.

Both the unbalance problem and the third-harmonic problem can be solved using one of two techniques:

1. *Solidly ground the neutrals of the transformers*, especially the primary winding's neutral. This connection permits the additive third-harmonic components to cause a current flow in the neutral instead of building up large voltages. The neutral also provides a return path for any current imbalances in the load.
2. *Add a third (tertiary) winding connected in* Δ to the transformer bank. If a third Δ-connected winding is added to the transformer, then the third-harmonic

components of voltage in the Δ will add up, causing a circulating current flow within the winding. This suppresses the third-harmonic components of voltage in the same manner as grounding the transformer neutrals.

The Δ-connected tertiary windings need not even be brought out of the transformer case, but they often are used to supply lights and auxiliary power within the substation where it is located. The tertiary windings must be large enough to handle the circulating currents, so they are usually made about one-third the power rating of the two main windings.

One or the other of these correction techniques *must* be used any time a Y–Y transformer is installed. In practice, very few Y–Y transformers are used, since the same jobs can be done by one of the other types of three-phase transformers.

WYE-DELTA CONNECTION. The Y–Δ connection of three-phase transformers is shown in Figure 2–38b. In this connection, the primary line voltage is related to the primary phase voltage by $V_{LP} = \sqrt{3}V_{\phi P}$, while the secondary line voltage is equal to the secondary phase voltage $V_{LS} = V_{\phi S}$. The voltage ratio of each phase is

$$\frac{V_{\phi P}}{V_{\phi S}} = a$$

so the overall relationship between the line voltage on the primary side of the bank and the line voltage on the secondary side of the bank is

$$\frac{V_{LP}}{V_{LS}} = \frac{\sqrt{3}V_{\phi P}}{V_{\phi S}}$$

$$\boxed{\frac{V_{LP}}{V_{LS}} = \sqrt{3}a \qquad \text{Y–}\Delta} \tag{2–88}$$

The Y–Δ connection has no problem with third-harmonic components in its voltages, since they are consumed in a circulating current on the Δ side. This connection is also more stable with respect to unbalanced loads, since the Δ partially redistributes any imbalance that occurs.

This arrangement does have one problem, though. Because of the connection, the secondary voltage is shifted 30° relative to the primary voltage of the transformer. The fact that a phase shift has occurred can cause problems in paralleling the secondaries of two transformer banks together. The phase angles of transformer secondaries must be equal if they are to be paralleled, which means that attention must be paid to the direction of the 30° phase shift occurring in each transformer bank to be paralleled together.

In the United States, it is customary to make the secondary voltage lag the primary voltage by 30°. Although this is the standard, it has not always been observed, and older installations must be checked very carefully before a new transformer is paralleled with them, to make sure that their phase angles match.

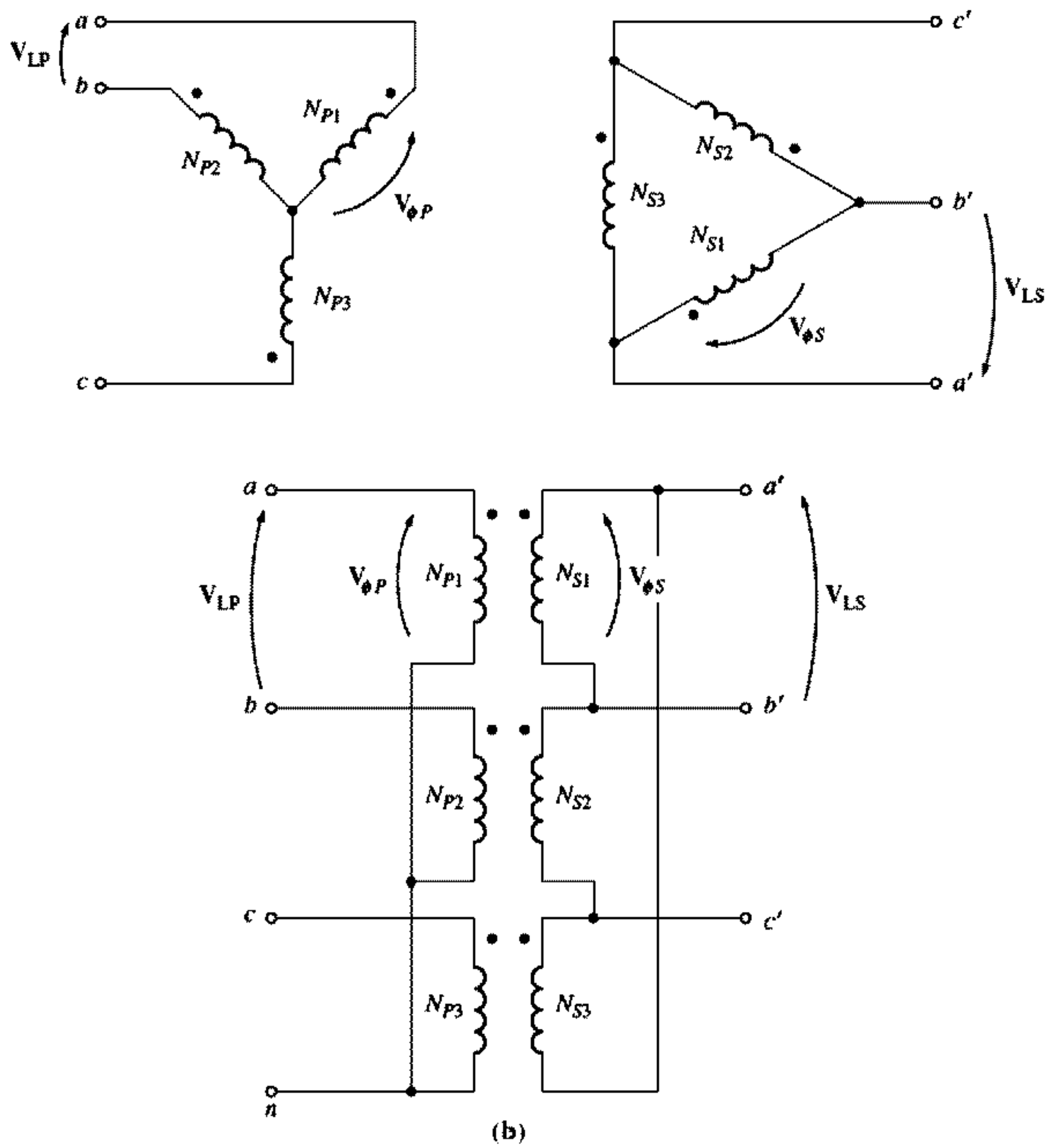
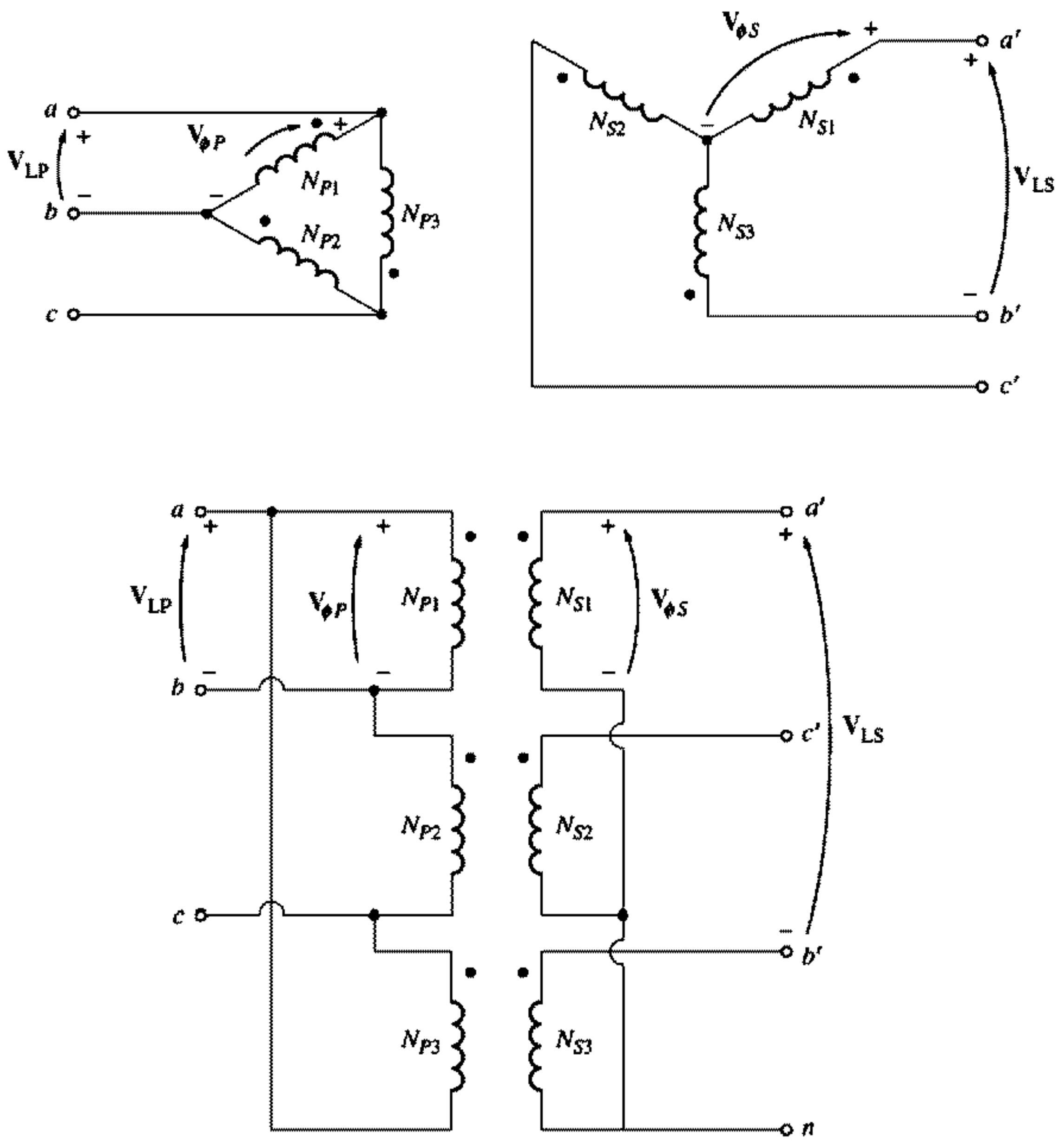


FIGURE 2–38
(b) Y–Δ *(continued)*

The connection shown in Figure 2–38b will cause the secondary voltage to be lagging if the system phase sequence is *abc*. If the system phase sequence is *acb*, then the connection shown in Figure 2–38b will cause the secondary voltage to be leading the primary voltage by 30°.

DELTA–WYE CONNECTION. A Δ–Y connection of three-phase transformers is shown in Figure 2–38c. In a Δ–Y connection, the primary line voltage is equal to the primary-phase voltage $V_{LP} = V_{\phi P}$, while the secondary voltages are related by $V_{LS} = \sqrt{3} V_{\phi S}$. Therefore, the line-to-line voltage ratio of this transformer connection is



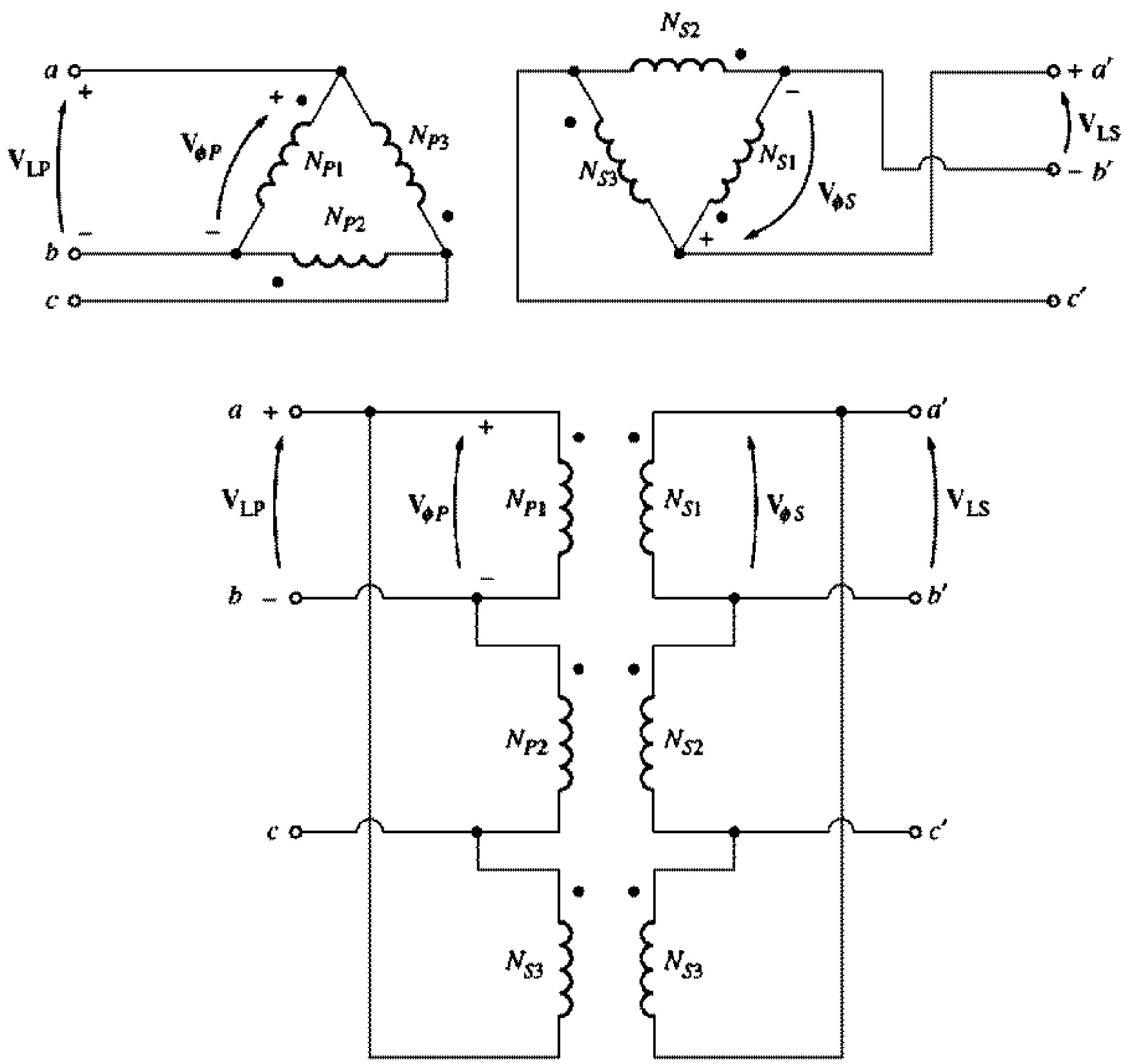
(c)

FIGURE 2–38
(c) Δ–Y *(continued)*

$$\frac{V_{LP}}{V_{LS}} = \frac{V_{\phi P}}{\sqrt{3}V_{\phi S}}$$

$$\boxed{\frac{V_{LP}}{V_{LS}} = \frac{\sqrt{3}}{a} \qquad \Delta\text{–Y}} \tag{2–89}$$

This connection has the same advantages and the same phase shift as the Y–Δ transformer. The connection shown in Figure 2–38c makes the secondary voltage lag the primary voltage by 30°, as before.



(d)

FIGURE 2–38
(d) Δ–Δ *(concluded)*

DELTA–DELTA CONNECTION. The Δ–Δ connection is shown in Figure 2–38d. In a Δ–Δ connection, $V_{LP} = V_{\phi P}$ and $V_{LS} = V_{\phi S}$, so the relationship between primary and secondary line voltages is

$$\boxed{\frac{V_{LP}}{V_{LS}} = \frac{V_{\phi P}}{V_{\phi S}} = a \qquad \Delta-\Delta} \tag{2–90}$$

This transformer has no phase shift associated with it and no problems with unbalanced loads or harmonics.

The Per-Unit System for Three-Phase Transformers

The per-unit system of measurements applies just as well to three-phase transformers as to single-phase transformers. The single-phase base equations (2–53) to (2–56) apply to three-phase systems on a *per-phase* basis. If the total base voltampere value of the transformer bank is called S_{base}, then the base voltampere value of one of the transformers $S_{1\phi,\text{base}}$ is

$$S_{1\phi,\text{base}} = \frac{S_{\text{base}}}{3} \tag{2–91}$$

and the base phase current and impedance of the transformer are

$$I_{\phi,\text{base}} = \frac{S_{1\phi,\text{base}}}{V_{\phi,\text{base}}} \tag{2–92a}$$

$$\boxed{I_{\phi,\text{base}} = \frac{S_{\text{base}}}{3\,V_{\phi,\text{base}}}} \tag{2–92b}$$

$$Z_{\text{base}} = \frac{(V_{\phi,\text{base}})^2}{S_{1\phi,\text{base}}} \tag{2–93a}$$

$$\boxed{Z_{\text{base}} = \frac{3(V_{\phi,\text{base}})^2}{S_{\text{base}}}} \tag{2–93b}$$

Line quantities on three-phase transformer banks can also be represented in the per-unit system. The relationship between the base line voltage and the base phase voltage of the transformer depends on the connection of windings. If the windings are connected in delta, $V_{L,\text{base}} = V_{\phi,\text{base}}$, while if the windings are connected in wye, $V_{L,\text{base}} = \sqrt{3}V_{\phi,\text{base}}$. The base line current in a three-phase transformer bank is given by

$$I_{L,\text{base}} = \frac{S_{\text{base}}}{\sqrt{3}V_{L,\text{base}}} \tag{2–94}$$

The application of the per-unit system to three-phase transformer problems is similar to its application in the single-phase examples already given.

Example 2–9. A 50-kVA 13,800/208-V Δ-Y distribution transformer has a resistance of 1 percent and a reactance of 7 percent per unit.

(a) What is the transformer's phase impedance referred to the high-voltage side?
(b) Calculate this transformer's voltage regulation at full load and 0.8 PF lagging, using the calculated high-side impedance.
(c) Calculate this transformer's voltage regulation under the same conditions, using the per-unit system.

Solution

(a) The high-voltage side of this transformer has a base line voltage of 13,800 V and a base apparent power of 50 kVA. Since the primary is Δ-connected, its phase voltage is equal to its line voltage. Therefore, its base impedance is

$$Z_{\text{base}} = \frac{3(V_{\phi,\text{base}})^2}{S_{\text{base}}} \tag{2–93b}$$

$$= \frac{3(13{,}800\text{ V})^2}{50{,}000\text{ VA}} = 11{,}426\ \Omega$$

The per-unit impedance of the transformer is

$$Z_{eq} = 0.01 + j0.07\text{ pu}$$

so the high-side impedance in ohms is

$$Z_{eq} = Z_{eq,pu} Z_{base}$$
$$= (0.01 + j0.07\text{ pu})(11{,}426\ \Omega) = 114.2 + j800\ \Omega$$

(b) To calculate the voltage regulation of a three-phase transformer bank, determine the voltage regulation of any single transformer in the bank. The voltages on a single transformer are phase voltages, so

$$\text{VR} = \frac{V_{\phi P} - aV_{\phi S}}{aV_{\phi S}} \times 100\%$$

The rated transformer phase voltage on the primary is 13,800 V, so the rated phase current on the primary is given by

$$I_\phi = \frac{S}{3V_\phi}$$

The rated apparent power $S = 50$ kVA, so

$$I_\phi = \frac{50{,}000\text{ VA}}{3(13{,}800\text{ V})} = 1.208\text{ A}$$

The rated phase voltage on the secondary of the transformer is 208 V/ $\sqrt{3}$ = 120 V. When referred to the high-voltage side of the transformer, this voltage becomes $V'_{\phi S} = aV_{\phi S}$ = 13,800 V. Assume that the transformer secondary is operating at the rated voltage and current, and find the resulting primary phase voltage:

$$\mathbf{V}_{\phi P} = a\mathbf{V}_{\phi S} + R_{eq}\mathbf{I}_\phi + jX_{eq}\mathbf{I}_\phi$$
$$= 13{,}800\angle 0^\circ\text{ V} + (114.2\ \Omega)(1.208\angle -36.87^\circ\text{ A}) + (j800\ \Omega)(1.208\angle -36.87^\circ\text{ A})$$
$$= 13{,}800 + 138\angle -36.87^\circ + 966.4\angle 53.13^\circ$$
$$= 13{,}800 + 110.4 - j82.8 + 579.8 + j773.1$$
$$= 14{,}490 + j690.3 = 14{,}506\angle 2.73^\circ\text{ V}$$

Therefore,

$$\text{VR} = \frac{V_{\phi P} - aV_{\phi S}}{aV_{\phi S}} \times 100\%$$
$$= \frac{14{,}506 - 13{,}800}{13{,}800} \times 100\% = 5.1\%$$

(c) In the per-unit system, the output voltage is $1\angle 0^\circ$, and the current is $1\angle -36.87^\circ$. Therefore, the input voltage is

$$V_P = 1\angle 0^\circ + (0.01)(1\angle -36.87^\circ) + (j0.07)(1\angle -36.87^\circ)$$
$$= 1 + 0.008 - j0.006 + 0.042 + j0.056$$
$$= 1.05 + j0.05 = 1.051\angle 2.73^\circ$$

The voltage regulation is

$$\mathrm{VR} = \frac{1.051 - 1.0}{1.0} \times 100\% = 5.1\%$$

Of course, the voltage regulation of the transformer bank is the same whether the calculations are done in actual ohms or in the per-unit system.

2.11 THREE-PHASE TRANSFORMATION USING TWO TRANSFORMERS

In addition to the standard three-phase transformer connections, there are ways to perform three-phase transformation with only two transformers. All techniques that do so involve a reduction in the power-handling capability of the transformers, but they may be justified by certain economic situations.

Some of the more important two-transformer connections are

1. The open-Δ (or V–V) connection
2. The open-Y–open-Δ connection
3. The Scott-T connection
4. The three-phase T connection

Each of these transformer connections is described below.

The Open-Δ (or V–V) Connection

In some situations a full transformer bank may not be used to accomplish three-phase transformation. For example, suppose that a Δ–Δ transformer bank composed of separate transformers has a damaged phase that must be removed for repair. The resulting situation is shown in Figure 2–39. If the two remaining secondary voltages are $\mathbf{V}_A = V\angle 0°$ and $\mathbf{V}_A = V\angle 120°$ V, then the voltage across the gap where the third transformer used to be is given by

$$\begin{aligned}
\mathbf{V}_C &= -\mathbf{V}_A - \mathbf{V}_B \\
&= -V\angle 0° - V\angle -120° \\
&= -V - (-0.5V - j0.866V) \\
&= -0.5V + j0.866V \\
&= V\angle 120° \qquad \mathrm{V}
\end{aligned}$$

This is exactly the same voltage that would be present if the third transformer were still there. Phase C is sometimes called a ***ghost phase***. Thus, the open-delta connection lets a transformer bank get by with only two transformers, allowing some power flow to continue even with a damaged phase removed.

How much apparent power can the bank supply with one of its three transformers removed? At first, it seems that it could supply two-thirds of its rated

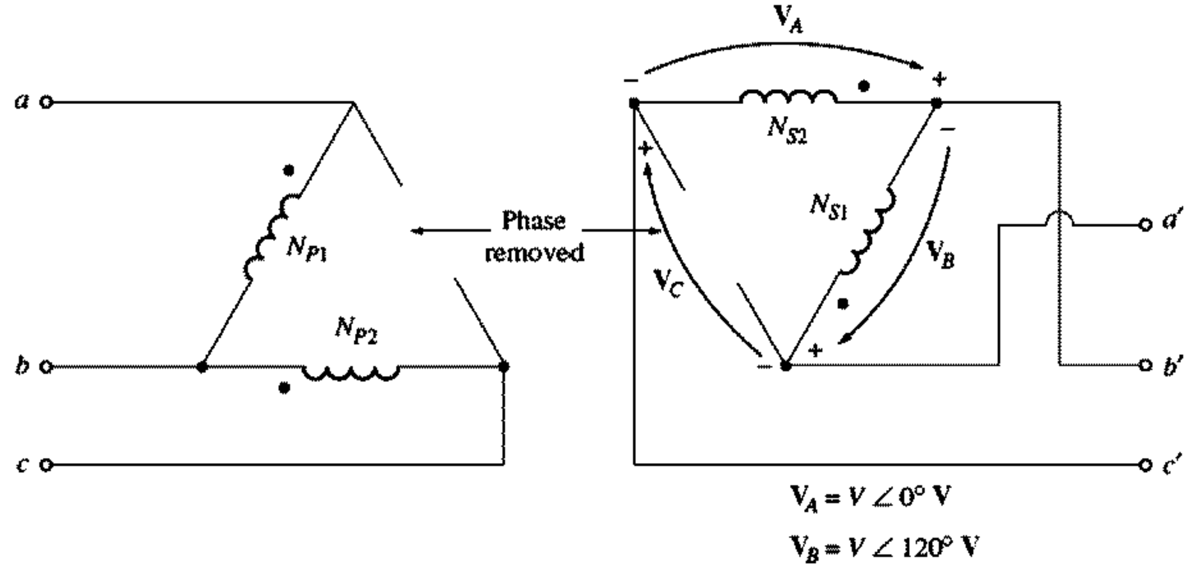


FIGURE 2–39
The open-Δ or V–V transformer connection.

apparent power, since two-thirds of the transformers are still present. Things are not quite that simple, though. To understand what happens when a transformer is removed, see Figure 2–40.

Figure 2–40a shows the transformer bank in normal operation connected to a resistive load. If the rated voltage of one transformer in the bank is V_ϕ and the rated current is I_ϕ, then the maximum power that can be supplied to the load is

$$P = 3V_\phi I_\phi \cos\theta$$

The angle between the voltage V_ϕ and the current I_ϕ in each phase is 0° so the total power supplied by the transformer is

$$\begin{aligned} P &= 3V_\phi I_\phi \cos\theta \\ &= 3V_\phi I_\phi \end{aligned} \tag{2–95}$$

The open-delta transformer is shown in Figure 2–40*b*. It is important to note the angles on the voltages and currents in this transformer bank. Because one of the transformer phases is missing, the transmission line current is now equal to the phase current in each transformer, and the currents and voltages in the transformer bank differ in angle by 30°. Since the current and voltage angles differ in each of the two transformers, it is necessary to examine each transformer individually to determine the maximum power it can supply. For transformer 1, the voltage is at an angle of 150° and the current is at an angle of 120°, so the expression for the maximum power in transformer 1 is

$$\begin{aligned} P_1 &= 3V_\phi I_\phi \cos(150° - 120°) \\ &= 3V_\phi I_\phi \cos 30° \\ &= \frac{\sqrt{3}}{2} V_\phi I_\phi \end{aligned} \tag{2–96}$$

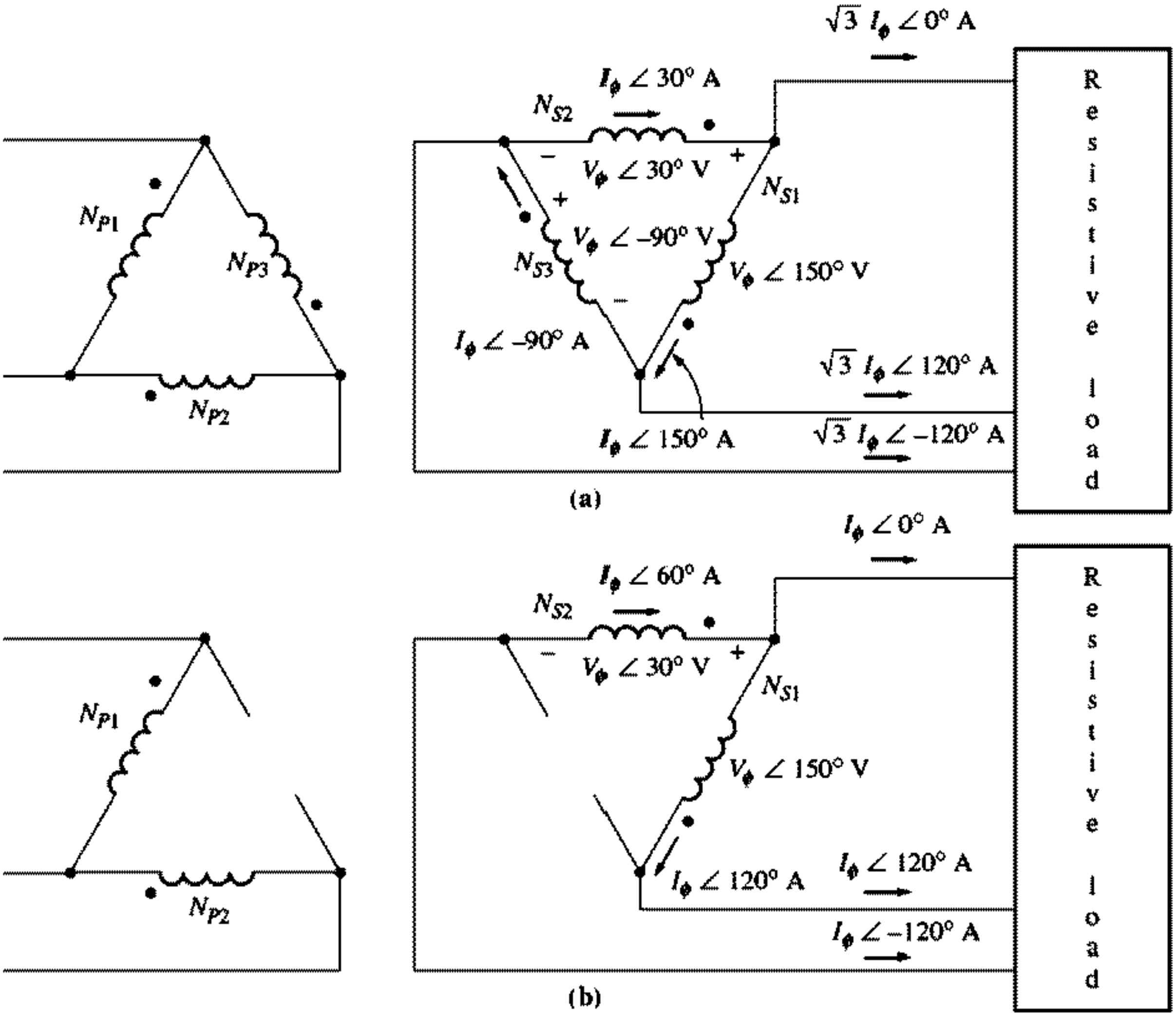


FIGURE 2–40
(a) Voltages and currents in a Δ–Δ transformer bank. (b) Voltages and currents in an open-Δ transformer bank.

For transformer 2, the voltage is at an angle of 30° and the current is at an angle of 60°, so its maximum power is

$$\begin{aligned} P_2 &= 3V_\phi I_\phi \cos(30° - 60°) \\ &= 3V_\phi I_\phi \cos(-30°) \\ &= \frac{\sqrt{3}}{2} V_\phi I_\phi \end{aligned} \tag{2–97}$$

Therefore, the total maximum power of the open-delta bank is given by

$$P = \sqrt{3} V_\phi I_\phi \tag{2–98}$$

The rated current is the same in each transformer whether there are two or three of them, and the voltage is the same on each transformer; so the ratio of the output power available from the open-delta bank to the output power available from the normal three-phase bank is

$$\frac{P_{\text{open}\,\Delta}}{P_{3\text{ phase}}} = \frac{\sqrt{3}V_\phi I_\phi}{3V_\phi I_\phi} = \frac{1}{\sqrt{3}} = 0.577 \tag{2–99}$$

The available power out of the open-delta bank is only 57.7 percent of the original bank's rating.

A good question that could be asked is: What happens to the rest of the open-delta bank's rating? After all, the total power that the two transformers together can produce is two-thirds that of the original bank's rating. To find out, examine the reactive power of the open-delta bank. The reactive power of transformer 1 is

$$\begin{aligned} Q_1 &= 3V_\phi I_\phi \sin(150^\circ - 120^\circ) \\ &= 3V_\phi I_\phi \sin 30^\circ \\ &= \tfrac{1}{2} V_\phi I_\phi \end{aligned}$$

The reactive power of transformer 2 is

$$\begin{aligned} Q_2 &= 3V_\phi I_\phi \sin(30^\circ - 60^\circ) \\ &= 3V_\phi I_\phi \sin(-30^\circ) \\ &= -\tfrac{1}{2} V_\phi I_\phi \end{aligned}$$

Thus one transformer is producing reactive power which the other one is consuming. It is this exchange of energy between the two transformers that limits the power output to 57.7 percent of the *original bank's rating* instead of the otherwise expected 66.7 percent.

An alternative way to look at the rating of the open-delta connection is that 86.6 percent of the rating *of the two remaining transformers* can be used.

Open-delta connections are used occasionally when it is desired to supply a small amount of three-phase power to an otherwise single-phase load. In such a case, the connection in Figure 2–41 can be used, where transformer T_2 is much larger than transformer T_1.

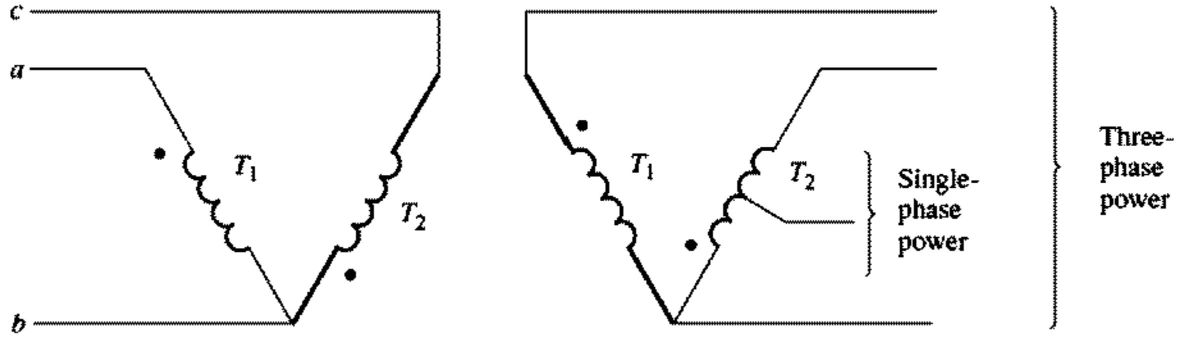


FIGURE 2–41

Using an open-Δ transformer connection to supply a small amount of three-phase power along with a lot of single-phase power. Transformer T_2 is much larger than transformer T_1.

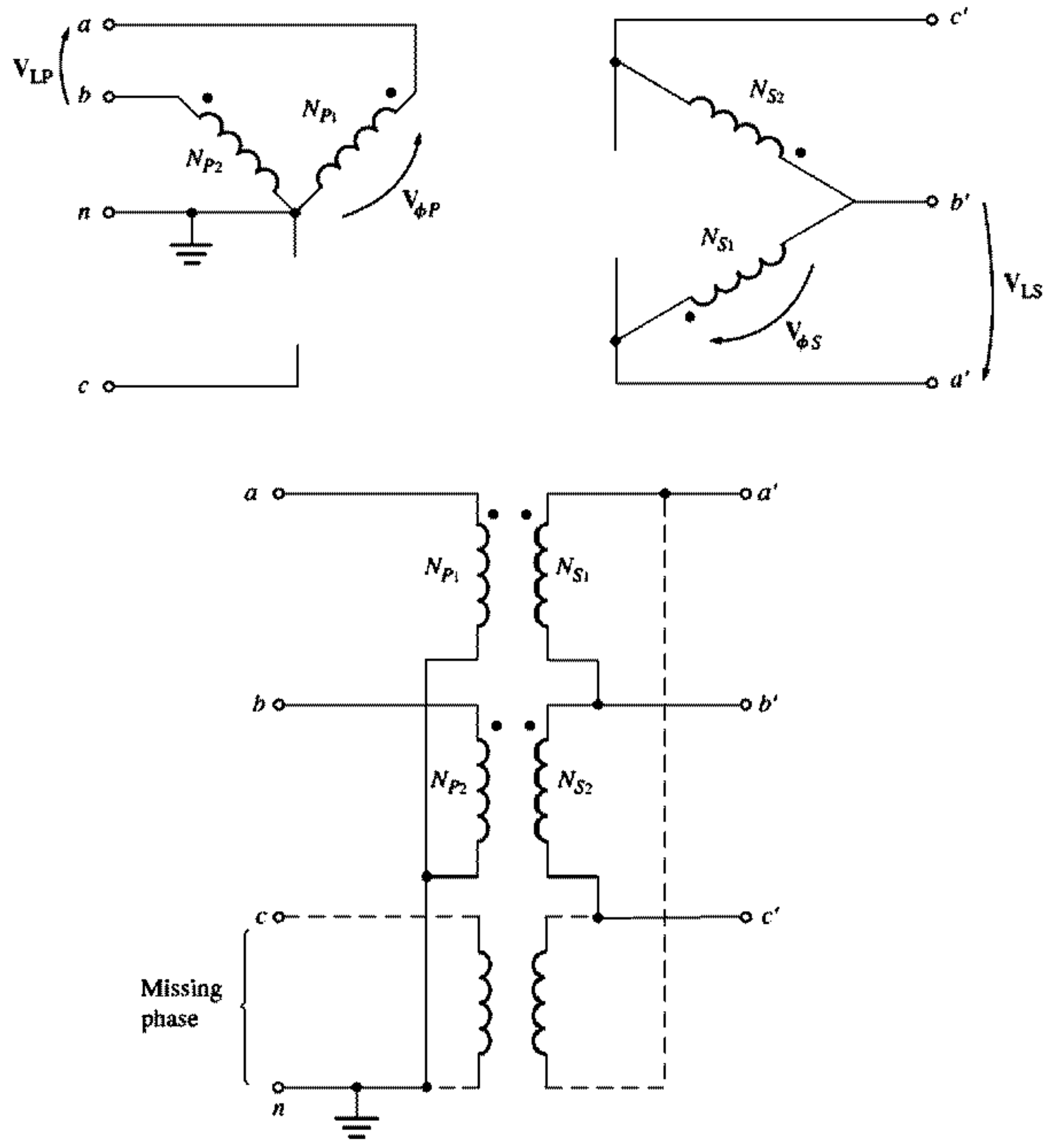


FIGURE 2-42
The open-Y–open-Δ transformer connection and wiring diagram. Note that this connection is identical to the Y–Δ connection in Figure 3–38*b*, except for the absence of the third transformer and the presence of the neutral lead.

The Open-Wye–Open-Delta Connection

The open-wye–open-delta connection is very similar to the open-delta connection except that the primary voltages are derived from two phases and the neutral. This type of connection is shown in Figure 2–42. It is used to serve small commercial customers needing three-phase service in rural areas where all three phases are not yet present on the power poles. With this connection, a customer can get three-phase service in a makeshift fashion until demand requires installation of the third phase on the power poles.

A major disadvantage of this connection is that a very large return current must flow in the neutral of the primary circuit.

The Scott-T Connection

The Scott-T connection is a way to derive two phases 90° apart from a three-phase power supply. In the early history of ac power transmission, two-phase and three-phase power systems were quite common. In those days, it was routinely necessary to interconnect two- and three-phase power systems, and the Scott-T transformer connection was developed for that purpose.

Today, two-phase power is primarily limited to certain control applications, but the Scott T is still used to produce the power needed to operate them.

The Scott T consists of two single-phase transformers with identical ratings. One has a tap on its primary winding at 86.6 percent of full-load voltage. They are connected as shown in Figure 2–43a. The 86.6 percent tap of transformer T_2 is connected to the center tap of transformer T_1. The voltages applied to the primary winding are shown in Figure 2–43b, and the resulting voltages applied to the primaries of the two transformers are shown in Figure 2–43c. Since these voltages are 90° apart, they result in a two-phase output.

It is also possible to convert two-phase power into three-phase power with this connection, but since there are very few two-phase generators in use, this is rarely done.

The Three-Phase T Connection

The Scott-T connection uses two transformers to convert *three-phase power* to *two-phase power* at a different voltage level. By a simple modification of that connection, the same two transformers can also convert *three-phase power* to *three-phase power* at a different voltage level. Such a connection is shown in Figure 2–44. Here both the primary and the secondary windings of transformer T_2 are tapped at the 86.6 percent point, and the taps are connected to the center taps of the corresponding windings on transformer T_1. In this connection T_1 is called the *main transformer* and T_2 is called the *teaser transformer*.

As in the Scott T, the three-phase input voltage produces two voltages 90° apart on the primary windings of the transformers. These primary voltages produce secondary voltages which are also 90° apart. Unlike the Scott T, though, the secondary voltages are recombined into a three-phase output.

One major advantage of the three-phase T connection over the other three-phase two-transformer connections (the open-delta and open-wye–open-delta) is that a neutral can be connected to both the primary side and the secondary side of the transformer bank. This connection is sometimes used in self-contained three-phase distribution transformers, since its construction costs are lower than those of a full three-phase transformer bank.

Since the bottom parts of the teaser transformer windings are not used on either the primary or the secondary sides, they could be left off with no change in performance. This is, in fact, typically done in distribution transformers.

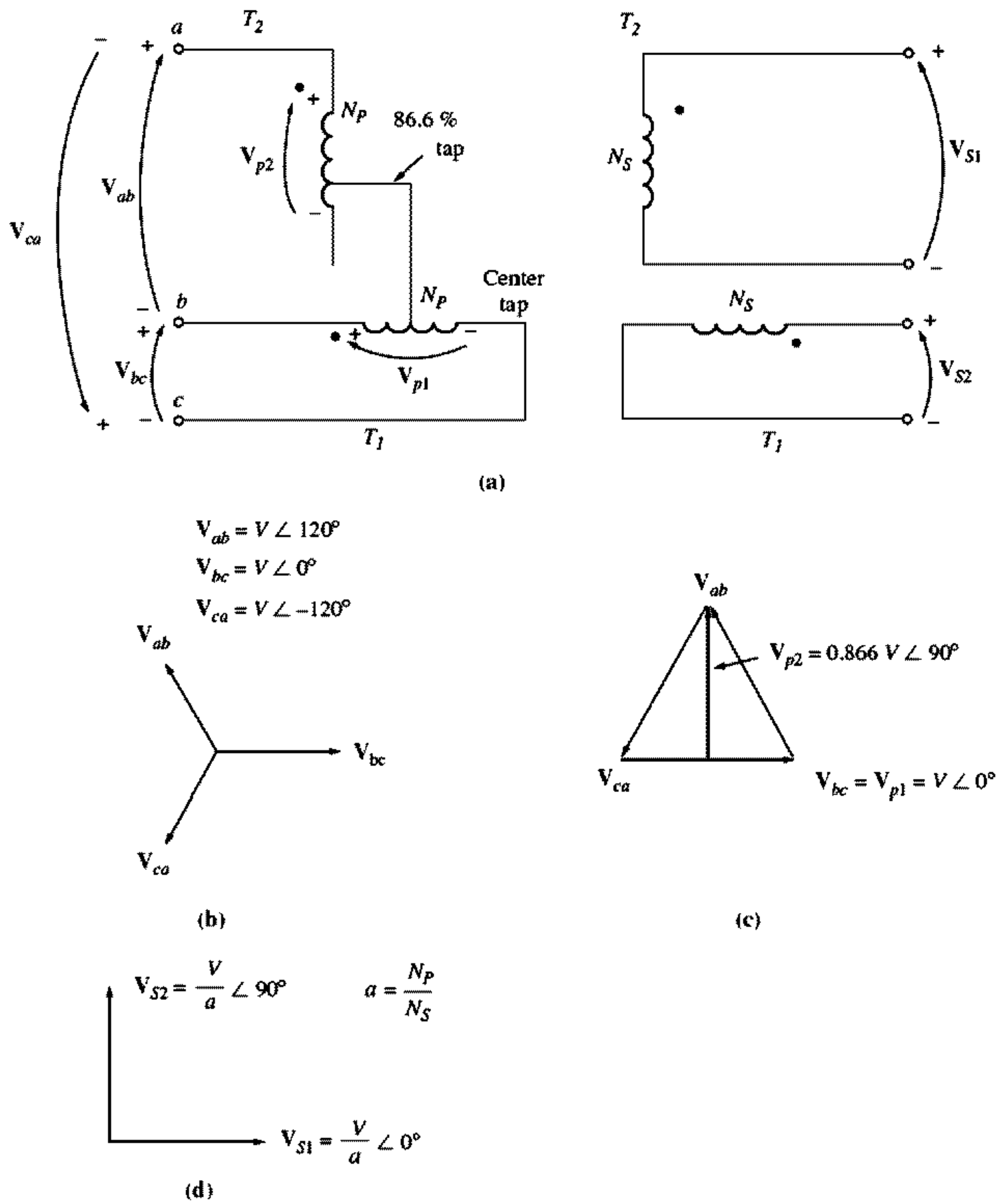


FIGURE 2–43
The Scott-T transformer connection. (a) Wiring diagram; (b) the three-phase input voltages; (c) the voltages on the transformer primary windings; (d) the two-phase secondary voltages.

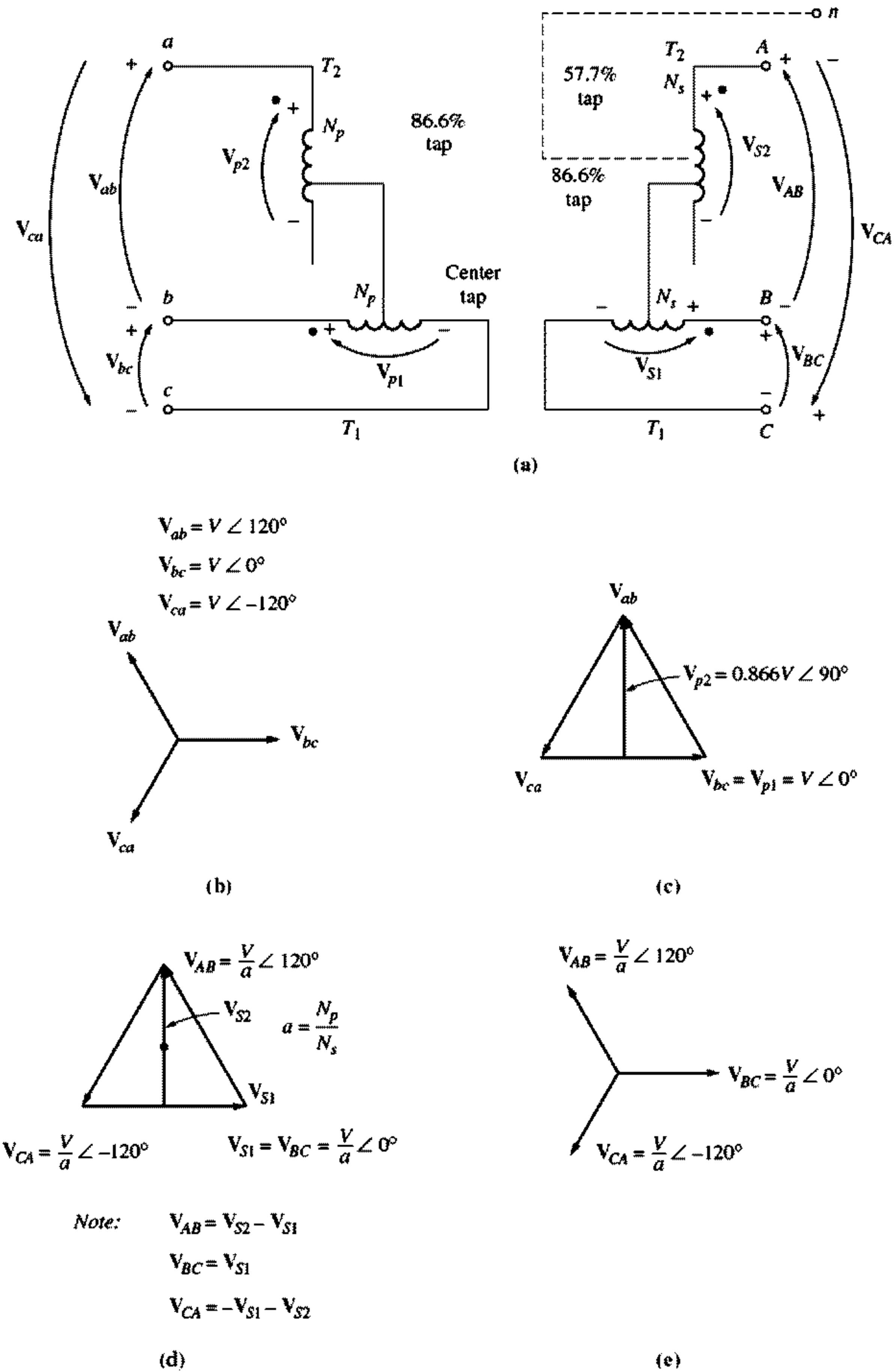


FIGURE 2–44
The three-phase T transformer connection. (a) Wiring diagram; (b) the three-phase input voltages; (c) the voltages on the transformer primary windings; (d) the voltages on the transformer secondary windings; (e) the resulting three-phase secondary voltages.

2.12 TRANSFORMER RATINGS AND RELATED PROBLEMS

Transformers have four major ratings: apparent power, voltage, current, and frequency. This section examines the ratings of a transformer and explains why they are chosen the way they are. It also considers the related question of the current inrush that occurs when a transformer is first connected to the line.

The Voltage and Frequency Ratings of a Transformer

The voltage rating of a transformer serves two functions. One is to protect the winding insulation from breakdown due to an excessive voltage applied to it. This is not the most serious limitation in practical transformers. The second function is related to the magnetization curve and magnetization current of the transformer. Figure 2–11 shows a magnetization curve for a transformer. If a steady-state voltage

$$v(t) = V_M \sin \omega t \qquad \text{V}$$

is applied to a transformer's primary winding, the flux of the transformer is given by

$$\phi(t) = \frac{1}{N_P} \int v(t)\, dt$$

$$= \frac{1}{N_P} \int V_M \sin \omega t \, dt$$

$$\boxed{\phi(t) = -\frac{V_M}{\omega N_P} \cos \omega t} \tag{2–100}$$

If the applied voltage $v(t)$ is increased by 10 percent, the resulting maximum flux in the core also increases by 10 percent. Above a certain point on the magnetization curve, though, a 10 percent increase in flux requires an increase in magnetization current *much* larger than 10 percent. This concept is illustrated in Figure 2–45. As the voltage increases, the high-magnetization currents soon become unacceptable. The maximum applied voltage (and therefore the rated voltage) is set by the maximum acceptable magnetization current in the core.

Notice that voltage and frequency are related in a reciprocal fashion if the maximum flux is to be held constant:

$$\phi_{max} = \frac{V_{max}}{\omega N_P} \tag{2–101}$$

Thus, *if a 60-Hz transformer is to be operated on 50 Hz, its applied voltage must also be reduced by one-sixth* or the peak flux in the core will be too high. This reduction in applied voltage with frequency is called *derating*. Similarly, a 50-Hz transformer may be operated at a 20 percent higher voltage on 60 Hz if this action does not cause insulation problems.

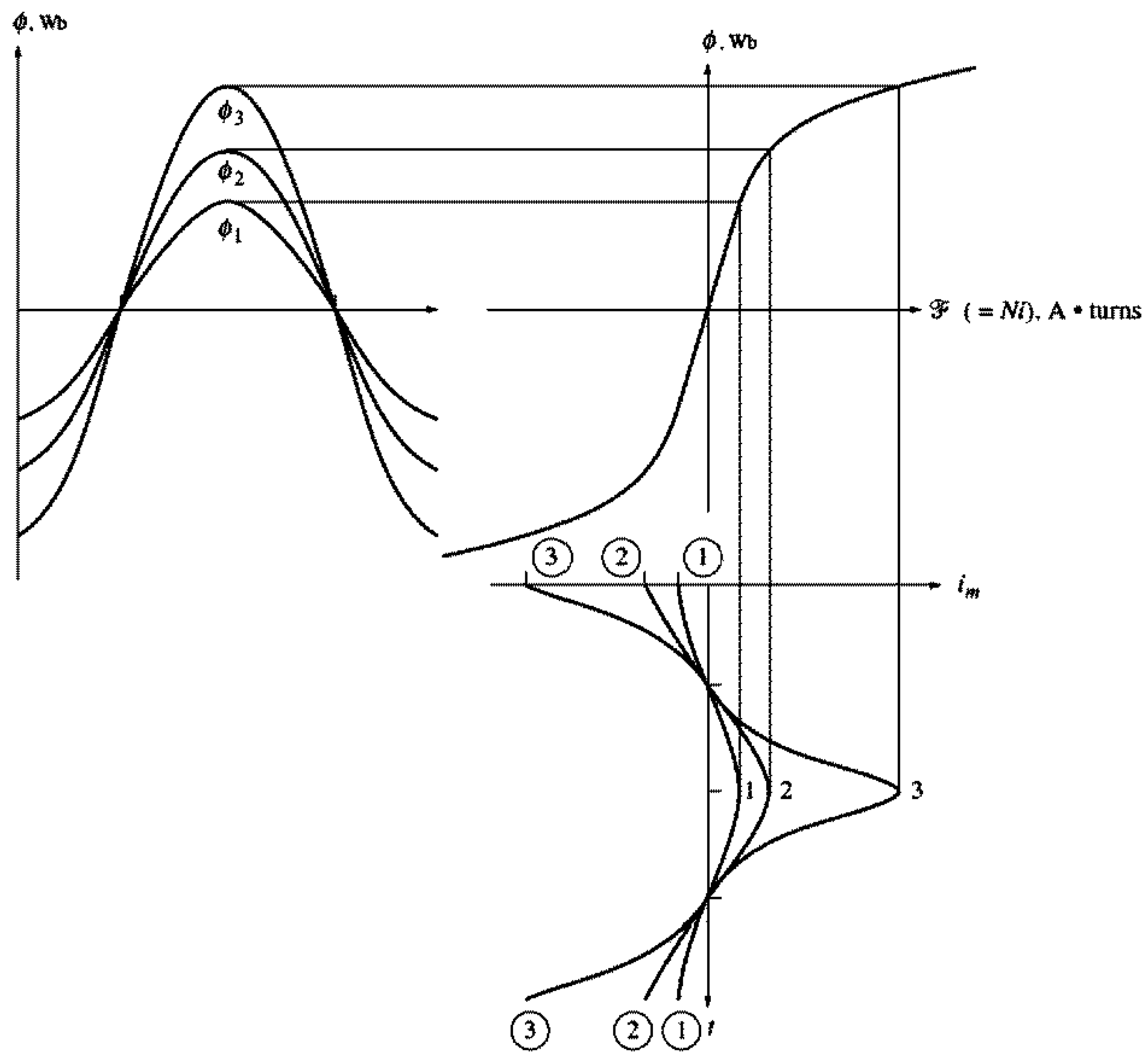


FIGURE 2–45
The effect of the peak flux in a transformer core upon the required magnetization current.

Example 2–10. A 1-kVA, 230/115-V, 60-Hz single-phase transformer has 850 turns on the primary winding and 425 turns on the secondary winding. The magnetization curve for this transformer is shown in Figure 2–46.

(*a*) Calculate and plot the magnetization current of this transformer when it is run at 230 V on a 60-Hz power source. What is the rms value of the magnetization current?

(*b*) Calculate and plot the magnetization current of this transformer when it is run at 230 V on a 50-Hz power source. What is the rms value of the magnetization current? How does this current compare to the magnetization current at 60 Hz?

Solution

The best way to solve this problem is to calculate the flux as a function of time for this core, and then use the magnetization curve to transform each flux value to a corresponding magnetomotive force. The magnetizing current can then be determined from the equation

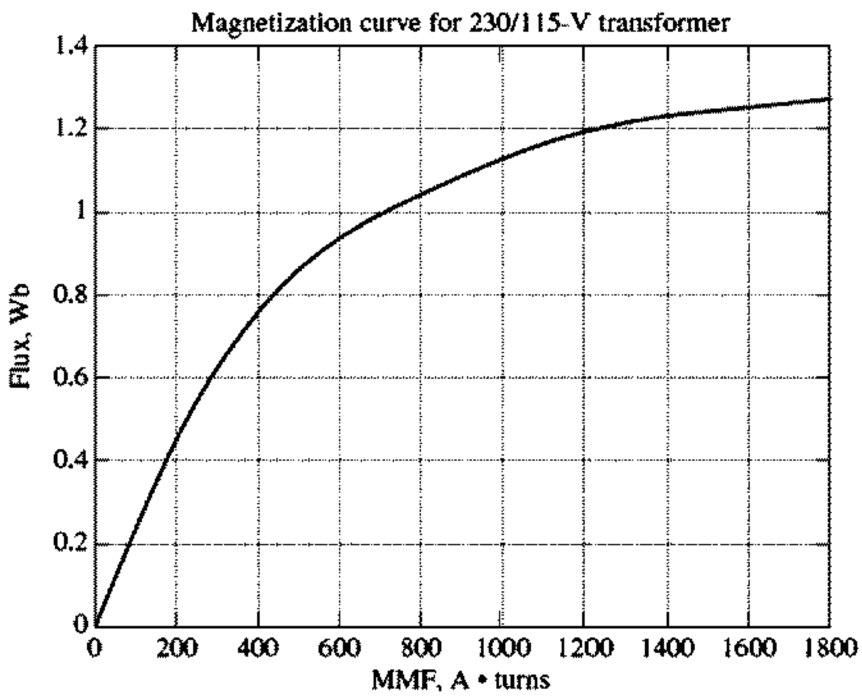


FIGURE 2–46
Magnetization curve for the 230/115-V transformer of Example 2–10.

$$i = \frac{\mathcal{F}}{N_P} \tag{2–102}$$

Assuming that the voltage applied to the core is $v(t) = V_M \sin \omega t$ volts, the flux in the core as a function of time is given by Equation (2–101):

$$\boxed{\phi(t) = -\frac{V_M}{\omega N_P} \cos \omega t} \tag{2–100}$$

The magnetization curve for this transformer is available electronically in a file called `mag_curve_1.dat`. This file can be used by MATLAB to translate these flux values into corresponding mmf values, and Equation (2–102) can be used to find the required magnetization current values. Finally, the rms value of the magnetization current can be calculated from the equation

$$I_{\text{rms}} = \sqrt{\frac{1}{T}\int_0^T i^2\, dt} \tag{2–103}$$

A MATLAB program to perform these calculations is shown below:

```
% M-file: mag_current.m
% M-file to calculate and plot the magnetization
% current of a 230/115 transformer operating at
% 230 volts and 50/60 Hz. This program also
% calculates the rms value of the mag. current.

% Load the magnetization curve. It is in two
% columns, with the first column being mmf and
% the second column being flux.
load mag_curve_1.dat;
mmf_data = mag_curve_1(:,1);
```

```
flux_data = mag_curve_1(:,2);

% Initialize values
VM = 325;                          % Maximum voltage (V)
NP = 850;                          % Primary turns

% Calculate angular velocity for 60 Hz
freq = 60;                         % Freq (Hz)
w = 2 * pi * freq;

% Calculate flux versus time
time = 0:1/3000:1/30;              % 0 to 1/30 sec
flux = -VM/(w*NP) * cos(w .* time);

% Calculate the mmf corresponding to a given flux
% using the flux's interpolation function.
mmf = interp1(flux_data,mmf_data,flux);

% Calculate the magnetization current
im = mmf / NP;

% Calculate the rms value of the current
irms = sqrt(sum(im.^2)/length(im));
disp(['The rms current at 60 Hz is ', num2str(irms)]);

% Plot the magnetization current.
figure(1)
subplot(2,1,1);
plot(time,im);
title ('\bfMagnetization Current at 60 Hz');
xlabel ('\bfTime (s)');
ylabel ('\bf\itI_{m} \rm(A)');
axis([0 0.04 -2 2]);
grid on;

% Calculate angular velocity for 50 Hz
freq = 50;                           % Freq (Hz)
w = 2 * pi * freq;

% Calculate flux versus time
time = 0:1/2500:1/25;                % 0 to 1/25 sec
flux = -VM/(w*NP) * cos(w .* time);

% Calculate the mmf corresponding to a given flux
% using the flux's interpolation function.
mmf = interp1(flux_data,mmf_data,flux);

% Calculate the magnetization current
im = mmf / NP;

% Calculate the rms value of the current
irms = sqrt(sum(im.^2)/length(im));
disp(['The rms current at 50 Hz is ', num2str(irms)]);
```

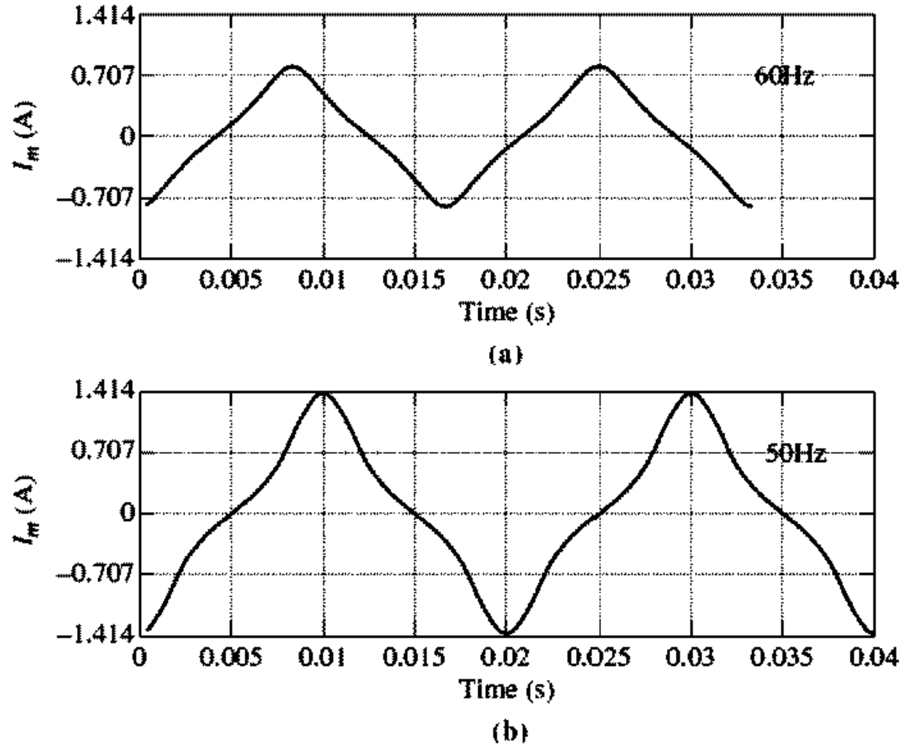


FIGURE 2–47
(a) Magnetization current for the transformer operating at 60 Hz. (b) Magnetization current for the transformer operating at 50 Hz.

```
% Plot the magnetization current.
subplot(2,1,2);
plot(time,im);
title ('\bfMagnetization Current at 50 Hz');
xlabel ('\bfTime (s)');
ylabel ('\bf\itI_{m} \rm(A)');
axis([0 0.04 -2 2]);
grid on;
```

When this program executes, the results are

```
» mag_current
The rms current at 60 Hz is 0.4894
The rms current at 50 Hz is 0.79252
```

The resulting magnetization currents are shown in Figure 2–47. Note that the rms magnetization current increases by more than 60 percent when the frequency changes from 60 Hz to 50 Hz.

The Apparent Power Rating of a Transformer

The principal purpose of the apparent power rating of a transformer is that, together with the voltage rating, it sets the current flow through the transformer windings. The current flow is important because it controls the i^2R losses in transformer, which in turn control the heating of the transformer coils. It is the heating that is critical, since overheating the coils of a transformer *drastically* shortens the life of its insulation.

The actual voltampere rating of a transformer may be more than a single value. In real transformers, there may be a voltampere rating for the transformer by itself, and another (higher) rating for the transformer with forced cooling. The key idea behind the power rating is that the hot-spot temperature in the transformer windings *must* be limited to protect the life of the transformer.

If a transformer's voltage is reduced for any reason (e.g., if it is operated at a lower frequency than normal), then the transformer's voltampere rating must be reduced by an equal amount. If this is not done, then the current in the transformer's windings will exceed the maximum permissible level and cause overheating.

The Problem of Current Inrush

A problem related to the voltage level in the transformer is the problem of current inrush at starting. Suppose that the voltage

$$v(t) = V_M \sin(\omega t + \theta) \qquad \text{V} \tag{2–104}$$

is applied at the moment the transformer is first connected to the power line. The maximum flux height reached on the first half-cycle of the applied voltage depends on the phase of the voltage at the time the voltage is applied. If the initial voltage is

$$v(t) = V_M \sin(\omega t + 90^\circ) = V_M \cos \omega t \qquad \text{V} \tag{2–105}$$

and if the initial flux in the core is zero, then the maximum flux during the first half-cycle will just equal the maximum flux at steady state:

$$\phi_{\max} = \frac{V_{\max}}{\omega N_P} \tag{2–101}$$

This flux level is just the steady-state flux, so it causes no special problems. But if the applied voltage happens to be

$$v(t) = V_M \sin \omega t \qquad \text{V}$$

the maximum flux during the first half-cycle is given by

$$\begin{aligned} \phi(t) &= \frac{1}{N_P} \int_0^{\pi/\omega} V_M \sin \omega t \, dt \\ &= -\frac{V_M}{\omega N_P} \cos \omega t \Big|_0^{\pi/\omega} \\ &= -\frac{V_M}{\omega N_P} [(-1) - (1)] \end{aligned}$$

$$\boxed{\phi_{\max} = \frac{2V_{\max}}{\omega N_P}} \tag{2–106}$$

This maximum flux is twice as high as the normal steady-state flux. If the magnetization curve in Figure 2–11 is examined, it is easy to see that doubling the

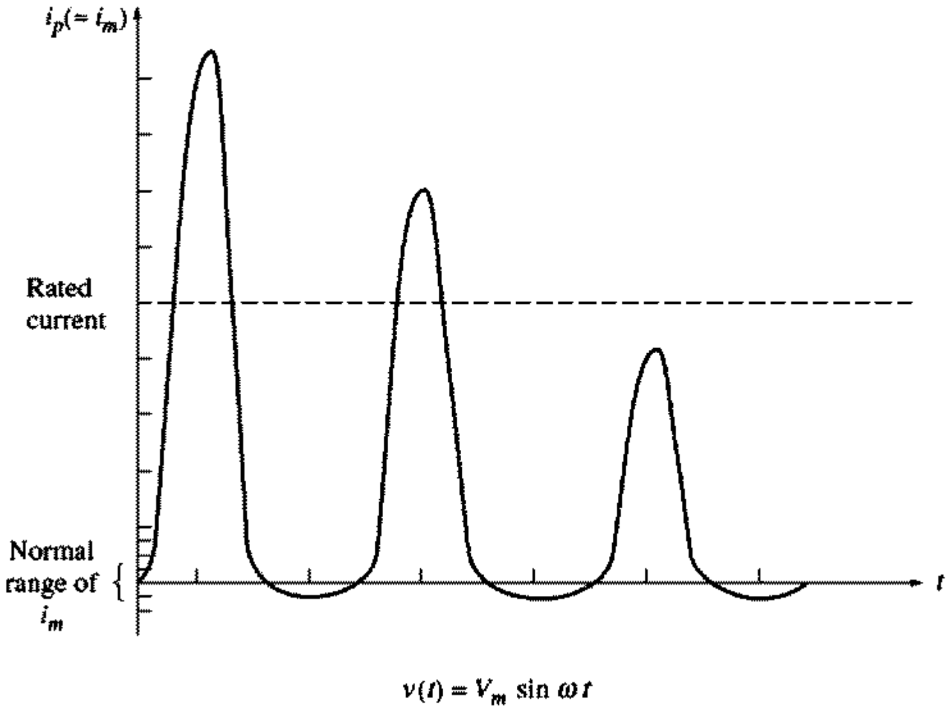


FIGURE 2–48
The current inrush due to a transformer's magnetization current on starting.

maximum flux in the core results in an *enormous* magnetization current. In fact, for part of the cycle, the transformer looks like a short circuit, and a very large current flows (see Figure 2–48).

For any other phase angle of the applied voltage between 90°, which is no problem, and 0°, which is the worst case, there is some excess current flow. The applied phase angle of the voltage is not normally controlled on starting, so there can be huge inrush currents during the first several cycles after the transformer is connected to the line. The transformer and the power system to which it is connected must be able to withstand these currents.

The Transformer Nameplate

A typical nameplate from a distribution transformer is shown in Figure 2–49. The information on such a nameplate includes rated voltage, rated kilovoltamperes, rated frequency, and the transformer per-unit series impedance. It also shows the voltage ratings for each tap on the transformer and the wiring schematic of the transformer.

Nameplates such as the one shown also typically include the transformer type designation and references to its operating instructions.

2.13 INSTRUMENT TRANSFORMERS

Two special-purpose transformers are used with power systems for taking measurements. One is the potential transformer, and the other is the current transformer.

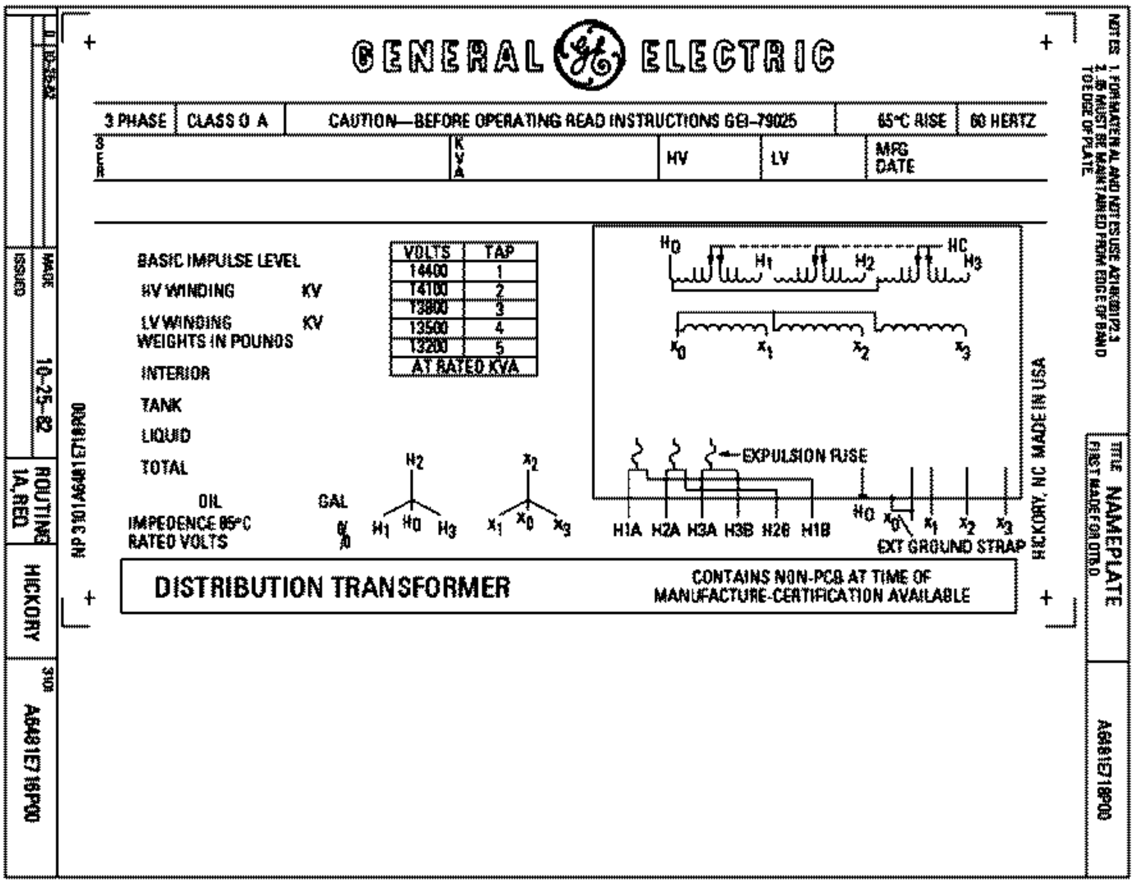


FIGURE 2–49
A sample distribution transformer nameplate. Note the ratings listed: voltage, frequency, apparent power, and tap settings. (*Courtesy of General Electric Company.*)

A *potential transformer* is a specially wound transformer with a high-voltage primary and a low-voltage secondary. It has a very low power rating, and its sole purpose is to provide a *sample* of the power system's voltage to the instruments monitoring it. Since the principal purpose of the transformer is voltage sampling, it must be very accurate so as not to distort the true voltage values too badly. Potential transformers of several *accuracy classes* may be purchased depending on how accurate the readings must be for a given application.

Current transformers sample the current in a line and reduce it to a safe and measurable level. A diagram of a typical current transformer is shown in Figure 2–50. The current transformer consists of a secondary winding wrapped around a ferromagnetic ring, with the single primary line running through the center of the ring. The ferromagnetic ring holds and concentrates a small sample of the flux from the primary line. That flux then induces a voltage and current in the secondary winding.

A current transformer differs from the other transformers described in this chapter in that its windings are *loosely coupled.* Unlike all the other transformers, the mutual flux ϕ_M in the current transformer is smaller than the leakage flux ϕ_L. Because of the loose coupling, the voltage and current ratios of Equations (2–1) to (2–5) do not apply to a current transformer. Nevertheless, the secondary current

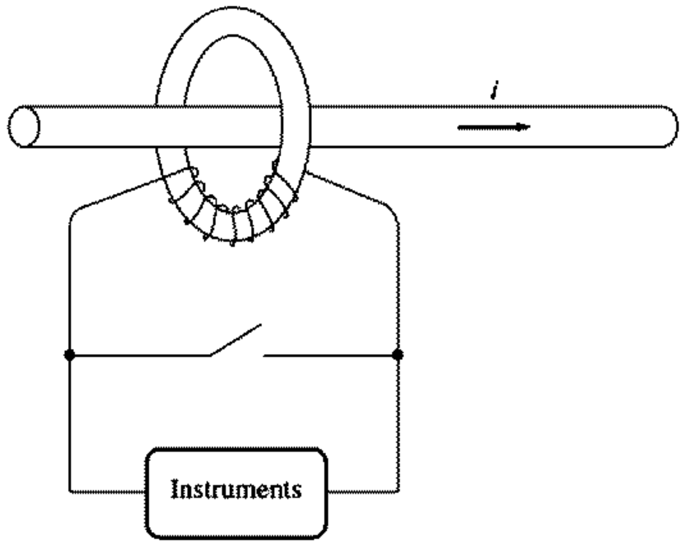


FIGURE 2–50
Sketch of a current transformer.

in a current transformer is directly proportional to the much larger primary current, and the device can provide an accurate sample of a line's current for measurement purposes.

Current transformer ratings are given as ratios of primary to secondary current. A typical current transformer ratio might be 600:5,800:5, or 1000:5. A 5-A rating is standard on the secondary of a current transformer.

It is important to keep a current transformer short-circuited at all times, since extremely high voltages can appear across its open secondary terminals. In fact, most relays and other devices using the current from a current transformer have a *shorting interlock* which must be shut before the relay can be removed for inspection or adjustment. Without this interlock, very dangerous high voltages will appear at the secondary terminals as the relay is removed from its socket.

2.14 SUMMARY

A transformer is a device for converting electric energy at one voltage level to electric energy at another voltage level through the action of a magnetic field. It plays an extremely important role in modern life by making possible the economical long-distance transmission of electric power.

When a voltage is applied to the primary of a transformer, a flux is produced in the core as given by Faraday's law. The changing flux in the core then induces a voltage in the secondary winding of the transformer. Because transformer cores have very high permeability, the net magnetomotive force required in the core to produce its flux is very small. Since the net magnetomotive force is very small, the primary circuit's magnetomotive force must be approximately equal and opposite to the secondary circuit's magnetomotive force. This fact yields the transformer current ratio.

A real transformer has leakage fluxes that pass through either the primary or the secondary winding, but not both. In addition there are hysteresis, eddy current, and copper losses. These effects are accounted for in the equivalent circuit of the transformer. Transformer imperfections are measured in a real transformer by its voltage regulation and its efficiency.

The per-unit system of measurement is a convenient way to study systems containing transformers, because in this system the different system voltage levels disappear. In addition, the per-unit impedances of a transformer expressed to its own ratings base fall within a relatively narrow range, providing a convenient check for reasonableness in problem solutions.

An autotransformer differs from a regular transformer in that the two windings of the autotransformer are connected. The voltage on one side of the transformer is the voltage across a single winding, while the voltage on the other side of the transformer is the sum of the voltages across *both* windings. Because only a portion of the power in an autotransformer actually passes through the windings, an autotransformer has a power rating advantage compared to a regular transformer of equal size. However, the connection destroys the electrical isolation between a transformer's primary and secondary sides.

The voltage levels of three-phase circuits can be transformed by a proper combination of two or three transformers. Potential transformers and current transformers can sample the voltages and currents present in a circuit. Both devices are very common in large power distribution systems.

QUESTIONS

2–1. Is the turns ratio of a transformer the same as the ratio of voltages across the transformer? Why or why not?

2–2. Why does the magnetization current impose an upper limit on the voltage applied to a transformer core?

2–3. What components compose the excitation current of a transformer? How are they modeled in the transformer's equivalent circuit?

2–4. What is the leakage flux in a transformer? Why is it modeled in a transformer equivalent circuit as an inductor?

2–5. List and describe the types of losses that occur in a transformer.

2–6. Why does the power factor of a load affect the voltage regulation of a transformer?

2–7. Why does the short-circuit test essentially show only i^2R losses and not excitation losses in a transformer?

2–8. Why does the open-circuit test essentially show only excitation losses and not i^2R losses?

2–9. How does the per-unit system of measurement eliminate the problem of different voltage levels in a power system?

2–10. Why can autotransformers handle more power than conventional transformers of the same size?

2–11. What are transformer taps? Why are they used?

2–12. What are the problems associated with the Y–Y three-phase transformer connection?

2–13. What is a TCUL transformer?

2–14. How can three-phase transformation be accomplished using only two transformers? What types of connections can be used? What are their advantages and disadvantages?

2–15. Explain why the open-Δ transformer connection is limited to supplying 57.7 percent of a normal Δ–Δ transformer bank's load.

2–16. Can a 60-Hz transformer be operated on a 50-Hz system? What actions are necessary to enable this operation?

2–17. What happens to a transformer when it is first connected to a power line? Can anything be done to mitigate this problem?

2–18. What is a potential transformer? How is it used?

2–19. What is a current transformer? How is it used?

2–20. A distribution transformer is rated at 18 kVA, 20,000/480 V, and 60 Hz. Can this transformer safely supply 15 kVA to a 415-V load at 50 Hz? Why or why not?

2–21. Why does one hear a hum when standing near a large power transformer?

PROBLEMS

2–1. The secondary winding of a transformer has a terminal voltage of $v_S(t) = 282.8 \sin 377t$ V. The turns ratio of the transformer is 100:200 ($a = 0.50$). If the secondary current of the transformer is $i_S(t) = 7.07 \sin (377t - 36.87°)$ A, what is the primary current of this transformer? What are its voltage regulation and efficiency? The impedances of this transformer referred to the primary side are

$$R_{eq} = 0.20\ \Omega \qquad R_C = 300\ \Omega$$
$$X_{eq} = 0.750\ \Omega \qquad X_M = 80\ \Omega$$

2–2. A 20-kVA 8000/480-V distribution transformer has the following resistances and reactances:

$$R_P = 32\ \Omega \qquad R_S = 0.05\ \Omega$$
$$X_P = 45\ \Omega \qquad R_S = 0.06\ \Omega$$
$$R_C = 250\ \text{k}\Omega \qquad X_M = 30\ \text{k}\Omega$$

The excitation branch impedances are given referred to the high-voltage side of the transformer.

(*a*) Find the equivalent circuit of this transformer referred to the high-voltage side.
(*b*) Find the per-unit equivalent circuit of this transformer.
(*c*) Assume that this transformer is supplying rated load at 480 V and 0.8 PF lagging. What is this transformer's input voltage? What is its voltage regulation?
(*d*) What is the transformer's efficiency under the conditions of part (*c*)?

2–3. A 1000-VA 230/115-V transformer has been tested to determine its equivalent circuit. The results of the tests are shown below.

Open-circuit test	Short-circuit test
$V_{OC} = 230$ V	$V_{SC} = 19.1$ V
$I_{OC} = 0.45$ A	$I_{SC} = 8.7$ A
$P_{OC} = 30$ W	$P_{SC} = 42.3$ W

All data given were taken from the primary side of the transformer.

(*a*) Find the equivalent circuit of this transformer referred to the low-voltage side of the transformer.

(*b*) Find the transformer's voltage regulation at rated conditions and (1) 0.8 PF lagging, (2) 1.0 PF, (3) 0.8 PF leading.

(*c*) Determine the transformer's efficiency at rated conditions and 0.8 PF lagging.

2–4. A single-phase power system is shown in Figure P2–1. The power source feeds a 100-kVA 14/2.4-kV transformer through a feeder impedance of $40.0 + j150\ \Omega$. The transformer's equivalent series impedance referred to its low-voltage side is $0.12 + j0.5\ \Omega$. The load on the transformer is 90 kW at 0.80 PF lagging and 2300 V.

(*a*) What is the voltage at the power source of the system?

(*b*) What is the voltage regulation of the transformer?

(*c*) How efficient is the overall power system?

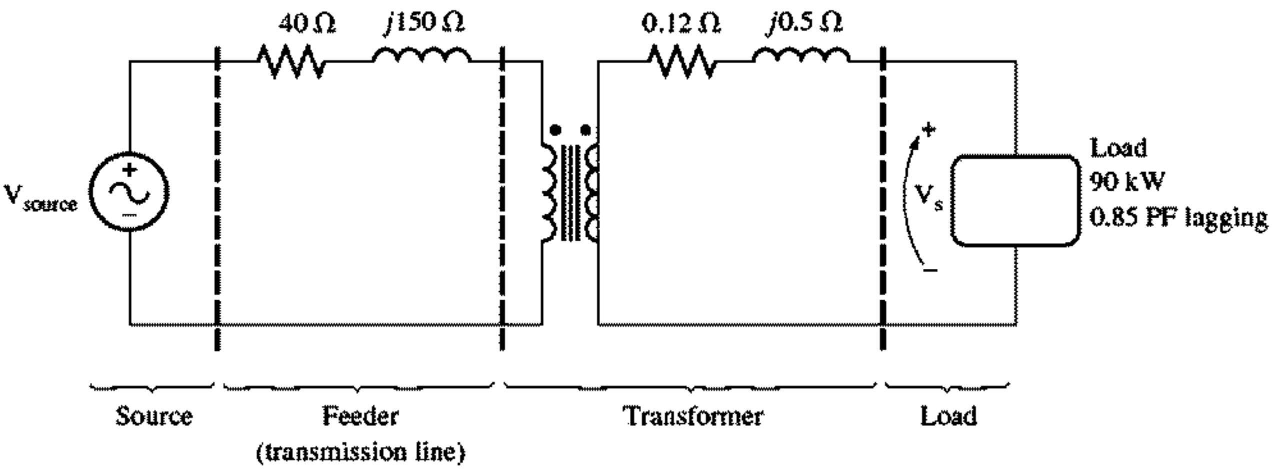


FIGURE P2–1
The circuit of Problem 2–4.

2–5. When travelers from the United States and Canada visit Europe, they encounter a different power distribution system. Wall voltages in North America are 120 V rms at 60 Hz, while typical wall voltages in Europe are 220 to 240 V at 50 Hz. Many travelers carry small step-up/step-down transformers so that they can use their appliances in the countries that they are visiting. A typical transformer might be rated at 1 kVA and 120/240 V. It has 500 turns of wire on the 120-V side and 1000 turns of wire on the 240-V side. The magnetization curve for this transformer is shown in Figure P2–2, and can be found in file `p22.mag` at this book's website.

(*a*) Suppose that this transformer is connected to a 120-V, 60-Hz power source with no load connected to the 240-V side. Sketch the magnetization current that would flow in the transformer. (Use MATLAB to plot the current accurately, if it is available.) What is the rms amplitude of the magnetization current? What percentage of full-load current is the magnetization current?

(*b*) Now suppose that this transformer is connected to a 240-V, 50-Hz power source with no load connected to the 120-V side. Sketch the magnetization current that would flow in the transformer. (Use MATLAB to plot the current accurately, if it is available.) What is the rms amplitude of the magnetization current? What percentage of full-load current is the magnetization current?

(*c*) In which case is the magnetization current a higher percentage of full-load current? Why?

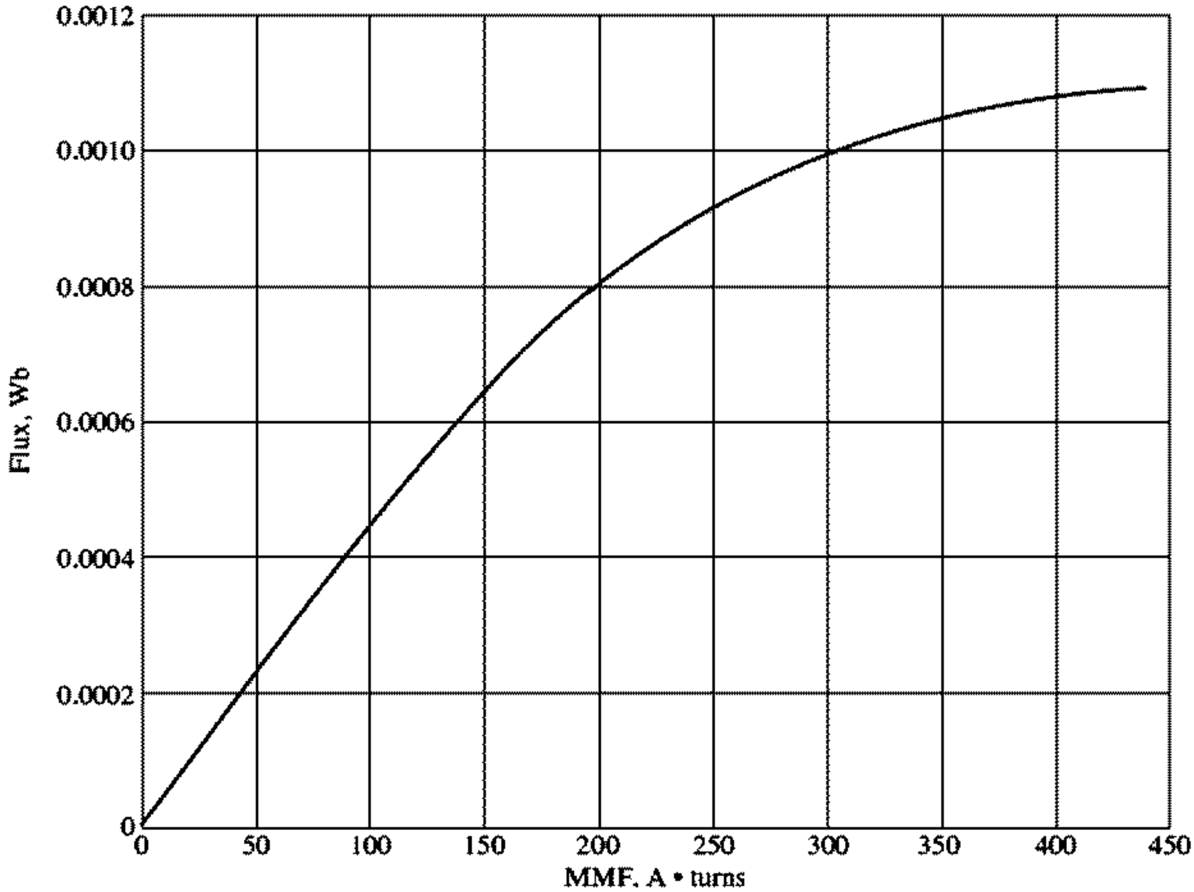


FIGURE P2–2
Magnetization curve for the transformer of Problem 2–5.

2–6. A 15-kVA 8000/230-V distribution transformer has an impedance referred to the primary of $80 + j300\ \Omega$. The components of the excitation branch referred to the primary side are $R_C = 350\ \text{k}\Omega$ and $X_M = 70\ \text{k}\Omega$.

(*a*) If the primary voltage is 7967 V and the load impedance is $Z_L = 3.0 + j1.5\ \Omega$, what is the secondary voltage of the transformer? What is the voltage regulation of the transformer?

(*b*) If the load is disconnected and a capacitor of $-j4.0\ \Omega$ is connected in its place, what is the secondary voltage of the transformer? What is its voltage regulation under these conditions?

2–7. A 5000-kVA 230/13.8-kV single-phase power transformer has a per-unit resistance of 1 percent and a per-unit reactance of 5 percent (data taken from the transformer's nameplate). The open-circuit test performed on the low-voltage side of the transformer yielded the following data:

$$V_{OC} = 13.8\ \text{kV} \qquad I_{OC} = 15.1\ \text{A} \qquad P_{OC} = 44.9\ \text{kW}$$

(*a*) Find the equivalent circuit referred to the low-voltage side of this transformer.

(*b*) If the voltage on the secondary side is 13.8 kV and the power supplied is 4000 kW at 0.8 PF lagging, find the voltage regulation of the transformer. Find its efficiency.

2–8. A 200-MVA, 15/200-kV single-phase power transformer has a per-unit resistance of 1.2 percent and a per-unit reactance of 5 percent (data taken from the transformer's nameplate). The magnetizing impedance is $j80$ per unit.

(a) Find the equivalent circuit referred to the low-voltage side of this transformer.

(b) Calculate the voltage regulation of this transformer for a full-load current at power factor of 0.8 lagging.

(c) Assume that the primary voltage of this transformer is a constant 15 kV, and plot the secondary voltage as a function of load current for currents from no load to full load. Repeat this process for power factors of 0.8 lagging, 1.0, and 0.8 leading.

2–9. A three-phase transformer bank is to handle 600 kVA and have a 34.5/13.8-kV voltage ratio. Find the rating of each individual transformer in the bank (high voltage, low voltage, turns ratio, and apparent power) if the transformer bank is connected to (*a*) Y–Y, (*b*) Y–Δ, (*c*) Δ–Y, (*d*) Δ–Δ, (*e*) open Δ, (*f*) open Y–open Δ.

2–10. A 13,800/480-V three-phase Y–Δ-connected transformer bank consists of three identical 100-kVA 7967/480-V transformers. It is supplied with power directly from a large constant-voltage bus. In the short-circuit test, the recorded values on the high-voltage side for one of these transformers are

$$V_{SC} = 560\ \text{V} \qquad I_{SC} = 12.6\ \text{A} \qquad P_{SC} = 3300\ \text{W}$$

(a) If this bank delivers a rated load at 0.85 PF lagging and rated voltage, what is the line-to-line voltage on the high-voltage side of the transformer bank?

(b) What is the voltage regulation under these conditions?

(c) Assume that the primary voltage of this transformer is a constant 13.8 kV, and plot the secondary voltage as a function of load current for currents from no-load to full-load. Repeat this process for power factors of 0.85 lagging, 1.0, and 0.85 leading.

(d) Plot the voltage regulation of this transformer as a function of load current for currents from no-load to full-load. Repeat this process for power factors of 0.85 lagging, 1.0, and 0.85 leading.

2–11. A 100,000-kVA, 230/115-kV Δ–Δ three-phase power transformer has a resistance of 0.02 pu and a reactance of 0.055 pu. The excitation branch elements are $R_C = 110$ pu and $X_M = 20$ pu.

(a) If this transformer supplies a load of 80 MVA at 0.85 PF lagging, draw the phasor diagram of one phase of the transformer.

(b) What is the voltage regulation of the transformer bank under these conditions?

(c) Sketch the equivalent circuit referred to the low-voltage side of one phase of this transformer. Calculate all the transformer impedances referred to the low-voltage side.

2–12. An autotransformer is used to connect a 13.2-kV distribution line to a 13.8-kV distribution line. It must be capable of handling 2000 kVA. There are three phases, connected Y–Y with their neutrals solidly grounded.

(a) What must the N_C/N_{SE} turns ratio be to accomplish this connection?

(b) How much apparent power must the windings of each autotransformer handle?

(c) If one of the autotransformers were reconnected as an ordinary transformer, what would its ratings be?

2–13. Two phases of a 13.8-kV three-phase distribution line serve a remote rural road (the neutral is also available). A farmer along the road has a 480-V feeder supplying

120 kW at 0.8 PF lagging of three-phase loads, plus 50 kW at 0.9 PF lagging of single-phase loads. The single-phase loads are distributed evenly among the three phases. Assuming that the open-Y–open-Δ connection is used to supply power to his farm, find the voltages and currents in each of the two transformers. Also find the real and reactive powers supplied by each transformer. Assume the transformers are ideal.

2–14. A 13.2-kV single-phase generator supplies power to a load through a transmission line. The load's impedance is $Z_{load} = 500 \angle 36.87° \ \Omega$, and the transmission line's impedance is $Z_{line} = 60 \angle 53.1° \ \Omega$.

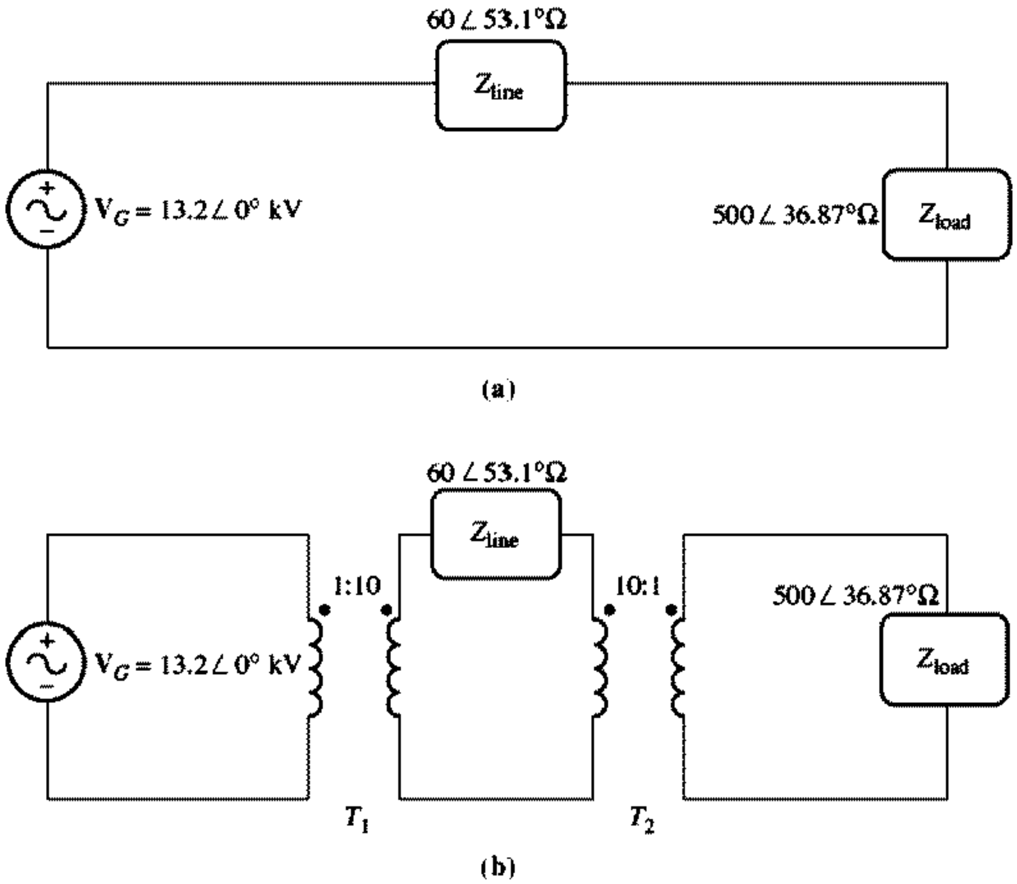


FIGURE P2–3
Circuits for Problem 2–14: (a) without transformers and (b) with transformers.

(*a*) If the generator is directly connected to the load (Figure P2–3a), what is the ratio of the load voltage to the generated voltage? What are the transmission losses of the system?

(*b*) If a 1:10 step-up transformer is placed at the output of the generator and a 10:1 transformer is placed at the load end of the transmission line, what is the new ratio of the load voltage to the generated voltage? What are the transmission losses of the system now? (*Note*: The transformers may be assumed to be ideal.)

2–15. A 5000-VA, 480/120-V conventional transformer is to be used to supply power from a 600-V source to a 120-V load. Consider the transformer to be ideal, and assume that all insulation can handle 600 V.

(*a*) Sketch the transformer connection that will do the required job.

(*b*) Find the kilovoltampere rating of the transformer in the configuration.

(*c*) Find the maximum primary and secondary currents under these conditions.

2–16. A 5000-VA, 480/120-V conventional transformer is to be used to supply power from a 600-V source to a 480-V load. Consider the transformer to be ideal, and assume that all insulation can handle 600 V. Answer the questions of Problem 2–15 for this transformer.

2–17. Prove the following statement: If a transformer having a series impedance Z_{eq} is connected as an autotransformer, its per-unit series impedance Z'_{eq} as an autotransformer will be

$$Z'_{eq} = \frac{N_{SE}}{N_{SE} + N_C} Z_{eq}$$

Note that this expression is the reciprocal of the autotransformer power advantage.

2–18. Three 25-kVA, 24,000/277-V distribution transformers are connected in Δ–Y. The open-circuit test was performed on the low-voltage side of this transformer bank, and the following data were recorded:

$$V_{line,OC} = 480 \text{ V} \qquad I_{line,OC} = 4.10 \text{ A} \qquad P_{3\phi,OC} = 945 \text{ W}$$

The short-circuit test was performed on the high-voltage side of this transformer bank, and the following data were recorded:

$$V_{line,SC} = 1600 \text{ V} \qquad I_{line,SC} = 2.00 \text{ A} \qquad P_{3\phi,SC} = 1150 \text{ W}$$

(*a*) Find the per-unit equivalent circuit of this transformer bank.
(*b*) Find the voltage regulation of this transformer bank at the rated load and 0.90 PF lagging.
(*c*) What is the transformer bank's efficiency under these conditions?

2–19. A 20-kVA, 20,000/480-V, 60-Hz distribution transformer is tested with the following results:

Open-circuit test (measured from secondary side)	Short-circuit test (measured from primary side)
$V_{OC} = 480$ V	$V_{SC} = 1130$ V
$I_{OC} = 1.60$ A	$I_{SC} = 1.00$ A
$P_{OC} = 305$ W	$P_{SC} = 260$ W

(*a*) Find the per-unit equivalent circuit for this transformer at 60 Hz.
(*b*) What would the rating of this transformer be if it were operated on a 50-Hz power system?
(*c*) Sketch the per-unit equivalent circuit of this transformer referred to the primary side *if it is operating at 50 Hz.*

2–20. Prove that the three-phase system of voltages on the secondary of the Y–Δ transformer shown in Figure 2–38b lags the three-phase system of voltages on the primary of the transformer by 30°.

2–21. Prove that the three-phase system of voltages on the secondary of the Δ–Y transformer shown in Figure 2–38c lags the three-phase system of voltages on the primary of the transformer by 30°.

2–22. A single-phase 10-kVA, 480/120-V transformer is to be used as an autotransformer tying a 600-V distribution line to a 480-V load. When it is tested as a conventional transformer, the following values are measured on the primary (480-V) side of the transformer:

Open-circuit test	Short-circuit test
$V_{OC} = 480$ V	$V_{SC} = 10.0$ V
$I_{OC} = 0.41$ A	$I_{SC} = 10.6$ A
$P_{OC} = 38$ W	$P_{SC} = 26$ W

(a) Find the per-unit equivalent circuit of this transformer when it is connected in the conventional manner. What is the efficiency of the transformer at rated conditions and unity power factor? What is the voltage regulation at those conditions?

(b) Sketch the transformer connections when it is used as a 600/480-V step-down autotransformer.

(c) What is the kilovoltampere rating of this transformer when it is used in the autotransformer connection?

(d) Answer the questions in *a* for the autotransformer connection.

2–23. Figure P2–4 shows a power system consisting of a three-phase 480-V 60-Hz generator supplying two loads through a transmission line with a pair of transformers at either end.

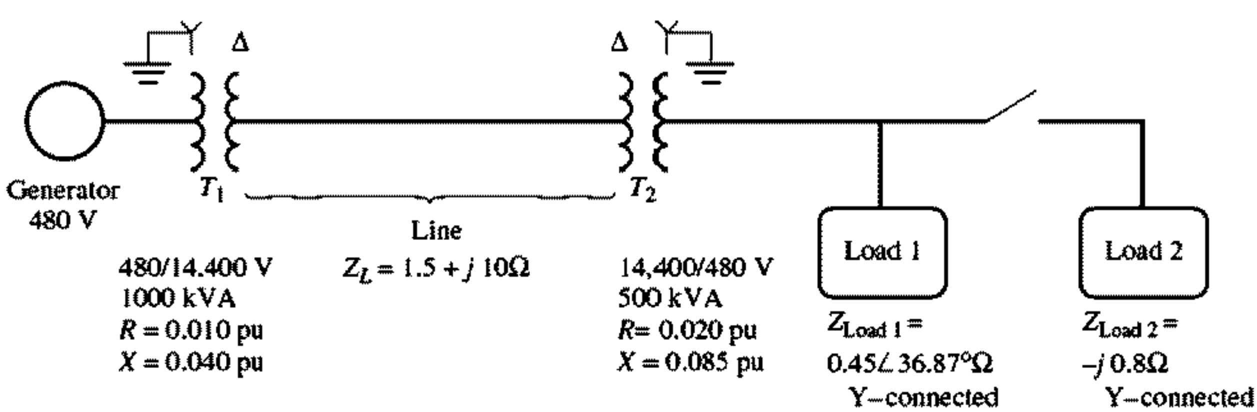


FIGURE P2–4

A one-line diagram of the power system of Problem 2–23. Note that some impedance values are given in the per-unit system, while others are given in ohms.

(a) Sketch the per-phase equivalent circuit of this power system.

(b) With the switch opened, find the real power *P*, reactive power *Q*, and apparent power *S* supplied by the generator. What is the power factor of the generator?

(c) With the switch closed, find the real power *P*, reactive power *Q*, and apparent power *S* supplied by the generator. What is the power factor of the generator?

(d) What are the transmission losses (transformer plus transmission line losses) in this system with the switch open? With the switch closed? What is the effect of adding load 2 to the system?

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