CHAPTER 4

AC MACHINERY FUNDAMENTALS

AC machines are generators that convert mechanical energy to ac electrical energy and motors that convert ac electrical energy to mechanical energy. The fundamental principles of ac machines are very simple, but unfortunately, they are somewhat obscured by the complicated construction of real machines. This chapter will first explain the principles of ac machine operation using simple examples, and then consider some of the complications that occur in real ac machines.

There are two major classes of ac machines—synchronous machines and induction machines. *Synchronous machines* are motors and generators whose magnetic field current is supplied by a separate dc power source, while *induction machines* are motors and generators whose field current is supplied by magnetic induction (transformer action) into their field windings. The field circuits of most synchronous and induction machines are located on their rotors. This chapter covers some of the fundamentals common to both types of three-phase ac machines. Synchronous machines will be covered in detail in Chapters 5 and 6, and induction machines will be covered in Chapter 7.

4.1 A SIMPLE LOOP IN A UNIFORM MAGNETIC FIELD

We will start our study of ac machines with a simple loop of wire rotating within a uniform magnetic field. A loop of wire in a uniform magnetic field is the simplest possible machine that produces a sinusoidal ac voltage. This case is not representative of real ac machines, since the flux in real ac machines is not constant in either magnitude or direction. However, the factors that control the voltage and torque on the loop will be the same as the factors that control the voltage and torque in real ac machines.

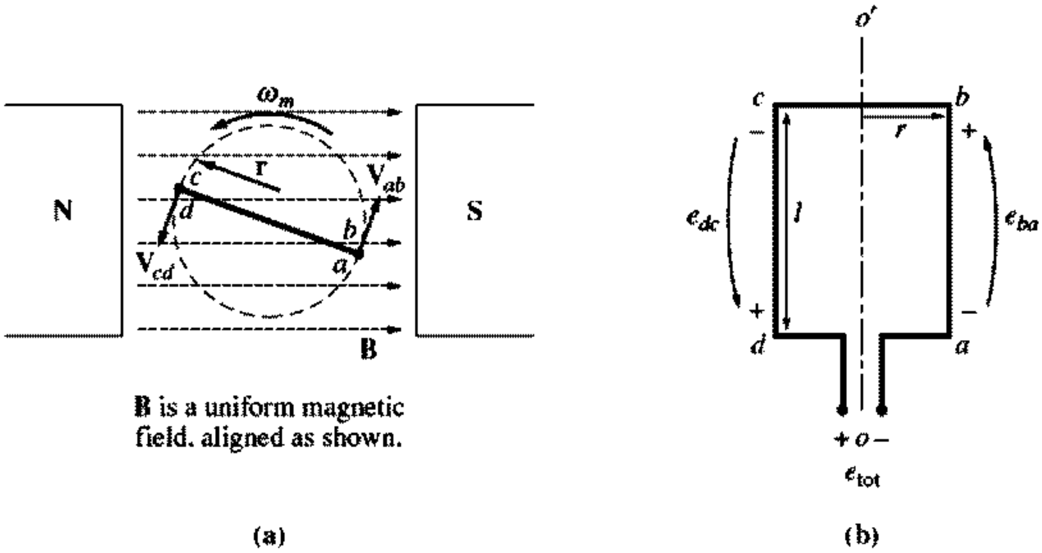


FIGURE 4–1
A simple rotating loop in a uniform magnetic field. (a) Front view; (b) view of coil.

Figure 4–1 shows a simple machine consisting of a large stationary magnet producing an essentially constant and uniform magnetic field and a rotating loop of wire within that field. The rotating part of the machine is called the *rotor*, and the stationary part of the machine is called the *stator*. We will now determine the voltages present in the rotor as it rotates within the magnetic field.

The Voltage Induced in a Simple Rotating Loop

If the rotor of this machine is rotated, a voltage will be induced in the wire loop. To determine the magnitude and shape of the voltage, examine Figure 4–2. The loop of wire shown is rectangular, with sides *ab* and *cd* perpendicular to the plane of the page and with sides *bc* and *da* parallel to the plane of the page. The magnetic field is constant and uniform, pointing from left to right across the page.

To determine the total voltage e_{tot} on the loop, we will examine each segment of the loop separately and sum all the resulting voltages. The voltage on each segment is given by Equation (1–45):

$$e_{ind} = (\mathbf{v} \times \mathbf{B}) \cdot \mathbf{l} \tag{1–45}$$

1. *Segment ab.* In this segment, the velocity of the wire is tangential to the path of rotation, while the magnetic field **B** points to the right, as shown in Figure 4–2b. The quantity **v** × **B** points into the page, which is the same direction as segment *ab*. Therefore, the induced voltage on this segment of the wire is

$$\begin{aligned} e_{ba} &= (\mathbf{v} \times \mathbf{B}) \cdot \mathbf{l} \\ &= vBl \sin \theta_{ab} \qquad \text{into the page} \end{aligned} \tag{4–1}$$

2. *Segment bc.* In the first half of this segment, the quantity **v** × **B** points into the page, and in the second half of this segment, the quantity **v** × **B** points out of

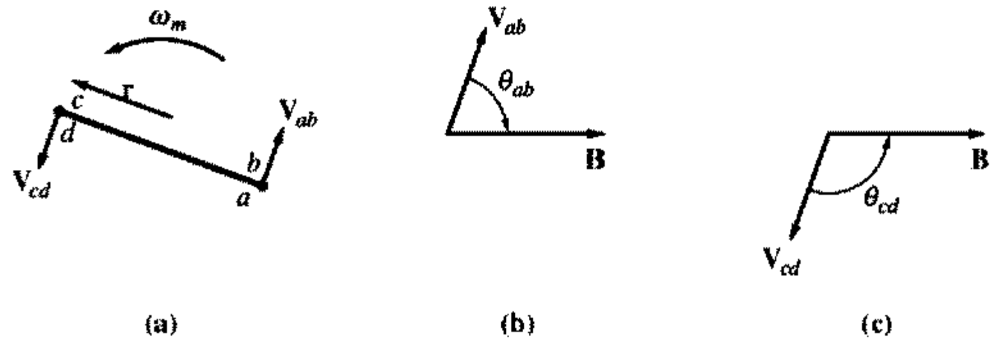


FIGURE 4–2
(a) Velocities and orientations of the sides of the loop with respect to the magnetic field. (b) The direction of motion with respect to the magnetic field for side *ab*. (c) The direction of motion with respect to the magnetic field for side *cd*.

the page. Since the length **l** is in the plane of the page, $\mathbf{v} \times \mathbf{B}$ is perpendicular to **l** for both portions of the segment. Therefore the voltage in segment *bc* will be zero:

$$e_{cb} = 0 \tag{4–2}$$

3. *Segment cd.* In this segment, the velocity of the wire is tangential to the path of rotation, while the magnetic field **B** points to the right, as shown in Figure 4–2c. The quantity $\mathbf{v} \times \mathbf{B}$ points into the page, which is the same direction as segment *cd*. Therefore, the induced voltage on this segment of the wire is

$$\begin{aligned} e_{dc} &= (\mathbf{v} \times \mathbf{B}) \cdot \mathbf{l} \\ &= vBl \sin \theta_{cd} \qquad \text{out of the page} \end{aligned} \tag{4–3}$$

4. *Segment da.* Just as in segment *bc*, $\mathbf{v} \times \mathbf{B}$ is perpendicular to **l**. Therefore the voltage in this segment will be zero too:

$$e_{ad} = 0 \tag{4–4}$$

The total induced voltage on the loop e_{ind} is the sum of the voltages on each of its sides:

$$\begin{aligned} e_{\text{ind}} &= e_{ba} + e_{cb} + e_{dc} + e_{ad} \\ &= vBl \sin \theta_{ab} + vBl \sin \theta_{cd} \end{aligned} \tag{4–5}$$

Note that $\theta_{ab} = 180° - \theta_{cd}$, and recall the trigonometric identity $\sin \theta = \sin (180° - \theta)$. Therefore, the induced voltage becomes

$$e_{\text{ind}} = 2vBL \sin \theta \tag{4–6}$$

The resulting voltage e_{ind} is shown as a function of time in Figure 4–3.

There is an alternative way to express Equation (4–6), which clearly relates the behavior of the single loop to the behavior of larger, real ac machines. To derive this alternative expression, examine Figure 4–2 again. If the loop is rotating at a constant angular velocity ω, then angle θ of the loop will increase linearly with time. In other words,

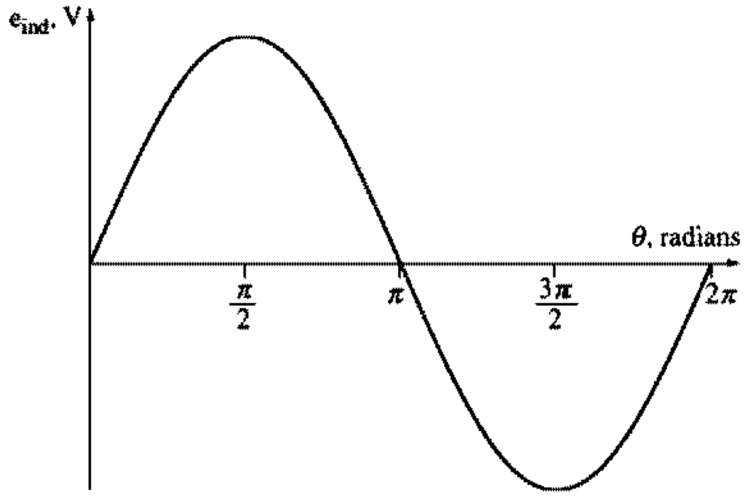


FIGURE 4–3
Plot of e_{ind} versus θ.

$$\theta = \omega t$$

Also, the tangential velocity v of the edges of the loop can be expressed as

$$v = r\omega \tag{4–7}$$

where r is the radius from axis of rotation out to the edge of the loop and ω is the angular velocity of the loop. Substituting these expressions into Equation (4–6) gives

$$e_{ind} = 2r\omega Bl \sin \omega t \tag{4–8}$$

Notice also from Figure 4–1b that the area A of the loop is just equal to $2rl$. Therefore,

$$e_{ind} = AB\omega \sin \omega t \tag{4–9}$$

Finally, note that the maximum flux through the loop occurs when the loop is perpendicular to the magnetic flux density lines. This flux is just the product of the loop's surface area and the flux density through the loop.

$$\phi_{max} = AB \tag{4–10}$$

Therefore, the final form of the voltage equation is

$$\boxed{e_{ind} = \phi_{max}\omega \sin \omega t} \tag{4–11}$$

Thus, *the voltage generated in the loop is a sinusoid whose magnitude is equal to the product of the flux inside the machine and the speed of rotation of the machine.* This is also true of real ac machines. In general, the voltage in any real machine will depend on three factors:

1. The flux in the machine
2. The speed of rotation
3. A constant representing the construction of the machine (the number of loops, etc.)

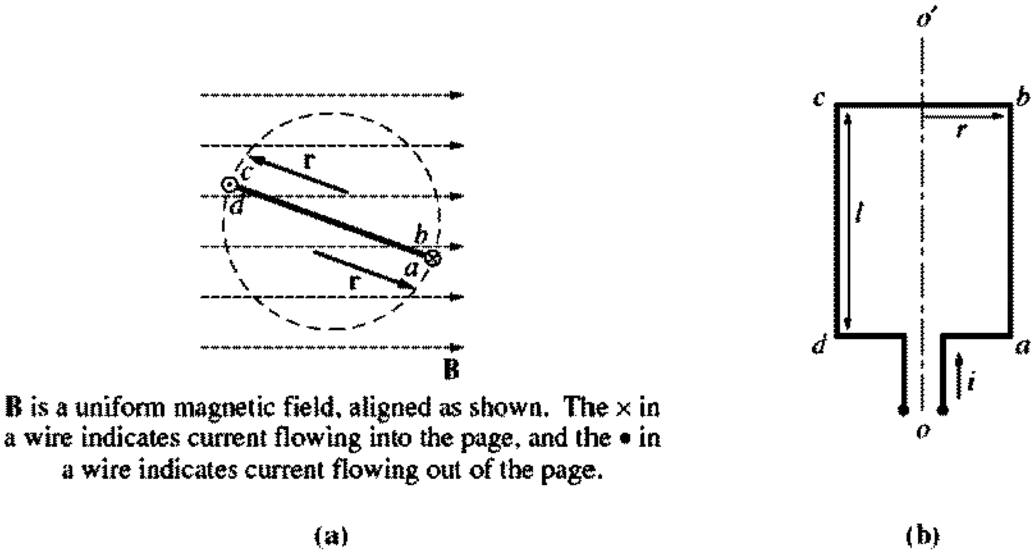


FIGURE 4–4
A current-carrying loop in a uniform magnetic field. (a) Front view; (b) view of coil.

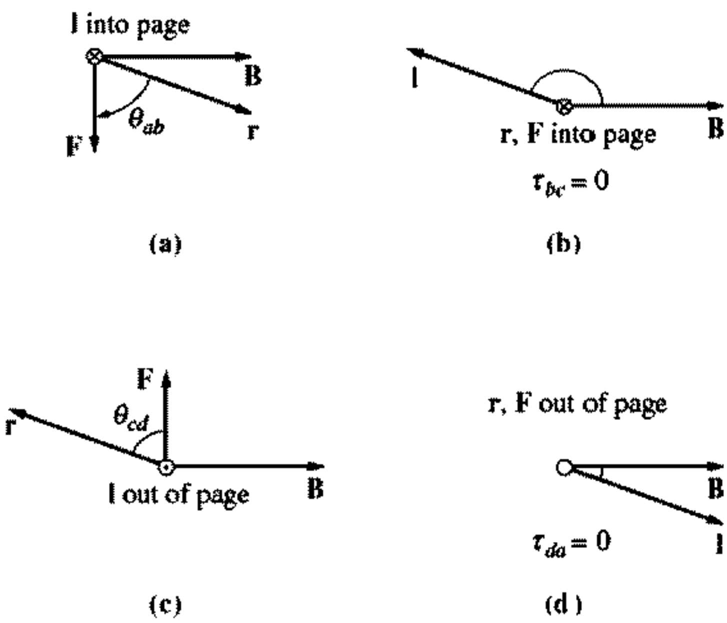


FIGURE 4–5
(a) Derivation of force and torque on segment *ab*. (b) Derivation of force and torque on segment *bc*. (c) Derivation of force and torque on segment *cd*. (d) Derivation of force and torque on segment *da*.

The Torque Induced in a Current-Carrying Loop

Now assume that the rotor loop is at some arbitrary angle θ with respect to the magnetic field, and that a current i is flowing in the loop, as shown in Figure 4–4. If a current flows in the loop, then a torque will be induced on the wire loop. To determine the magnitude and direction of the torque, examine Figure 4–5. The force on each segment of the loop will be given by Equation (1–43),

$$\mathbf{F} = i(\mathbf{l} \times \mathbf{B}) \qquad (1\text{–}43)$$

where i = magnitude of current in the segment
$\mathbf{l}$ = length of the segment, with direction of $\mathbf{l}$ defined to be in the direction of current flow
$\mathbf{B}$ = magnetic flux density vector

The torque on that segment will then be given by

$$\begin{aligned}\tau &= \text{(force applied)(perpendicular distance)} \\ &= (F)\,(r \sin \theta) \\ &= rF \sin \theta \end{aligned} \tag{1–6}$$

where θ is the angle between the vector $\mathbf{r}$ and the vector $\mathbf{F}$. The direction of the torque is clockwise if it would tend to cause a clockwise rotation and counterclockwise if it would tend to cause a counterclockwise rotation.

1. *Segment ab.* In this segment, the direction of the current is into the page, while the magnetic field $\mathbf{B}$ points to the right, as shown in Figure 4–5a. The quantity $\mathbf{l} \times \mathbf{B}$ points down. Therefore, the induced force on this segment of the wire is

$$\begin{aligned}\mathbf{F} &= i(\mathbf{l} \times \mathbf{B}) \\ &= ilB \qquad \text{down}\end{aligned}$$

   The resulting torque is

$$\begin{aligned}\tau_{ab} &= (F)\,(r \sin \theta_{ab}) \\ &= rilB \sin \theta_{ab} \qquad \text{clockwise}\end{aligned} \tag{4–12}$$

2. *Segment bc.* In this segment, the direction of the current is in the plane of the page, while the magnetic field $\mathbf{B}$ points to the right, as shown in Figure 4–5b. The quantity $\mathbf{l} \times \mathbf{B}$ points into the page. Therefore, the induced force on this segment of the wire is

$$\begin{aligned}\mathbf{F} &= i(\mathbf{l} \times \mathbf{B}) \\ &= ilB \qquad \text{into the page}\end{aligned}$$

   For this segment, the resulting torque is 0, since vectors $\mathbf{r}$ and $\mathbf{l}$ are parallel (both point into the page), and the angle θ_{bc} is 0.

$$\begin{aligned}\tau_{bc} &= (F)\,(r \sin \theta_{ab}) \\ &= 0\end{aligned} \tag{4–13}$$

3. *Segment cd.* In this segment, the direction of the current is out of the page, while the magnetic field $\mathbf{B}$ points to the right, as shown in Figure 4–5c. The quantity $\mathbf{l} \times \mathbf{B}$ points up. Therefore, the induced force on this segment of the wire is

$$\begin{aligned}\mathbf{F} &= i(\mathbf{l} \times \mathbf{B}) \\ &= ilB \qquad \text{up}\end{aligned}$$

The resulting torque is

$$\begin{aligned} \tau_{cd} &= (F)\,(r \sin \theta_{cd}) \\ &= rilB \sin \theta_{cd} \qquad \text{clockwise} \end{aligned} \tag{4–14}$$

4. *Segment da.* In this segment, the direction of the current is in the plane of the page, while the magnetic field **B** points to the right, as shown in Figure 4–5d. The quantity **l** × **B** points out of the page. Therefore, the induced force on this segment of the wire is

$$\begin{aligned} \mathbf{F} &= i(\mathbf{l} \times \mathbf{B}) \\ &= ilB \qquad \text{out of the page} \end{aligned}$$

For this segment, the resulting torque is 0, since vectors **r** and **l** are parallel (both point out of the page), and the angle θ_{da} is 0.

$$\begin{aligned} \tau_{da} &= (F)\,(r \sin \theta_{da}) \\ &= 0 \end{aligned} \tag{4–15}$$

The total induced torque on the loop τ_{ind} is the sum of the torques on each of its sides:

$$\begin{aligned} \tau_{\text{ind}} &= \tau_{ab} + \tau_{bc} + \tau_{cd} + \tau_{da} \\ &= rilB \sin \theta_{ab} + rilB \sin \theta_{cd} \end{aligned} \tag{4–16}$$

Note that $\theta_{ab} = \theta_{cd}$, so the induced torque becomes

$$\tau_{\text{ind}} = 2rilB \sin \theta \tag{4–17}$$

The resulting torque τ_{ind} is shown as a function of angle in Figure 4–6. Note that the torque is maximum when the plane of the loop is parallel to the magnetic field, and the torque is zero when the plane of the loop is perpendicular to the magnetic field.

There is an alternative way to express Equation (4–17), which clearly relates the behavior of the single loop to the behavior of larger, real ac machines. To derive this alternative expression, examine Figure 4–7. If the current in the loop is as shown in the figure, that current will generate a magnetic flux density $\mathbf{B}_{\text{loop}}$ with the direction shown. The magnitude of $\mathbf{B}_{\text{loop}}$ will be

$$B_{\text{loop}} = \frac{\mu i}{G}$$

where G is a factor that depends on the geometry of the loop.* Also, note that the area of the loop A is just equal to $2rl$. Substituting these two equations into Equation (4–17) yields the result

$$\tau_{\text{ind}} = \frac{AG}{\mu} B_{\text{loop}} B_S \sin \theta \tag{4–18}$$

*If the loop were a circle, then $G = 2r$, where r is the radius of the circle, so $B_{\text{loop}} = \mu i/2r$. For a rectangular loop, the value of G will vary depending on the exact length-to-width ratio of the loop.

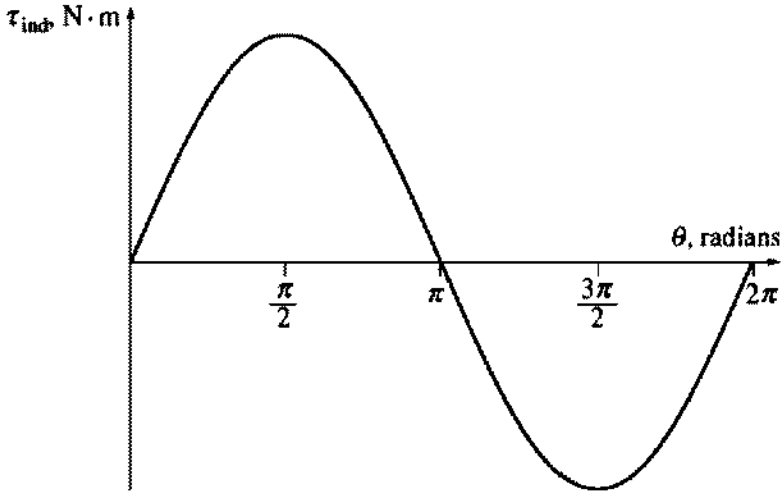


FIGURE 4–6
Plot of τ_{ind} versus θ.

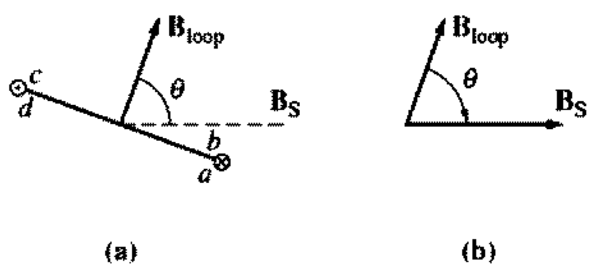


FIGURE 4–7
Derivation of the induced torque equation. (a) The current in the loop produces a magnetic flux density $\mathbf{B}_{loop}$ perpendicular to the plane of the loop; (b) geometric relationship between $\mathbf{B}_{loop}$ and $\mathbf{B}_S$.

$$= kB_{loop}B_S \sin\theta \tag{4–19}$$

where $k = AG/\mu$ is a factor depending on the construction of the machine, B_S is used for the stator magnetic field to distinguish it from the magnetic field generated by the rotor, and θ is the angle between $\mathbf{B}_{loop}$ and $\mathbf{B}_S$. The angle between $\mathbf{B}_{loop}$ and $\mathbf{B}_S$ can be seen by trigonometric identities to be the same as the angle θ in Equation (4–17).

Both the magnitude and the direction of the induced torque can be determined by expressing Equation (4–19) as a cross product:

$$\boxed{\tau_{ind} = k\mathbf{B}_{loop} \times \mathbf{B}_S} \tag{4–20}$$

Applying this equation to the loop in Figure 4–7 produces a torque vector into the page, indicating that the torque is clockwise, with the magnitude given by Equation (4–19).

Thus, *the torque induced in the loop is proportional to the strength of the loop's magnetic field, the strength of the external magnetic field, and the sine of the angle between them.* This is also true of real ac machines. In general, the torque in any real machine will depend on four factors:

1. The strength of the rotor magnetic field
2. The strength of the external magnetic field
3. The sine of the angle between them
4. A constant representing the construction of the machine (geometry, etc.)

4.2 THE ROTATING MAGNETIC FIELD

In Section 4.1, we showed that if two magnetic fields are present in a machine, then a torque will be created which will tend to line up the two magnetic fields. If one magnetic field is produced by the stator of an ac machine and the other one is produced by the rotor of the machine, then a torque will be induced in the rotor which will cause the rotor to turn and align itself with the stator magnetic field.

If there were some way to make the stator magnetic field rotate, then the induced torque in the rotor would cause it to constantly "chase" the stator magnetic field around in a circle. This, in a nutshell, is the basic principle of all ac motor operation.

How can the stator magnetic field be made to rotate? The fundamental principle of ac machine operation is that *if a three-phase set of currents, each of equal magnitude and differing in phase by 120°, flows in a three-phase winding, then it will produce a rotating magnetic field of constant magnitude.* The three-phase winding consists of three separate windings spaced 120 electrical degrees apart around the surface of the machine.

The rotating magnetic field concept is illustrated in the simplest case by an empty stator containing just three coils, each 120° apart (see Figure 4–8a). Since such a winding produces only one north and one south magnetic pole, it is a two-pole winding.

To understand the concept of the rotating magnetic field, we will apply a set of currents to the stator of Figure 4–8 and see what happens at specific instants of time. Assume that the currents in the three coils are given by the equations

$$i_{aa'}(t) = I_M \sin \omega t \qquad \text{A} \tag{4–21a}$$

$$i_{bb'}(t) = I_M \sin (\omega t - 120°) \qquad \text{A} \tag{4–21b}$$

$$i_{cc'}(t) = I_M \sin (\omega t - 240°) \qquad \text{A} \tag{4–21c}$$

The current in coil *aa'* flows into the *a* end of the coil and out the *a'* end of the coil. It produces the magnetic field intensity

$$\mathbf{H}_{aa'}(t) = H_M \sin \omega t \angle 0° \qquad \text{A} \cdot \text{turns / m} \tag{4–22a}$$

where 0° is the spatial angle of the magnetic field intensity vector, as shown in Figure 4–8b. The direction of the magnetic field intensity vector $\mathbf{H}_{aa'}(t)$ is given by the right-hand rule: If the fingers of the right hand curl in the direction of the current flow in the coil, then the resulting magnetic field is in the direction that the thumb points. Notice that the magnitude of the magnetic field intensity vector $\mathbf{H}_{aa'}(t)$ varies sinusoidally in time, but the direction of $\mathbf{H}_{aa'}(t)$ is always constant. Similarly, the magnetic field intensity vectors $\mathbf{H}_{bb'}(t)$ and $\mathbf{H}_{cc'}(t)$ are

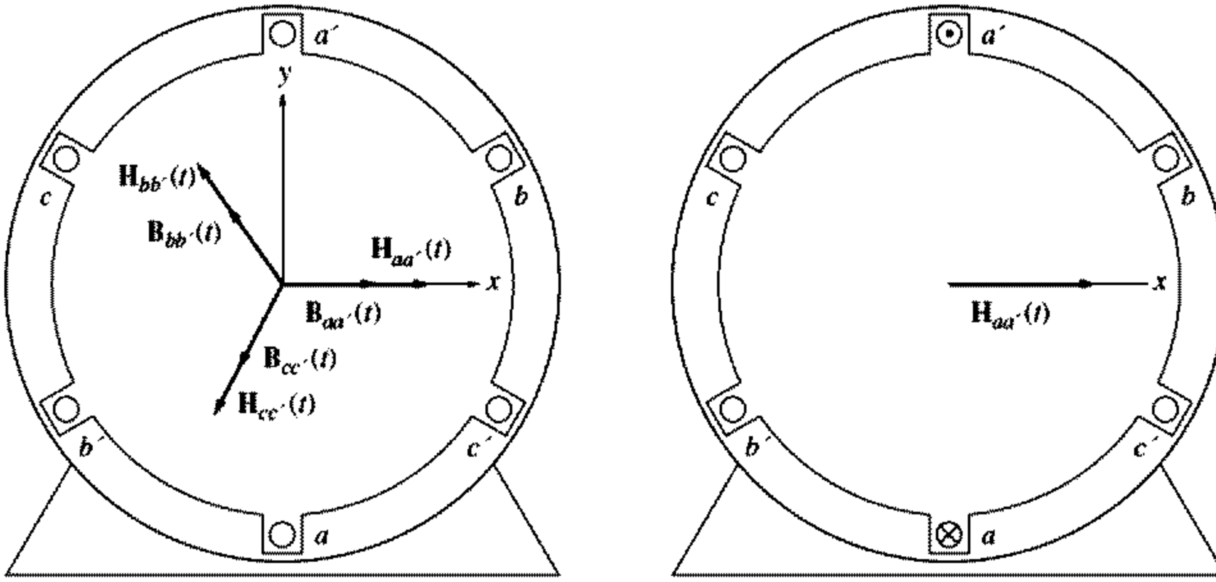


FIGURE 4–8
(a) A simple three-phase stator. Currents in this stator are assumed positive if they flow into the unprimed end and out the primed end of the coils. The magnetizing intensities produced by each coil are also shown. (b) The magnetizing intensity vector $\mathbf{H}_{aa'}(t)$ produced by a current flowing in coil aa'.

$$\mathbf{H}_{bb'}(t) = H_M \sin(\omega t - 120^\circ) \angle 120^\circ \quad \text{A} \bullet \text{turns / m} \quad (4\text{–}22b)$$

$$\mathbf{H}_{cc'}(t) = H_M \sin(\omega t - 240^\circ) \angle 240^\circ \quad \text{A} \bullet \text{turns / m} \quad (4\text{–}22c)$$

The flux densities resulting from these magnetic field intensities are given by Equation (1–21):

$$\mathbf{B} = \mu\mathbf{H} \quad (1\text{–}21)$$

They are

$$\mathbf{B}_{aa'}(t) = B_M \sin \omega t \angle 0^\circ \quad \text{T} \quad (4\text{–}23a)$$

$$\mathbf{B}_{bb'}(t) = B_M \sin(\omega t - 120^\circ) \angle 120^\circ \quad \text{T} \quad (4\text{–}23b)$$

$$\mathbf{B}_{cc'}(t) = B_M \sin(\omega t - 240^\circ) \angle 240^\circ \quad \text{T} \quad (4\text{–}23c)$$

where $B_M = \mu H_M$. The currents and their corresponding flux densities can be examined at specific times to determine the resulting net magnetic field in the stator.

For example, at time $\omega t = 0^\circ$, the magnetic field from coil aa' will be

$$\mathbf{B}_{aa'} = 0 \quad (4\text{–}24a)$$

The magnetic field from coil bb' will be

$$\mathbf{B}_{bb'} = B_M \sin(-120^\circ) \angle 120^\circ \quad (4\text{–}24b)$$

and the magnetic field from coil cc' will be

$$\mathbf{B}_{cc'} = B_M \sin(-240^\circ) \angle 240^\circ \quad (4\text{–}24c)$$

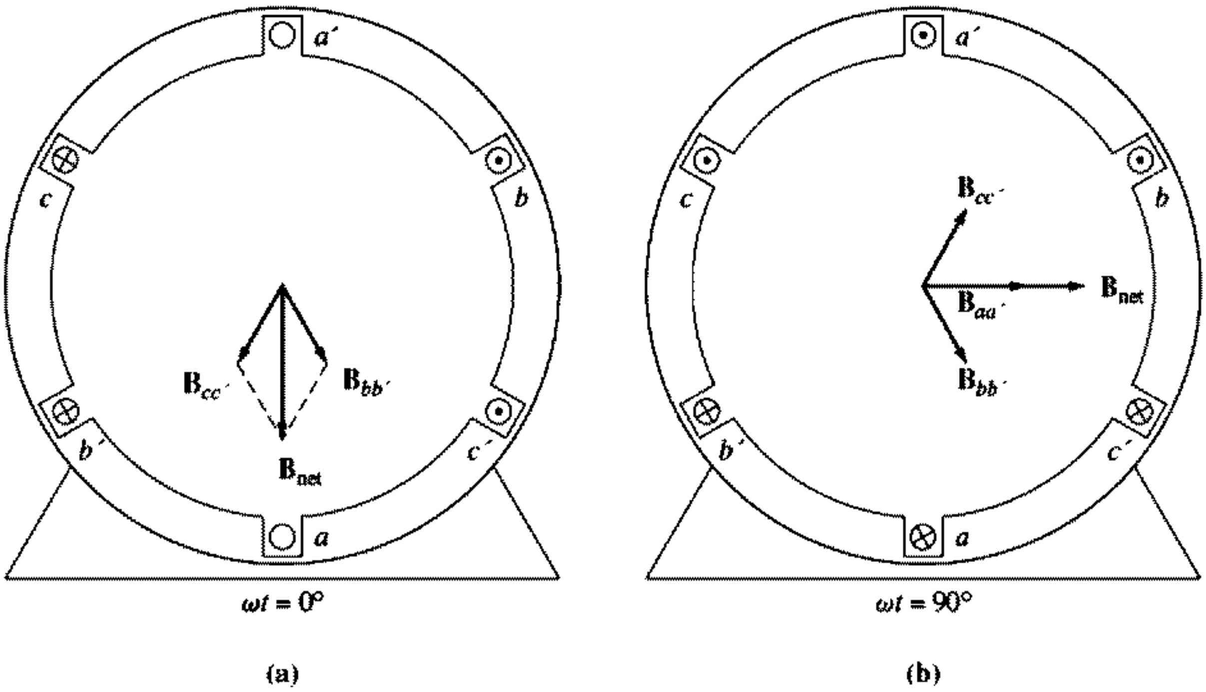


FIGURE 4–9
(a) The vector magnetic field in a stator at time $\omega t = 0°$. (b) The vector magnetic field in a stator at time $\omega t = 90°$.

The total magnetic field from all three coils added together will be

$$\begin{aligned}\mathbf{B}_{\text{net}} &= \mathbf{B}_{aa'} + \mathbf{B}_{bb'} + \mathbf{B}_{cc'} \\ &= 0 + \left(-\frac{\sqrt{3}}{2}B_M\right)\angle 120° + \left(\frac{\sqrt{3}}{2}B_M\right)\angle 240° \\ &= 1.5B_M \angle -90°\end{aligned}$$

The resulting net magnetic field is shown in Figure 4–9a.

As another example, look at the magnetic field at time $\omega t = 90°$. At that time, the currents are

$$\begin{aligned}i_{aa'} &= I_M \sin 90° \qquad \text{A} \\ i_{bb'} &= I_M \sin (-30°) \qquad \text{A} \\ i_{cc'} &= I_M \sin (-150°) \qquad \text{A}\end{aligned}$$

and the magnetic fields are

$$\begin{aligned}\mathbf{B}_{aa'} &= B_M \angle 0° \\ \mathbf{B}_{bb'} &= -0.5\, B_M \angle 120° \\ \mathbf{B}_{cc'} &= -0.5\, B_M \angle 240°\end{aligned}$$

The resulting net magnetic field is

$$\begin{aligned}\mathbf{B}_{\text{net}} &= \mathbf{B}_{aa'} + \mathbf{B}_{bb'} + \mathbf{B}_{cc'} \\ &= B_M \angle 0° + (-0.5B_M) \angle 120° + (-0.5B_M) \angle 240° \\ &= 1.5\, B_M \angle 0°\end{aligned}$$

The resulting magnetic field is shown in Figure 4–9b. Notice that although the *direction* of the magnetic field has changed, the *magnitude* is constant. The magnetic field is maintaining a constant magnitude while rotating in a counterclockwise direction.

Proof of the Rotating Magnetic Field Concept

At any time t, the magnetic field will have the same magnitude $1.5B_M$, and it will continue to rotate at angular velocity ω. A proof of this statement for all time t is now given.

Refer again to the stator shown in Figure 4–8. In the coordinate system shown in the figure, the x direction is to the right and the y direction is upward. The vector $\hat{\mathbf{x}}$ is the unit vector in the horizontal direction, and the vector $\hat{\mathbf{y}}$ is the unit vector in the vertical direction. To find the total magnetic flux density in the stator, simply add vectorially the three component magnetic fields and determine their sum.

The net magnetic flux density in the stator is given by

$$\begin{aligned}\mathbf{B}_{\text{net}}(t) &= \mathbf{B}_{aa'}(t) + \mathbf{B}_{bb'}(t) + \mathbf{B}_{cc'}(t)\\ &= B_M \sin \omega t \angle 0° + B_M \sin(\omega t - 120°) \angle 120° + B_M \sin(\omega t - 240°) \angle 240° \text{ T}\end{aligned}$$

Each of the three component magnetic fields can now be broken down into its x and y components.

$$\begin{aligned}\mathbf{B}_{\text{net}}(t) = {} & B_M \sin \omega t\, \hat{\mathbf{x}}\\ & - [0.5B_M \sin(\omega t - 120°)]\hat{\mathbf{x}} + \left[\frac{\sqrt{3}}{2} B_M \sin(\omega t - 120°)\right]\hat{\mathbf{y}}\\ & - [0.5B_M \sin(\omega t - 240°)]\hat{\mathbf{x}} - \left[\frac{\sqrt{3}}{2} B_M \sin(\omega t - 240°)\right]\hat{\mathbf{y}}\end{aligned}$$

Combining x and y components yields

$$\begin{aligned}\mathbf{B}_{\text{net}}(t) = {} & [B_M \sin \omega t - 0.5B_M \sin(\omega t - 120°) - 0.5B_M \sin(\omega t - 240°)]\,\hat{\mathbf{x}}\\ & + \left[\frac{\sqrt{3}}{2} B_M \sin(\omega t - 120°) - \frac{\sqrt{3}}{2} B_M \sin(\omega t - 240°)\right]\hat{\mathbf{y}}\end{aligned}$$

By the angle-addition trigonometric identities,

$$\begin{aligned}\mathbf{B}_{\text{net}}(t) = {} & \left[B_M \sin \omega t + \tfrac{1}{4}B_M \sin \omega t + \tfrac{\sqrt{3}}{4}B_M \cos \omega t + \tfrac{1}{4}B_M \sin \omega t - \tfrac{\sqrt{3}}{4}B_M \cos \omega t\right]\hat{\mathbf{x}}\\ & + \left[-\tfrac{\sqrt{3}}{4}B_M \sin \omega t - \tfrac{3}{4}B_M \cos \omega t + \tfrac{\sqrt{3}}{4}B_M \sin \omega t - \tfrac{3}{4}B_M \cos \omega t\right]\hat{\mathbf{y}}\end{aligned}$$

$$\boxed{\mathbf{B}_{\text{net}}(t) = (1.5B_M \sin \omega t)\hat{\mathbf{x}} - (1.5B_M \cos \omega t)\hat{\mathbf{y}}} \tag{4–25}$$

Equation (4–25) is the final expression for the net magnetic flux density. Notice that the magnitude of the field is a constant $1.5B_M$ and that the angle changes continually in a counterclockwise direction at angular velocity ω. Notice also that at

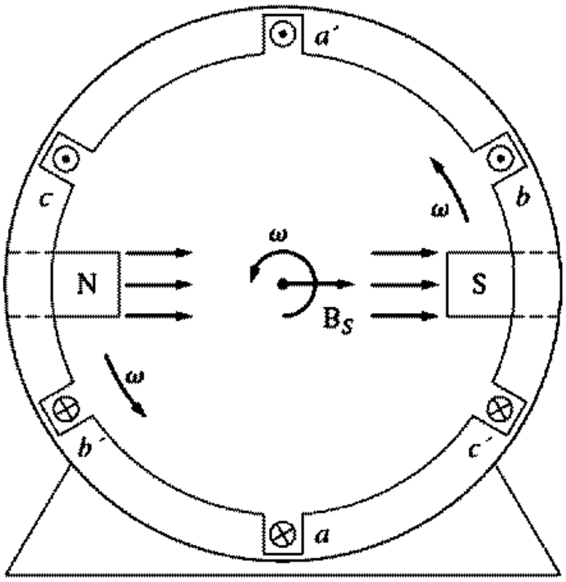


FIGURE 4–10
The rotating magnetic field in a stator represented as moving north and south stator poles.

$\omega t = 0°$, $\mathbf{B}_{net} = 1.5B_M \angle -90°$ and that at $\omega t = 90°$, $\mathbf{B}_{net} = 1.5B_M \angle 0°$. These results agree with the specific examples examined previously.

The Relationship between Electrical Frequency and the Speed of Magnetic Field Rotation

Figure 4–10 shows that the rotating magnetic field in this stator can be represented as a north pole (where the flux leaves the stator) and a south pole (where the flux enters the stator). These magnetic poles complete one mechanical rotation around the stator surface for each electrical cycle of the applied current. Therefore, the mechanical speed of rotation of the magnetic field in revolutions per second is equal to the electric frequency in hertz:

$$f_e = f_m \qquad \text{two poles} \tag{4–26}$$

$$\omega_e = \omega_m \qquad \text{two poles} \tag{4–27}$$

Here f_m and ω_m are the mechanical speed in revolutions per second and radians per second, while f_e and ω_e are the electrical speed in hertz and radians per second.

Notice that the windings on the two-pole stator in Figure 4–10 occur in the order (taken counterclockwise)

$$a\text{-}c'\text{-}b\text{-}a'\text{-}c\text{-}b'$$

What would happen in a stator if this pattern were repeated twice within it? Figure 4–11a shows such a stator. There, the pattern of windings (taken counterclockwise) is

$$a\text{-}c'\text{-}b\text{-}a'\text{-}c\text{-}b'\text{-}a\text{-}c'\text{-}b\text{-}a'\text{-}c\text{-}b'$$

which is just the pattern of the previous stator repeated twice. When a three-phase set of currents is applied to this stator, *two* north poles and *two* south poles are produced in the stator winding, as shown in Figure 4–11b. In this winding, a pole

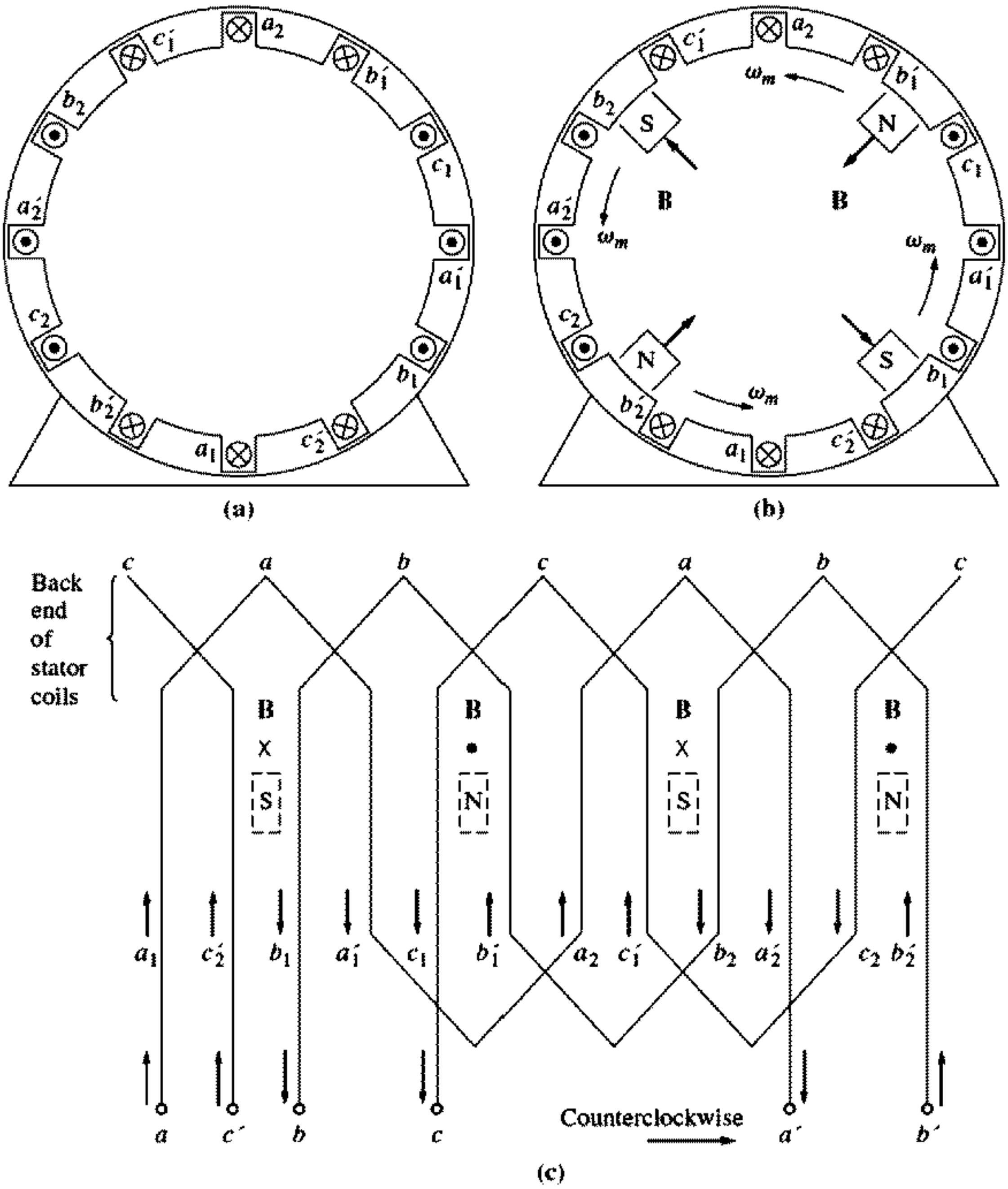


FIGURE 4–11
(a) A simple four-pole stator winding. (b) The resulting stator magnetic poles. Notice that there are moving poles of alternating polarity every 90° around the stator surface. (c) A winding diagram of the stator as seen from its inner surface, showing how the stator currents produce north and south magnetic poles.

moves only halfway around the stator surface in one electrical cycle. Since one electrical cycle is 360 electrical degrees, and since the mechanical motion is 180 mechanical degrees, the relationship between the electrical angle θ_e and the mechanical angle θ_m in this stator is

$$\theta_e = 2\theta_m \tag{4–28}$$

Thus for the four-pole winding, the electrical frequency of the current is twice the mechanical frequency of rotation:

$$f_e = 2f_m \quad \text{four poles} \tag{4–29}$$

$$\omega_e = 2\omega_m \quad \text{four poles} \tag{4–30}$$

In general, if the number of magnetic poles on an ac machine stator is P, then there are $P/2$ repetitions of the winding sequence a-c'-b-a'-c-b' around its inner surface, and the electrical and mechanical quantities on the stator are related by

$$\boxed{\theta_e = \frac{P}{2}\theta_m} \tag{4–31}$$

$$\boxed{f_e = \frac{P}{2}f_m} \tag{4–32}$$

$$\boxed{\omega_e = \frac{P}{2}\omega_m} \tag{4–33}$$

Also, noting that $f_m = n_m/60$, it is possible to relate the electrical frequency in hertz to the resulting mechanical speed of the magnetic fields in revolutions per minute. This relationship is

$$\boxed{f_e = \frac{n_m P}{120}} \tag{4–34}$$

Reversing the Direction of Magnetic Field Rotation

Another interesting fact can be observed about the resulting magnetic field. *If the current in any two of the three coils is swapped, the direction of the magnetic field's rotation will be reversed.* This means that it is possible to reverse the direction of rotation of an ac motor just by switching the connections on any two of the three coils. This result is verified below.

To prove that the direction of rotation is reversed, phases bb' and cc' in Figure 4–8 are switched and the resulting flux density $\mathbf{B}_{\text{net}}$ is calculated.

The net magnetic flux density in the stator is given by

$$\begin{aligned}\mathbf{B}_{\text{net}}(t) &= \mathbf{B}_{aa'}(t) + \mathbf{B}_{bb'}(t) + \mathbf{B}_{cc'}(t)\\ &= B_M \sin \omega t \angle 0^\circ + B_M \sin(\omega t - 240^\circ) \angle 120^\circ + B_M \sin(\omega t - 120^\circ) \angle 240^\circ \text{ T}\end{aligned}$$

Each of the three component magnetic fields can now be broken down into its x and y components:

$$\begin{aligned}\mathbf{B}_{\text{net}}(t) = {} & B_M \sin \omega t\, \hat{\mathbf{x}}\\ & - [0.5B_M \sin(\omega t - 240^\circ)]\hat{\mathbf{x}} + \left[\frac{\sqrt{3}}{2} B_M \sin(\omega t - 240^\circ)\right]\hat{\mathbf{y}}\\ & - [0.5B_M \sin(\omega t - 120^\circ)]\hat{\mathbf{x}} - \left[\frac{\sqrt{3}}{2} B_M \sin(\omega t - 120^\circ)\right]\hat{\mathbf{y}}\end{aligned}$$

Combining x and y components yields

$$\mathbf{B}_{\text{net}}(t) = [B_M \sin \omega t - 0.5B_M \sin(\omega t - 240°) - 0.5B_M \sin(\omega t - 120°)]\hat{\mathbf{x}}$$
$$+ \left[\frac{\sqrt{3}}{2} B_M \sin(\omega t - 240°) - \frac{\sqrt{3}}{2} B_M \sin(\omega t - 120°)\right]\hat{\mathbf{y}}$$

By the angle-addition trigonometric identities,

$$\mathbf{B}_{\text{net}}(t) = \left[B_M \sin \omega t + \frac{1}{4}B_M \sin \omega t - \frac{\sqrt{3}}{4}B_M \cos \omega t + \frac{1}{4}B_M \sin \omega t + \frac{\sqrt{3}}{4}B_M \cos \omega t\right]\hat{\mathbf{x}}$$
$$+ \left[-\frac{\sqrt{3}}{4}B_M \sin \omega t + \frac{3}{4}B_M \cos \omega t + \frac{\sqrt{3}}{4}B_M \sin \omega t + \frac{3}{4}B_M \cos \omega t\right]\hat{\mathbf{y}}$$

$$\boxed{\mathbf{B}_{\text{net}}(t) = (1.5B_M \sin \omega t)\hat{\mathbf{x}} + (1.5B_M \cos \omega t)\hat{\mathbf{y}}} \tag{4–35}$$

This time the magnetic field has the same magnitude but rotates in a clockwise direction. Therefore, *switching the currents in two stator phases reverses the direction of magnetic field rotation in an ac machine.*

Example 4–1. Create a MATLAB program that models the behavior of a rotating magnetic field in the three-phase stator shown in Figure 4–9.

Solution

The geometry of the loops in this stator is fixed as shown in Figure 4–9. The currents in the loops are

$$i_{aa'}(t) = I_M \sin \omega t \quad \text{A} \tag{4–21a}$$
$$i_{bb'}(t) = I_M \sin(\omega t - 120°) \quad \text{A} \tag{4–21b}$$
$$i_{cc'}(t) = I_M \sin(\omega t - 240°) \quad \text{A} \tag{4–21c}$$

and the resulting magnetic flux densities are

$$\mathbf{B}_{aa'}(t) = B_M \sin \omega t \angle 0° \quad \text{T} \tag{4–23a}$$
$$\mathbf{B}_{bb'}(t) = B_M \sin(\omega t - 120°) \angle 120° \quad \text{T} \tag{4–23b}$$
$$\mathbf{B}_{cc'}(t) = B_M \sin(\omega t - 240°) \angle 240° \quad \text{T} \tag{4–23c}$$
$$\phi = 2rlB = dlB$$

A simple MATLAB program that plots $\mathbf{B}_{aa'}$, $\mathbf{B}_{bb'}$, $\mathbf{B}_{cc'}$, and $\mathbf{B}_{\text{net}}$ as a function of time is shown below:

```
% M-file: mag_field.m
% M-file to calculate the net magnetic field produced
% by a three-phase stator.

% Set up the basic conditions
bmax = 1;                    % Normalize bmax to 1
freq = 60;                   % 60 Hz
w = 2*pi*freq;               % angular velocity (rad/s)

% First, generate the three component magnetic fields
t = 0:1/6000:1/60;
Baa = sin(w*t) .* (cos(0) + j*sin(0));
```

```
Bbb = sin(w*t-2*pi/3) .* (cos(2*pi/3) + j*sin(2*pi/3));
Bcc = sin(w*t+2*pi/3) .* (cos(-2*pi/3) + j*sin(-2*pi/3));

% Calculate Bnet
Bnet = Baa + Bbb + Bcc;

% Calculate a circle representing the expected maximum
% value of Bnet
circle = 1.5 * (cos(w*t) + j*sin(w*t));

% Plot the magnitude and direction of the resulting magnetic
% fields. Note that Baa is black, Bbb is blue, Bcc is
% magenta, and Bnet is red.
for ii = 1:length(t)

   % Plot the reference circle
   plot(circle,'k');
   hold on;

   % Plot the four magnetic fields
   plot([0 real(Baa(ii))],[0 imag(Baa(ii))],'k','LineWidth',2);
   plot([0 real(Bbb(ii))],[0 imag(Bbb(ii))],'b','LineWidth',2);
   plot([0 real(Bcc(ii))],[0 imag(Bcc(ii))],'m','LineWidth',2);
   plot([0 real(Bnet(ii))],[0 imag(Bnet(ii))],'r','LineWidth',3);
   axis square;
   axis([-2 2 -2 2]);
   drawnow;
   hold off;

end
```

When this program is executed, it draws lines corresponding to the three component magnetic fields as well as a line corresponding to the net magnetic field. Execute this program and observe the behavior of $\mathbf{B}_{net}$.

4.3 MAGNETOMOTIVE FORCE AND FLUX DISTRIBUTION ON AC MACHINES

In Section 4.2, the flux produced inside an ac machine was treated as if it were in free space. The direction of the flux density produced by a coil of wire was assumed to be perpendicular to the plane of the coil, with the direction of the flux given by the right-hand rule.

The *flux in a real machine does not behave in the simple manner assumed above,* since there is a ferromagnetic rotor in the center of the machine, with a small air gap between the rotor and the stator. The rotor can be cylindrical, like the one shown in Figure 4–12a, or it can have pole faces projecting out from its surface, as shown in Figure 4–12b. If the rotor is cylindrical, the machine is said to have ***nonsalient poles***; if the rotor has pole faces projecting out from it, the

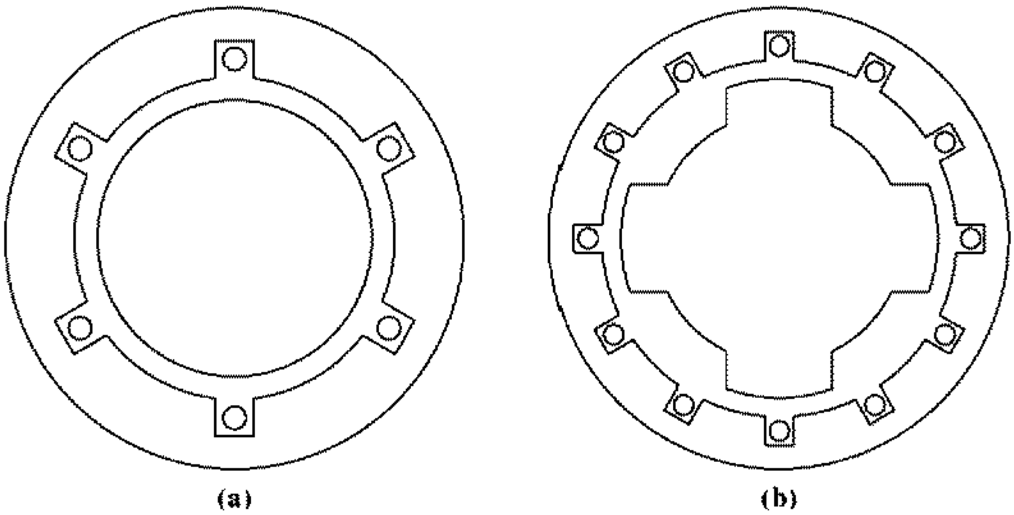


FIGURE 4–12
(a) An ac machine with a cylindrical or nonsalient-pole rotor. (b) An ac machine with a salient-pole rotor.

machine is said to have ***salient poles***. Cylindrical rotor or nonsalient-pole machines are easier to understand and analyze than salient-pole machines, and this discussion will be restricted to machines with cylindrical rotors. Machines with salient poles are discussed briefly in Appendix C and more extensively in References 1 and 2.

Refer to the cylindrical-rotor machine in Figure 4–12a. The reluctance of the air gap in this machine is much higher than the reluctances of either the rotor or the stator, so ***the flux density vector* B *takes the shortest possible path across the air gap*** and jumps perpendicularly between the rotor and the stator.

To produce a sinusoidal voltage in a machine like this, ***the magnitude of the flux density vector* B *must vary in a sinusoidal manner*** along the surface of the air gap. The flux density will vary sinusoidally only if the magnetizing intensity **H** (and magnetomotive force $\mathcal{F}$) varies in a sinusoidal manner along the surface of the air gap (see Figure 4–13).

The most straightforward way to achieve a sinusoidal variation of magnetomotive force along the surface of the air gap is to distribute the turns of the winding that produces the magnetomotive force in closely spaced slots around the surface of the machine and to vary the number of conductors in each slot in a sinusoidal manner. Figure 4–14a shows such a winding, and Figure 4–14b shows the magnetomotive force resulting from the winding. The number of conductors in each slot is given by the equation

$$n_C = N_C \cos \alpha \tag{4–36}$$

where N_C is the number of conductors at an angle of 0°. As Figure 4–14b shows, this distribution of conductors produces a close approximation to a sinusoidal distribution of magnetomotive force. Furthermore, the more slots there are around the surface of the machine and the more closely spaced the slots are, the better this approximation becomes.

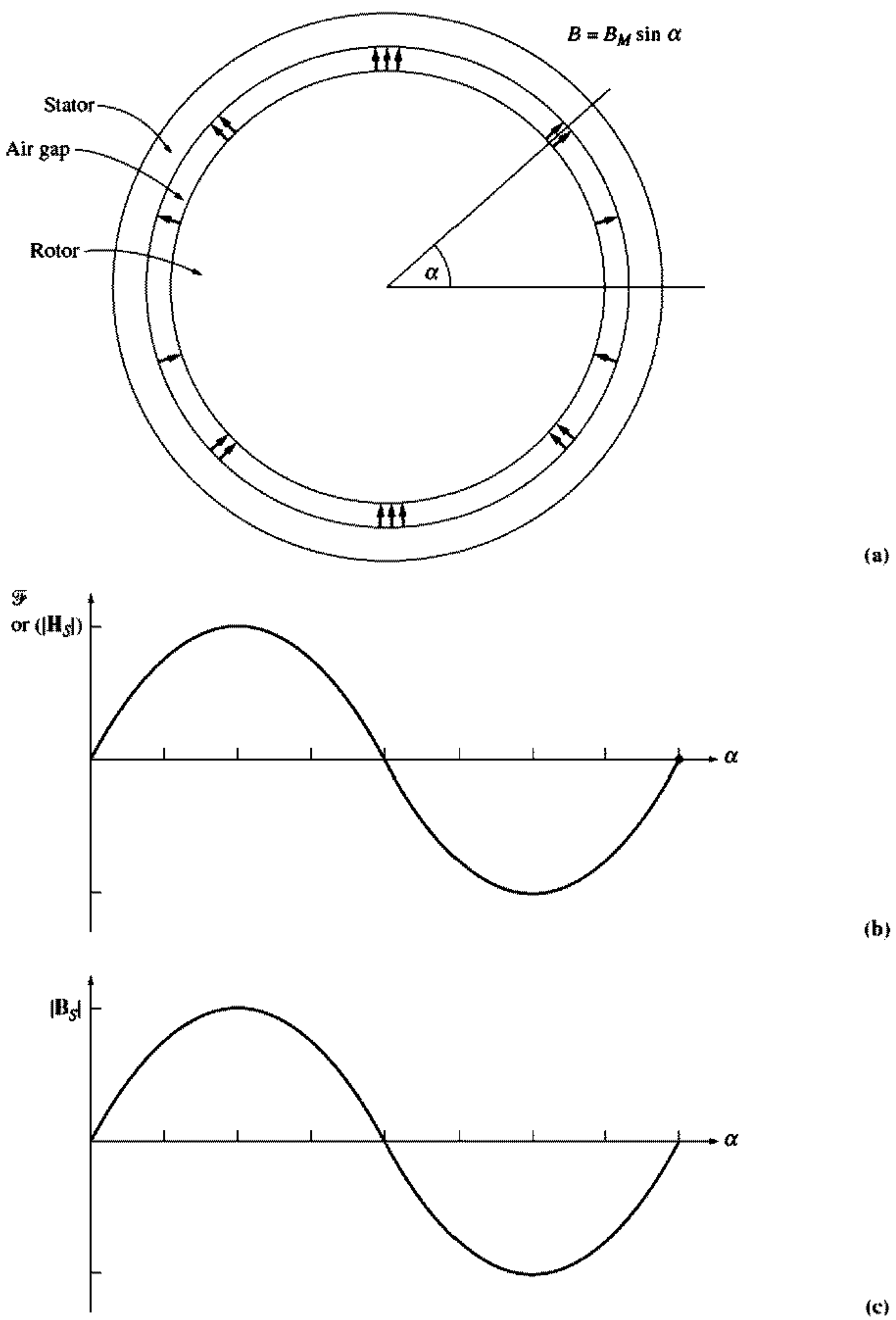


FIGURE 4–13
(a) A cylindrical rotor with sinusoidally varying air-gap flux density. (b) The magnetomotive force or magnetizing intensity as a function of angle α in the air gap. (c) The flux density as a function of angle α in the air gap.

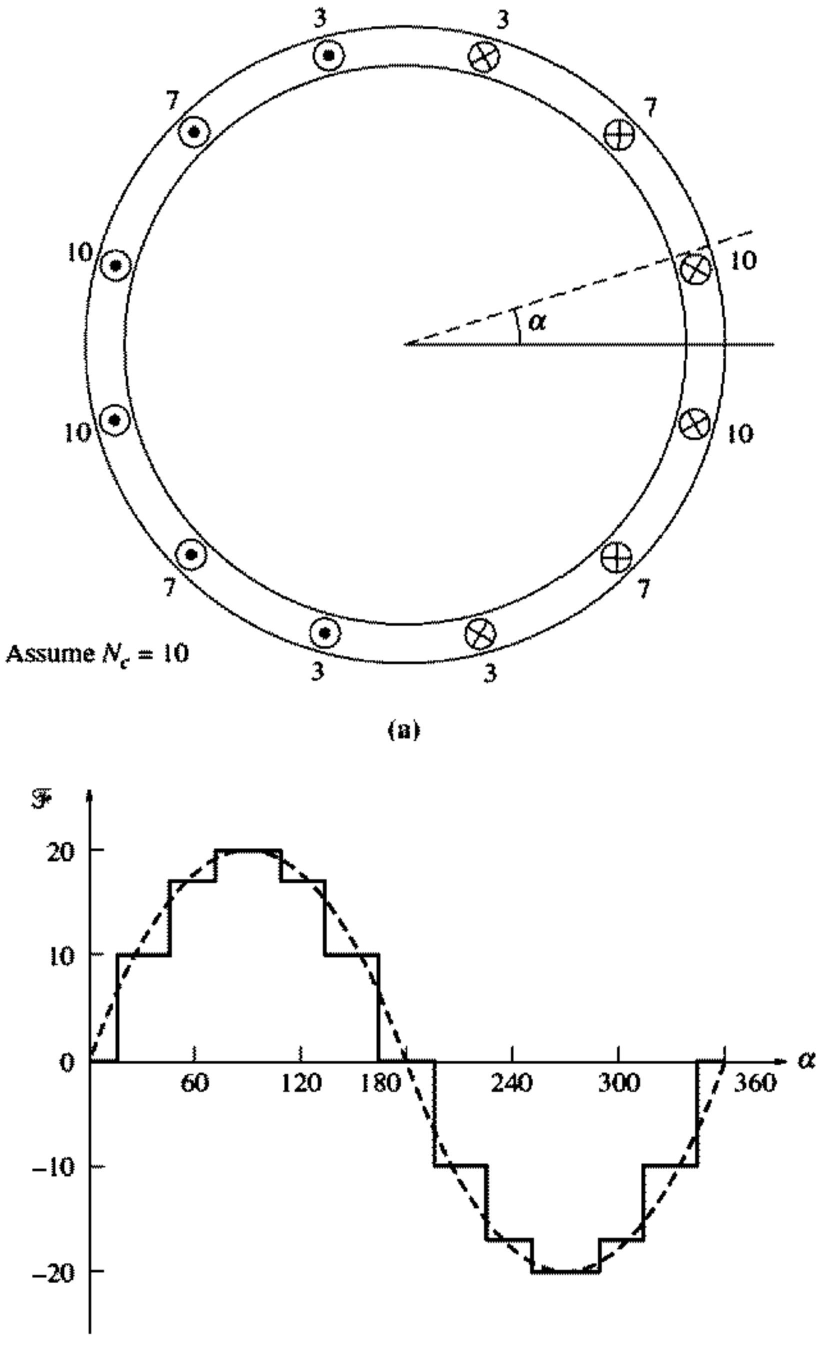


FIGURE 4–14
(a) An ac machine with a distributed stator winding designed to produce a sinusoidally varying air-gap flux density. The number of conductors in each slot is indicated on the diagram. (b) The magnetomotive force distribution resulting from the winding, compared to an ideal distribution.

In practice, it is not possible to distribute windings exactly in accordance with Equation (4–36), since there are only a finite number of slots in a real machine and since only integral numbers of conductors can be included in each slot. The resulting magnetomotive force distribution is only approximately sinusoidal, and higher-order harmonic components will be present. Fractional-pitch windings are used to suppress these unwanted harmonic components, as explained in Appendix B.1.

Furthermore, it is often convenient for the machine designer to include equal numbers of conductors in each slot instead of varying the number in accordance with Equation (4–36). Windings of this type are described in Appendix B.2; they have stronger high-order harmonic components than windings designed in accordance with Equation (4–36). The harmonic-suppression techniques of Appendix B.1 are especially important for such windings.

4.4 INDUCED VOLTAGE IN AC MACHINES

Just as a three-phase set of currents in a stator can produce a rotating magnetic field, a rotating magnetic field can produce a three-phase set of voltages in the coils of a stator. The equations governing the induced voltage in a three-phase stator will be developed in this section. To make the development easier, we will begin by looking at just one single-turn coil and then expand the results to a more general three-phase stator.

The Induced Voltage in a Coil on a Two-Pole Stator

Figure 4–15 shows a *rotating* rotor with a sinusoidally distributed magnetic field in the center of a *stationary* coil. Notice that this is the reverse of the situation studied in Section 4.1, which involved a stationary magnetic field and a rotating loop.

We will assume that the magnitude of the flux density vector **B** in the air gap between the rotor and the stator varies sinusoidally with mechanical angle, while the direction of **B** is always radially outward. This sort of flux distribution is the ideal to which machine designers aspire. (What happens when they don't achieve it is described in Appendix B.2.) If α is the angle measured from the direction of the peak rotor flux density, then the magnitude of the flux density vector **B** at a point around the *rotor* is given by

$$B = B_M \cos \alpha \tag{4–37a}$$

Note that at some locations around the air gap, the flux density vector will really point in toward the rotor; in those locations, the sign of Equation (4–37a) is negative. Since the rotor is itself rotating within the stator at an angular velocity ω_m, the magnitude of the flux density vector **B** at any angle α around the *stator* is given by

$$\boxed{B = B_M \cos(\omega t - \alpha)} \tag{4–37b}$$

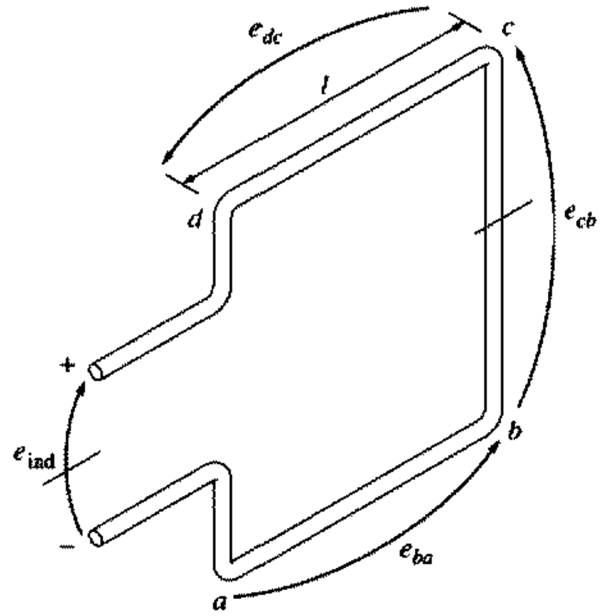
The equation for the induced voltage in a wire is

$$e = (\mathbf{v} \times \mathbf{B}) \cdot \mathbf{l} \tag{1–45}$$

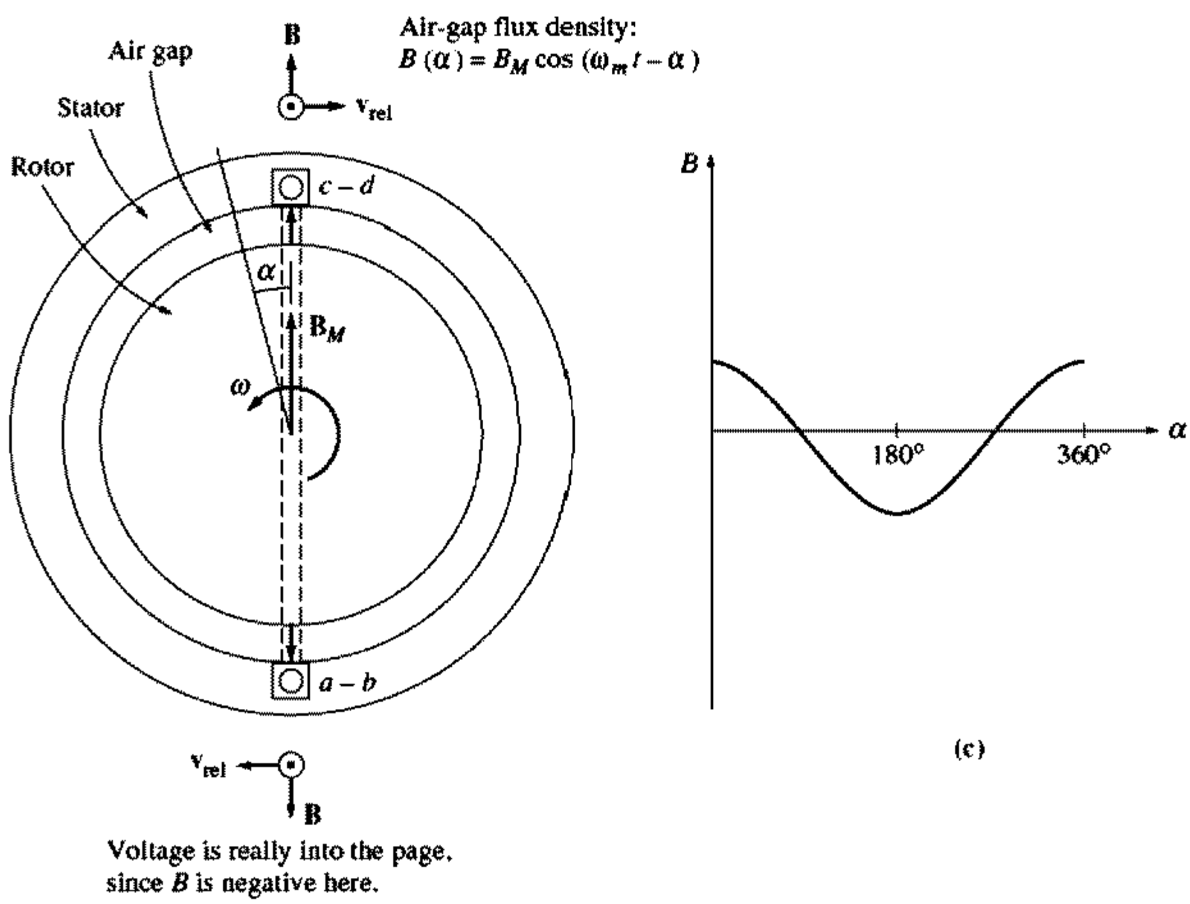
where $\mathbf{v}$ = velocity of the wire *relative to the magnetic field*

$\mathbf{B}$ = magnetic flux density vector

$\mathbf{l}$ = length of conductor in the magnetic field



(a)



(b)

(c)

FIGURE 4–15
(a) A rotating rotor magnetic field inside a stationary stator coil. Detail of coil. (b) The vector magnetic flux densities and velocities on the sides of the coil. The velocities shown are from a frame of reference in which the magnetic field is stationary. (c) The flux density distribution in the air gap.

However, this equation was derived for the case of a *moving wire* in a *stationary magnetic field*. In this case, the wire is stationary and the magnetic field is moving, so the equation does not directly apply. To use it, we must be in a frame of reference where the magnetic field appears to be stationary. If we "sit on the magnetic field" so that the field appears to be stationary, the sides of the coil will appear to go by at an apparent velocity $\mathbf{v}_{\text{rel}}$, and the equation can be applied. Figure 4–15b shows the vector magnetic field and velocities from the point of view of a stationary magnetic field and a moving wire.

The total voltage induced in the coil will be the sum of the voltages induced in each of its four sides. These voltages are determined below:

1. *Segment ab*. For segment ab, $\alpha = 180°$. Assuming that $\mathbf{B}$ is directed radially outward from the rotor, the angle between $\mathbf{v}$ and $\mathbf{B}$ in segment ab is 90°, while the quantity $\mathbf{v} \times \mathbf{B}$ is in the direction of $\mathbf{l}$, so

$$\begin{aligned} e_{ba} &= (\mathbf{v} \times \mathbf{B}) \cdot \mathbf{l} \\ &= vBl \qquad \text{directed out of the page} \\ &= -v[B_M \cos(\omega_m t - 180°)]l \\ &= -vB_M l \cos(\omega_m t - 180°) \end{aligned} \tag{4–38}$$

   where the minus sign comes from the fact that the voltage is built up with a polarity opposite to the assumed polarity.

2. Segment bc. The voltage on segment bc is zero, since the vector quantity $\mathbf{v} \times \mathbf{B}$ is perpendicular to $\mathbf{l}$, so

$$e_{cb} = (\mathbf{v} \times \mathbf{B}) \cdot \mathbf{l} = 0 \tag{4–39}$$

3. Segment cd. For segment cd, the angle $\alpha = 0°$. Assuming that $\mathbf{B}$ is directed radially outward from the rotor, the angle between $\mathbf{v}$ and $\mathbf{B}$ in segment cd is 90°, while the quantity $\mathbf{v} \times \mathbf{B}$ is in the direction of $\mathbf{l}$, so

$$\begin{aligned} e_{dc} &= (\mathbf{v} \times \mathbf{B}) \cdot \mathbf{l} \\ &= vBl \qquad \text{directed out of the page} \\ &= v(B_M \cos \omega_m t)l \\ &= vB_M l \cos \omega_m t \end{aligned} \tag{4–40}$$

4. Segment da. The voltage on segment da is zero, since the vector quantity $\mathbf{v} \times \mathbf{B}$ is perpendicular to $\mathbf{l}$, so

$$e_{ad} = (\mathbf{v} \times \mathbf{B}) \cdot \mathbf{l} = 0 \tag{4–41}$$

Therefore, the total voltage on the coil will be

$$\begin{aligned} e_{\text{ind}} &= e_{ba} + e_{dc} \\ &= -vB_M l \cos(\omega_m t - 180°) + vB_M l \cos \omega_m t \end{aligned} \tag{4–42}$$

Since $\cos \theta = -\cos(\theta - 180°)$,

$$e_{\text{ind}} = vB_M l \cos \omega_m t + vB_M l \cos \omega_m t$$
$$= 2vB_M l \cos \omega_m t \qquad (4\text{–}43)$$

Since the velocity of the end conductors is given by $v = r\omega_m$, Equation (4–43) can be rewritten as

$$e_{\text{ind}} = 2(r\omega_m)B_M l \cos \omega_m t$$
$$= 2rlB_M\omega_m \cos \omega_m t$$

Finally, the flux passing through the coil can be expressed as $\phi = 2rlB_m$ (see Problem 4–7), while $\omega_m = \omega_e = \omega$ for a two-pole stator, so the induced voltage can be expressed as

$$\boxed{e_{\text{ind}} = \phi\omega \cos \omega t} \qquad (4\text{–}44)$$

Equation (4–44) describes the voltage induced in a single-turn coil. If the coil in the stator has N_C turns of wire, then the total induced voltage of the coil will be

$$\boxed{e_{\text{ind}} = N_C\phi\omega \cos \omega t} \qquad (4\text{–}45)$$

Notice that the voltage produced in stator of this simple ac machine winding is sinusoidal with an amplitude which depends on the flux ϕ in the machine, the angular velocity ω of the rotor, and a constant depending on the construction of the machine (N_C in this simple case). This is the same as the result that we obtained for the simple rotating loop in Section 4.1.

Note that Equation (4–45) contains the term cos ωt instead of the sin ωt found in some of the other equations in this chapter. The cosine term has no special significance compared to the sine—it resulted from our choice of reference direction for α in this derivation. If the reference direction for α had been rotated by 90° we would have had a sin ωt term.

The Induced Voltage in a Three-Phase Set of Coils

If *three coils*, each of N_C turns, are placed around the rotor magnetic field as shown in Figure 4–16, then the voltages induced in each of them will be the same in magnitude but will differ in phase by 120°. The resulting voltages in each of the three coils are

$$e_{aa'}(t) = N_C\,\phi\omega \sin \omega t \quad \text{V} \qquad (4\text{–}46a)$$
$$e_{bb'}(t) = N_C\,\phi\omega \sin (\omega t - 120°) \quad \text{V} \qquad (4\text{–}46b)$$
$$e_{cc'}(t) = N_C\,\phi\omega \sin (\omega t - 240°) \quad \text{V} \qquad (4\text{–}46c)$$

Therefore, a three-phase set of currents can generate a uniform rotating magnetic field in a machine stator, and a uniform rotating magnetic field can generate a three-phase set of voltages in such a stator.

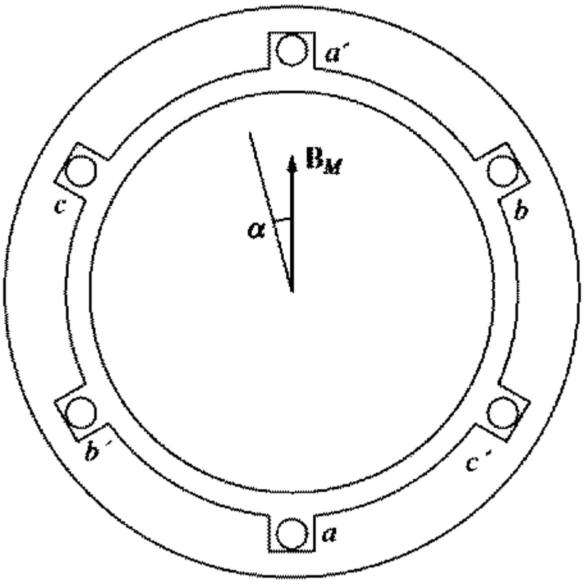


FIGURE 4–16
The production of three-phase voltages from three coils spaced 120° apart.

The RMS Voltage in a Three-Phase Stator

The peak voltage in any phase of a three-phase stator of this sort is

$$E_{max} = N_C \phi \omega \tag{4–47}$$

Since $\omega = 2\pi f$, this equation can also be written as

$$E_{max} = 2\pi N_C \phi f \tag{4–48}$$

Therefore, the rms voltage of any phase of this three-phase stator is

$$E_A = \frac{2\pi}{\sqrt{2}} N_C \phi f \tag{4–49}$$

$$\boxed{E_A = \sqrt{2}\pi N_C \phi f} \tag{4–50}$$

The rms voltage at the *terminals* of the machine will depend on whether the stator is Y- or Δ-connected. If the machine is Y-connected, then the terminal voltage will be $\sqrt{3}$ times E_A; if the machine is Δ-connected, then the terminal voltage will just be equal to E_A.

Example 4–2. The following information is known about the simple two-pole generator in Figure 4–16. The peak flux density of the rotor magnetic field is 0.2 T, and the mechanical rate of rotation of the shaft is 3600 r/min. The stator diameter of the machine is 0.5 m, its coil length is 0.3 m, and there are 15 turns per coil. The machine is Y-connected.

(a) What are the three phase voltages of the generator as a function of time?
(b) What is the rms phase voltage of this generator?
(c) What is the rms terminal voltage of this generator?

Solution

The flux in this machine is given by

$$\phi = 2rlB = dlB$$

where d is the diameter and l is the length of the coil. Therefore, the flux in the machine is given by

$$\phi = (0.5 \text{ m})(0.3 \text{ m})(0.2 \text{ T}) = 0.03 \text{ Wb}$$

The speed of the rotor is given by

$$\omega = (3600 \text{ r/min})(2\pi \text{ rad})(1 \text{ min}/60 \text{ s}) = 377 \text{ rad/s}$$

(a) The magnitudes of the peak phase voltages are thus

$$\begin{aligned} E_{max} &= N_C\phi\omega \\ &= (15 \text{ turns})(0.03 \text{ Wb})(377 \text{ rad/s}) = 169.7 \text{ V} \end{aligned}$$

and the three phase voltages are

$$\begin{aligned} e_{aa'}(t) &= 169.7 \sin 377t \quad \text{V} \\ e_{bb'}(t) &= 169.7 \sin (377t - 120°) \quad \text{V} \\ e_{cc'}(t) &= 169.7 \sin (377t - 240°) \quad \text{V} \end{aligned}$$

(b) The rms phase voltage of this generator is

$$E_A = \frac{E_{max}}{\sqrt{2}} = \frac{169.7 \text{ V}}{\sqrt{2}} = 120 \text{ V}$$

(c) Since the generator is Y-connected,

$$V_T = \sqrt{3}E_A = \sqrt{3}(120 \text{ V}) = 208 \text{ V}$$

4.5 INDUCED TORQUE IN AN AC MACHINE

In ac machines under normal operating conditions, there are two magnetic fields present—a magnetic field from the rotor circuit and another magnetic field from the stator circuit. The interaction of these two magnetic fields produces the torque in the machine, just as two permanent magnets near each other will experience a torque which causes them to line up.

Figure 4–17 shows a simplified ac machine with a sinusoidal stator flux distribution that peaks in the upward direction and a single coil of wire mounted on the rotor. The stator flux distribution in this machine is

$$B_S(\alpha) = B_S \sin \alpha \tag{4–51}$$

where B_S is the magnitude of the peak flux density; $B_S(\alpha)$ is positive when the flux density vector points radially outward from the rotor surface to the stator surface. How much torque is produced in the rotor of this simplified ac machine? To find out, we will analyze the force and torque on each of the two conductors separately.

The induced force on conductor 1 is

$$\begin{aligned} \mathbf{F} &= i(\mathbf{l} \times \mathbf{B}) \qquad\qquad (1–43) \\ &= ilB_S \sin \alpha \qquad \text{with direction as shown} \end{aligned}$$

The torque on the conductor is

$$\begin{aligned} \tau_{ind,1} &= (\mathbf{r} \times \mathbf{F}) \\ &= rilB_S \sin \alpha \qquad \text{counterclockwise} \end{aligned}$$

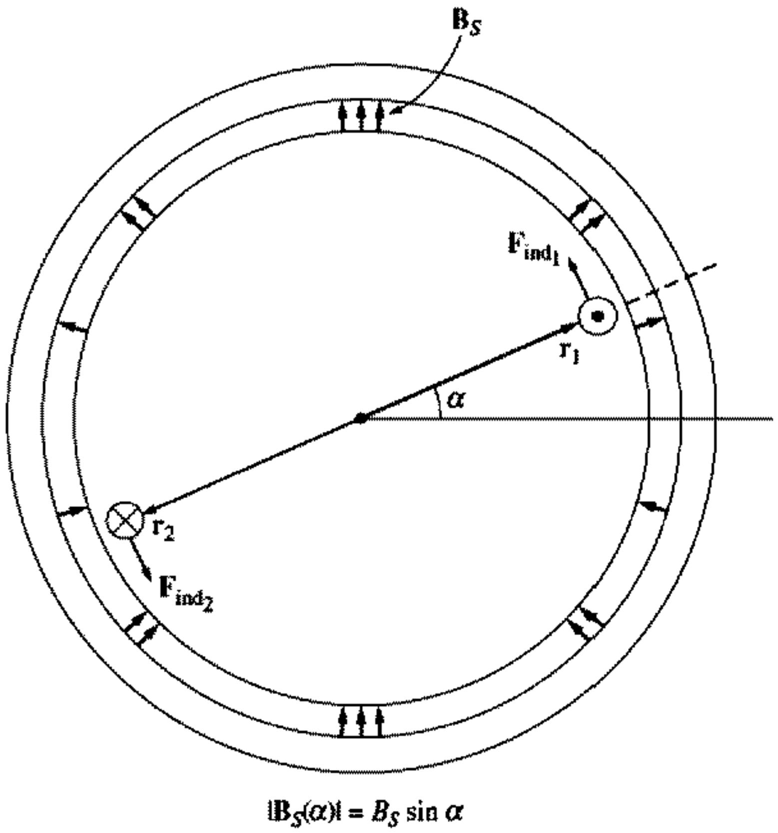


FIGURE 4–17
A simplified ac machine with a sinusoidal stator flux distribution and a single coil of wire mounted in the rotor.

The induced force on conductor 2 is

$$\mathbf{F} = i(\mathbf{l} \times \mathbf{B}) \tag{1–43}$$

$$= ilB_S \sin \alpha \qquad \text{with direction as shown}$$

The torque on the conductor is

$$\tau_{\text{ind.1}} = (\mathbf{r} \times \mathbf{F})$$

$$= rilB_S \sin \alpha \qquad \text{counterclockwise}$$

Therefore, the torque on the rotor loop is

$$\boxed{\tau_{\text{ind}} = 2rilB_S \sin \alpha \qquad \text{counterclockwise}} \tag{4–52}$$

Equation (4–52) can be expressed in a more convenient form by examining Figure 4–18 and noting two facts:

1. The current i flowing in the rotor coil produces a magnetic field of its own. The direction of the peak of this magnetic field is given by the right-hand rule, and the magnitude of its magnetizing intensity $\mathbf{H}_R$ is directly proportional to the current flowing in the rotor:

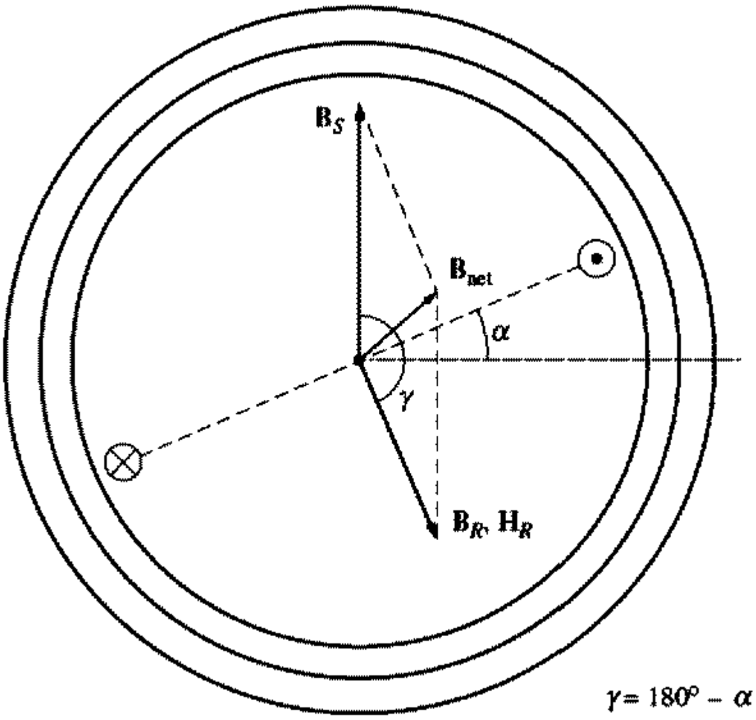


FIGURE 4–18
The components magnetic flux density inside the machine of Figure 4–17.

$$H_R = Ci \tag{4–53}$$

where C is a constant of proportionality.

2. The angle between the peak of the stator flux density $\mathbf{B}_S$ and the peak of the rotor magnetizing intensity $\mathbf{H}_R$ is γ. Furthermore,

$$\gamma = 180° - \alpha \tag{4–54}$$

$$\sin \gamma = \sin (180° - \alpha) = \sin \alpha \tag{4–55}$$

By combining these two observations, the torque on the loop can be expressed as

$$\tau_{\text{ind}} = KH_RB_S \sin \alpha \qquad \text{counterclockwise} \tag{4–56}$$

where K is a constant dependent on the construction of the machine. Note that both the magnitude and the direction of the torque can be expressed by the equation

$$\boxed{\tau_{\text{ind}} = K\mathbf{H}_R \times \mathbf{B}_S} \tag{4–57}$$

Finally, since $B_R = \mu H_R$, this equation can be reexpressed as

$$\boxed{\tau_{\text{ind}} = k\mathbf{B}_R \times \mathbf{B}_S} \tag{4–58}$$

where $k = K/\mu$. Note that in general k will not be constant, since the magnetic permeability μ varies with the amount of magnetic saturation in the machine.

Equation (4–58) is just the same as Equation (4–20), which we derived for the case of a single loop in a uniform magnetic field. It can apply to any ac machine, not just to the simple one-loop rotor just described. Only the constant k will differ from machine to machine. This equation will be used only for a *qualitative* study of torque in ac machines, so the actual value of k is unimportant for our purposes.

The net magnetic field in this machine is the vector sum of the rotor and stator fields (assuming no saturation):

$$\mathbf{B}_{\text{net}} = \mathbf{B}_R + \mathbf{B}_S \tag{4–59}$$

This fact can be used to produce an equivalent (and sometimes more useful) expression for the induced torque in the machine. From Equation (4–58)

$$\tau_{\text{ind}} = k\mathbf{B}_R \times \mathbf{B}_S \tag{4–58}$$

But from Equation (4–59), $\mathbf{B}_S = \mathbf{B}_{\text{net}} - \mathbf{B}_R$, so

$$\begin{aligned}\tau_{\text{ind}} &= k\mathbf{B}_R \times (\mathbf{B}_{\text{net}} - \mathbf{B}_R)\\ &= k(\mathbf{B}_R \times \mathbf{B}_{\text{net}}) - k(\mathbf{B}_R \times \mathbf{B}_R)\end{aligned}$$

Since the cross product of any vector with itself is zero, this reduces to

$$\boxed{\tau_{\text{ind}} = k\mathbf{B}_R \times \mathbf{B}_{\text{net}}} \tag{4–60}$$

so the induced torque can also be expressed as a cross product of $\mathbf{B}_R$ and $\mathbf{B}_{\text{net}}$ with the same constant k as before. The magnitude of this expression is

$$\boxed{\tau_{\text{ind}} = kB_R B_{\text{net}} \sin\delta} \tag{4–61}$$

where δ is the angle between $\mathbf{B}_R$ and $\mathbf{B}_{\text{net}}$.

Equations (4–58) to (4–61) will be used to help develop a qualitative understanding of the torque in ac machines. For example, look at the simple synchronous machine in Figure 4–19. Its magnetic fields are rotating in a counterclockwise direction. What is the direction of the torque on the shaft of the machine's rotor? By applying the right-hand rule to Equation (4–58) or (4–60), the induced torque is found to be clockwise, or opposite the direction of rotation of the rotor. Therefore, this machine must be acting as a generator.

4.6 WINDING INSULATION IN AN AC MACHINE

One of the most critical parts of an ac machine design is the insulation of its windings. If the insulation of a motor or generator breaks down, the machine shorts out. The repair of a machine with shorted insulation is quite expensive, if it is even possible. To prevent the winding insulation from breaking down as a result of overheating, it is necessary to limit the temperature of the windings. This can be partially done by providing a cooling air circulation over them, but ultimately the maximum winding temperature limits the maximum power that can be supplied continuously by the machine.

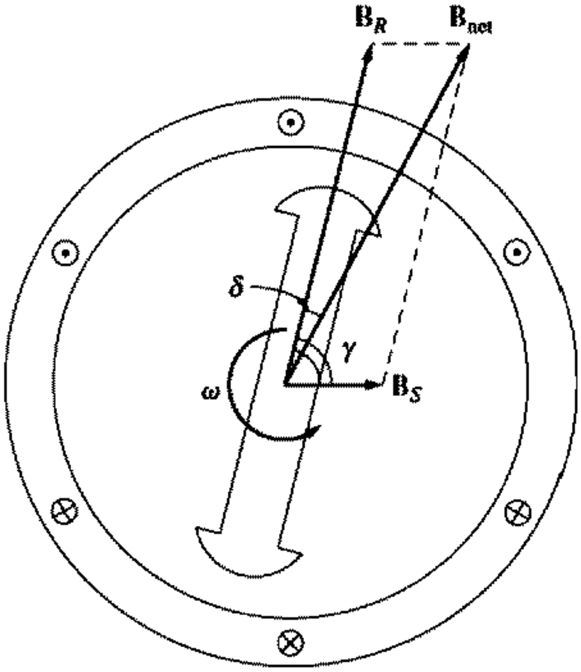


FIGURE 4–19
A simplified synchronous machine showing its rotor and stator magnetic fields.

Insulation rarely fails from immediate breakdown at some critical temperature. Instead, the increase in temperature produces a gradual degradation of the insulation, making it subject to failure from another cause such as shock, vibration, or electrical stress. There was an old rule of thumb that said that the life expectancy of a motor with a given type of insulation is halved for each 10 percent rise in temperature above the rated temperature of the winding. This rule still applies to some extent today.

To standardize the temperature limits of machine insulation, the National Electrical Manufacturers Association (NEMA) in the United States has defined a series of *insulation system classes*. Each insulation system class specifies the maximum temperature rise permissible for that class of insulation. There are three common NEMA insulation classes for integral-horsepower ac motors: B, F, and H. Each class represents a higher permissible winding temperature than the one before it. For example, the armature winding temperature rise above ambient temperature in one type of continuously operating ac induction motor must be limited to 80°C for class B, 105°C for class F, and 125°C for class H insulation.

The effect of operating temperature on insulation life for a typical machine can be quite dramatic. A typical curve is shown in Figure 4–20. This curve shows the mean life of a machine in thousands of hours versus the temperature of the windings, for several different insulation classes.

The specific temperature specifications for each type of ac motor and generator are set out in great detail in NEMA Standard MG1-1993, *Motors and Generators*. Similar standards have been defined by the International Electrotechnical Commission (IEC) and by various national standards organizations in other countries.

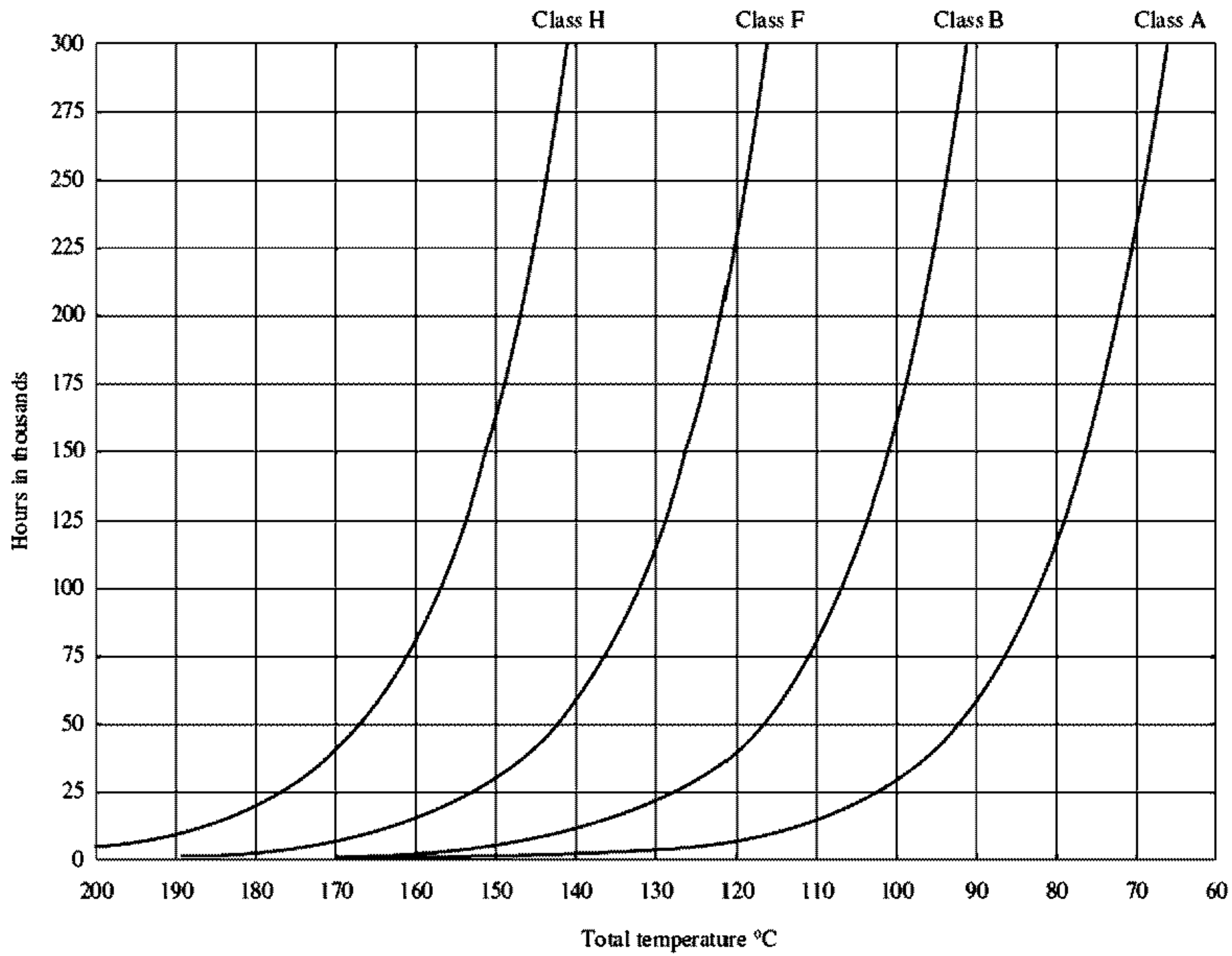


FIGURE 4–20
Plot of mean insulation life versus winding temperature for various insulation classes. *(Courtesy of Marathon Electric Company.)*

4.7 AC MACHINE POWER FLOWS AND LOSSES

AC generators take in mechanical power and produce electric power, while ac motors take in electric power and produce mechanical power. In either case, not all the power input to the machine appears in useful form at the other end—there is *always* some loss associated with the process.

The efficiency of an ac machine is defined by the equation

$$\boxed{\eta = \frac{P_{\text{out}}}{P_{\text{in}}} \times 100\%} \tag{4–62}$$

The difference between the input power and the output power of a machine is the losses that occur inside it. Therefore,

$$\boxed{\eta = \frac{P_{\text{in}} - P_{\text{loss}}}{P_{\text{in}}} \times 100\%} \tag{4–63}$$

The Losses in AC Machines

The losses that occur in ac machines can be divided into four basic categories:

1. Electrical or copper losses (I^2R losses)
2. Core losses
3. Mechanical losses
4. Stray load losses

ELECTRICAL OR COPPER LOSSES. Copper losses are the resistive heating losses that occur in the stator (armature) and rotor (field) windings of the machine. The stator copper losses (SCL) in a three-phase ac machine are given by the equation

$$\boxed{P_{\text{SCL}} = 3I_A^2R_A} \tag{4–64}$$

where I_A is the current flowing in each armature phase and R_A is the resistance of each armature phase.

The rotor copper losses (RCL) of a synchronous ac machine (induction machines will be considered separately in Chapter 7) are given by

$$\boxed{P_{\text{RCL}} = I_F^2R_F} \tag{4–65}$$

where I_F is the current flowing in the field winding on the rotor and R_F is the resistance of the field winding. The resistance used in these calculations is usually the winding resistance at normal operating temperature.

CORE LOSSES. The core losses are the hysteresis losses and eddy current losses occurring in the metal of the motor. These losses were described in Chapter 1.

These losses vary as the square of the flux density (B^2) and, for the stator, as the 1.5th power of the speed of rotation of the magnetic fields ($n^{1.5}$).

MECHANICAL LOSSES. The mechanical losses in an ac machine are the losses associated with mechanical effects. There are two basic types of mechanical losses: *friction* and *windage*. Friction losses are losses caused by the friction of the bearings in the machine, while windage losses are caused by the friction between the moving parts of the machine and the air inside the motor's casing. These losses vary as the cube of the speed of rotation of the machine.

The mechanical and core losses of a machine are often lumped together and called the *no-load rotational loss* of the machine. At no load, all the input power must be used to overcome these losses. Therefore, measuring the input power to the stator of an ac machine acting as a motor at no load will give an approximate value for these losses.

STRAY LOSSES (OR MISCELLANEOUS LOSSES). Stray losses are losses that cannot be placed in one of the previous categories. No matter how carefully losses are accounted for, some always escape inclusion in one of the above categories. All such losses are lumped into stray losses. For most machines, stray losses are taken by convention to be 1 percent of full load.

The Power-Flow Diagram

One of the most convenient techniques for accounting for power losses in a machine is the *power-flow diagram*. A power-flow diagram for an ac generator is shown in Figure 4–21a. In this figure, mechanical power is input into the machine, and then the stray losses, mechanical losses, and core loses are subtracted. After they have been subtracted, the remaining power is ideally converted from mechanical to electrical form at the point labeled P_{conv}. The mechanical power that is converted is given by

$$\boxed{P_{\text{conv}} = \tau_{\text{ind}}\omega_m} \tag{4–66}$$

and the same amount of electrical power is produced. However, this is not the power that appears at the machine's terminals. Before the terminals are reached, the electrical I^2R losses must be subtracted.

In the case of ac motors, this power-flow diagram is simply reversed. The power-flow diagram for a motor is shown in Figure 4–21b.

Example problems involving the calculation of ac motor and generator efficiencies will be given in the next three chapters.

4.8 VOLTAGE REGULATION AND SPEED REGULATION

Generators are often compared to each other using a figure of merit called *voltage regulation*. Voltage regulation (VR) is a measure of the ability of a generator to keep a constant voltage at its terminals as load varies. It is defined by the equation

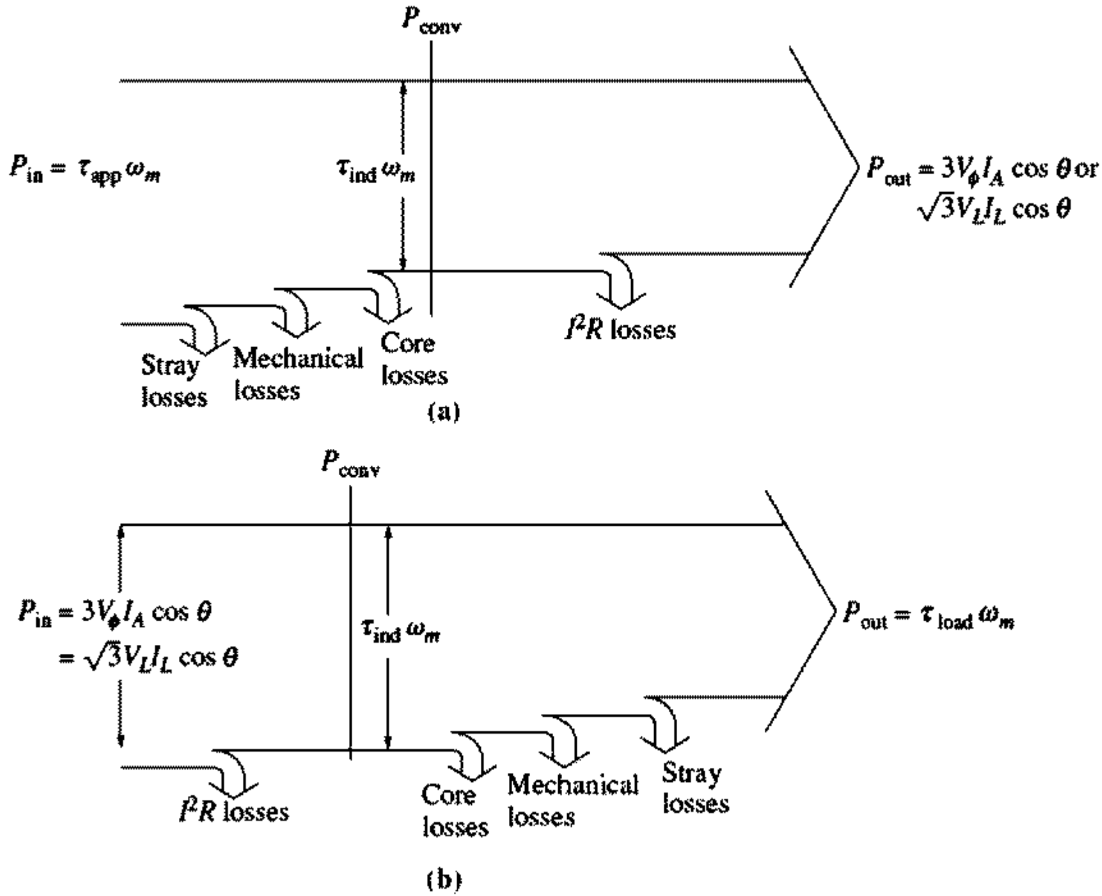


FIGURE 4–21
(a) The power-flow diagram of a three-phase ac generator. (b) The power-flow diagram of a three-phase ac motor.

$$\boxed{\mathrm{VR} = \frac{V_{\mathrm{nl}} - V_{\mathrm{fl}}}{V_{\mathrm{fl}}} \times 100\%} \qquad (4\text{–}67)$$

where V_{nl} is the no-load terminal voltage of the generator and V_{fl} is the full-load terminal voltage of the generator. It is a rough measure of the shape of the generator's voltage-current characteristic—a positive voltage regulation means a drooping characteristic, and a negative voltage regulation means a rising characteristic. A small VR is "better" in the sense that the voltage at the terminals of the generator is more constant with variations in load.

Similarly, motors are often compared to each other by using a figure of merit called *speed regulation*. Speed regulation (SR) is a measure of the ability of a motor to keep a constant shaft speed as load varies. It is defined by the equation

$$\boxed{\mathrm{SR} = \frac{n_{\mathrm{nl}} - n_{\mathrm{fl}}}{n_{\mathrm{fl}}} \times 100\%} \qquad (4\text{–}68)$$

or

$$\boxed{\mathrm{SR} = \frac{\omega_{\mathrm{nl}} - \omega_{\mathrm{fl}}}{\omega_{\mathrm{fl}}} \times 100\%} \qquad (4\text{–}69)$$

It is a rough measure of the shape of a motor's torque–speed characteristic—a positive speed regulation means that a motor's speed drops with increasing load, and a negative speed regulation means a motor's speed increases with increasing load. The magnitude of the speed regulation tells approximately how steep the slope of the torque-speed curve is.

4.9 SUMMARY

There are two major types of ac machines: synchronous machines and induction machines. The principal difference between the two types is that synchronous machines require a dc field current to be supplied to their rotors, while induction machines have the field current induced in their rotors by transformer action. They will be explored in detail in the next three chapters.

A three-phase system of currents supplied to a system of three coils spaced 120 electrical degrees apart on a stator will produce a uniform rotating magnetic field within the stator. The *direction of rotation* of the magnetic field can be *reversed* by simply swapping the connections to any two of the three phases. Conversely, a rotating magnetic field will produce a three-phase set of voltages within such a set of coils.

In stators of more than two poles, one complete mechanical rotation of the magnetic fields produces more than one complete electrical cycle. For such a stator, one mechanical rotation produces $P/2$ electrical cycles. Therefore, the electrical angle of the voltages and currents in such a machine is related to the mechanical angle of the magnetic fields by

$$\theta_e = \frac{P}{2}\theta_m$$

The relationship between the electrical frequency of the stator and the mechanical rate of rotation of the magnetic fields is

$$f_e = \frac{n_m P}{120}$$

The types of losses that occur in ac machines are electrical or copper losses (I^2R losses), core losses, mechanical losses, and stray losses. Each of these losses was described in this chapter, along with the definition of overall machine efficiency. Finally, voltage regulation was defined for generators as

$$\boxed{\mathrm{VR} = \frac{V_{\mathrm{nl}} - V_{\mathrm{fl}}}{V_{\mathrm{fl}}} \times 100\%}$$

and speed regulation was defined for motors as

$$\boxed{\mathrm{SR} = \frac{n_{\mathrm{nl}} - n_{\mathrm{fl}}}{n_{\mathrm{fl}}} \times 100\%}$$

QUESTIONS

4–1. What is the principal difference between a synchronous machine and an induction machine?

4–2. Why does switching the current flows in any two phases reverse the direction of rotation of a stator's magnetic field?

4–3. What is the relationship between electrical frequency and magnetic field speed for an ac machine?

4–4. What is the equation for the induced torque in an ac machine?

PROBLEMS

4–1. The simple loop rotating in a uniform magnetic field shown in Figure 4–1 has the following characteristics:

$$\mathbf{B} = 0.5 \text{ T to the right} \qquad r = 0.1 \text{ m}$$
$$l = 0.5 \text{ m} \qquad \omega = 103 \text{ rad/s}$$

(a) Calculate the voltage $e_{tot}(t)$ induced in this rotating loop.

(b) Suppose that a 5-Ω resistor is connected as a load across the terminals of the loop. Calculate the current that would flow through the resistor.

(c) Calculate the magnitude and direction of the induced torque on the loop for the conditions in *b*.

(d) Calculate the electric power being generated by the loop for the conditions in *b*.

(e) Calculate the mechanical power being consumed by the loop for the conditions in *b*. How does this number compare to the amount of electric power being generated by the loop?

4–2. Develop a table showing the speed of magnetic field rotation in ac machines of 2, 4, 6, 8, 10, 12, and 14 poles operating at frequencies of 50, 60, and 400 Hz.

4–3. A three-phase, four-pole winding is installed in 12 slots on a stator. There are 40 turns of wire in each slot of the windings. All coils in each phase are connected in series, and the three phases are connected in Δ. The flux per pole in the machine is 0.060 Wb, and the speed of rotation of the magnetic field is 1800 r/min.

(a) What is the frequency of the voltage produced in this winding?

(b) What are the resulting phase and terminal voltages of this stator?

4–4. A three-phase, Y-connected, 50-Hz, two-pole synchronous machine has a stator with 2000 turns of wire per phase. What rotor flux would be required to produce a terminal (line-to-line) voltage of 6 kV?

4–5. Modify the MATLAB problem in Example 4–1 by swapping the currents flowing in any two phases. What happens to the resulting net magnetic field?

4–6. If an ac machine has the rotor and stator magnetic fields shown in Figure P4–1, what is the direction of the induced torque in the machine? Is the machine acting as a motor or generator?

4–7. The flux density distribution over the surface of a two-pole stator of radius r and length l is given by

$$B = B_M \cos(\omega_m t - \alpha) \tag{4–37b}$$

Prove that the total flux under each pole face is

$$\phi = 2rlB_M$$

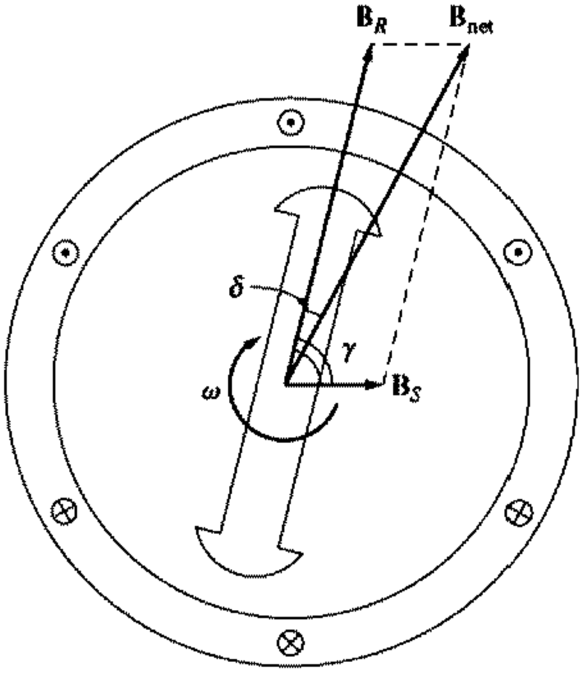


FIGURE P4–1
The ac machine of Problem 4–6.

4–8. In the early days of ac motor development, machine designers had great difficulty controlling the core losses (hysteresis and eddy currents) in machines. They had not yet developed steels with low hysteresis, and were not making laminations as thin as the ones used today. To help control these losses, early ac motors in the United States were run from a 25-Hz ac power supply, while lighting systems were run from a separate 60-Hz ac power supply.

(a) Develop a table showing the speed of magnetic field rotation in ac machines of 2, 4, 6, 8, 10, 12, and 14 poles operating at 25 Hz. What was the fastest rotational speed available to these early motors?

(b) For a given motor operating at a constant flux density B, how would the core losses of the motor running at 25 Hz compare to the core losses of the motor running at 60 Hz?

(c) Why did the early engineers provide a separate 60-Hz power system for lighting?

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