
CHAPTER 6

SYNCHRONOUS MOTORS

Synchronous motors are synchronous machines used to convert electrical power to mechanical power. This chapter explores the basic operation of synchronous motors and relates their behavior to that of synchronous generators.

6.1 BASIC PRINCIPLES OF MOTOR OPERATION

To understand the basic concept of a synchronous motor, look at Figure 6–1, which shows a two-pole synchronous motor. The field current I_F of the motor produces a steady-state magnetic field \mathbf{B}_R . A three-phase set of voltages is applied to the stator of the machine, which produces a three-phase current flow in the windings.

As was shown in Chapter 4, a three-phase set of currents in an armature winding produces a uniform rotating magnetic field \mathbf{B}_S . Therefore, there are two magnetic fields present in the machine, and the *rotor field will tend to line up with the stator field*, just as two bar magnets will tend to line up if placed near each other. Since the stator magnetic field is rotating, the rotor magnetic field (and the rotor itself) will constantly try to catch up. The larger the angle between the two magnetic fields (up to a certain maximum), the greater the torque on the rotor of the machine. The basic principle of synchronous motor operation is that the rotor “chases” the rotating stator magnetic field around in a circle, never quite catching up with it.

Since a synchronous motor is the same physical machine as a synchronous generator, all of the basic speed, power, and torque equations of Chapters 4 and 5 apply to synchronous motors also.

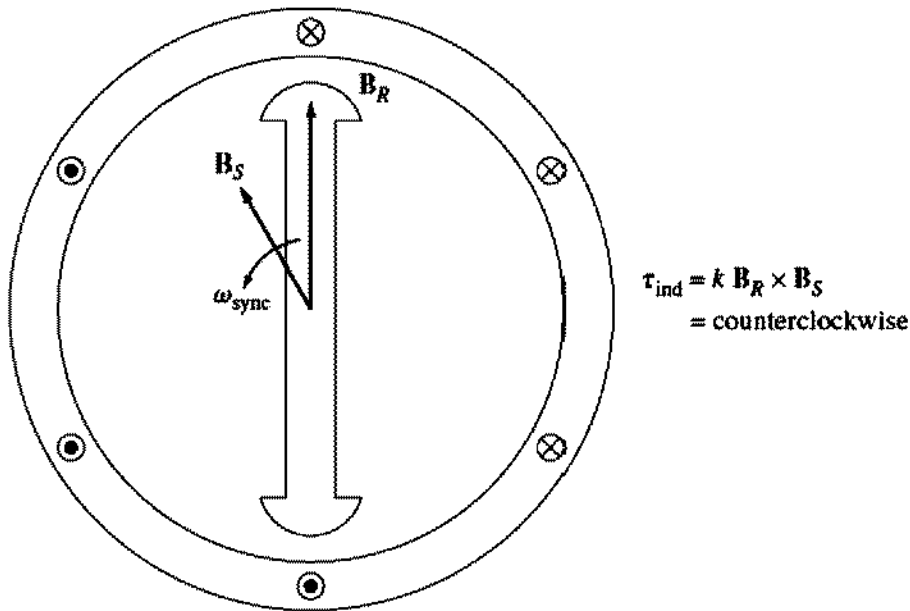


FIGURE 6-1
A two-pole synchronous motor.

The Equivalent Circuit of a Synchronous Motor

A synchronous motor is the same in all respects as a synchronous generator, except that the direction of power flow is reversed. Since the direction of power flow in the machine is reversed, the direction of current flow in the stator of the motor may be expected to reverse also. Therefore, the equivalent circuit of a synchronous motor is exactly the same as the equivalent circuit of a synchronous generator, *except* that the reference direction of I_A is *reversed*. The resulting full equivalent circuit is shown in Figure 6-2a, and the per-phase equivalent circuit is shown in Figure 6-2b. As before, the three phases of the equivalent circuit may be either Y- or Δ -connected.

Because of the change in direction of I_A , the Kirchhoff's voltage law equation for the equivalent circuit changes too. Writing a Kirchhoff's voltage law equation for the new equivalent circuit yields

$$\boxed{V_\phi = E_A + jX_S I_A + R_A I_A} \quad (6-1)$$

or

$$\boxed{E_A = V_\phi - jX_S I_A - R_A I_A} \quad (6-2)$$

This is exactly the same as the equation for a generator, except that the sign on the current term has been reversed.

The Synchronous Motor from a Magnetic Field Perspective

To begin to understand synchronous motor operation, take another look at a synchronous generator connected to an infinite bus. The generator has a prime mover

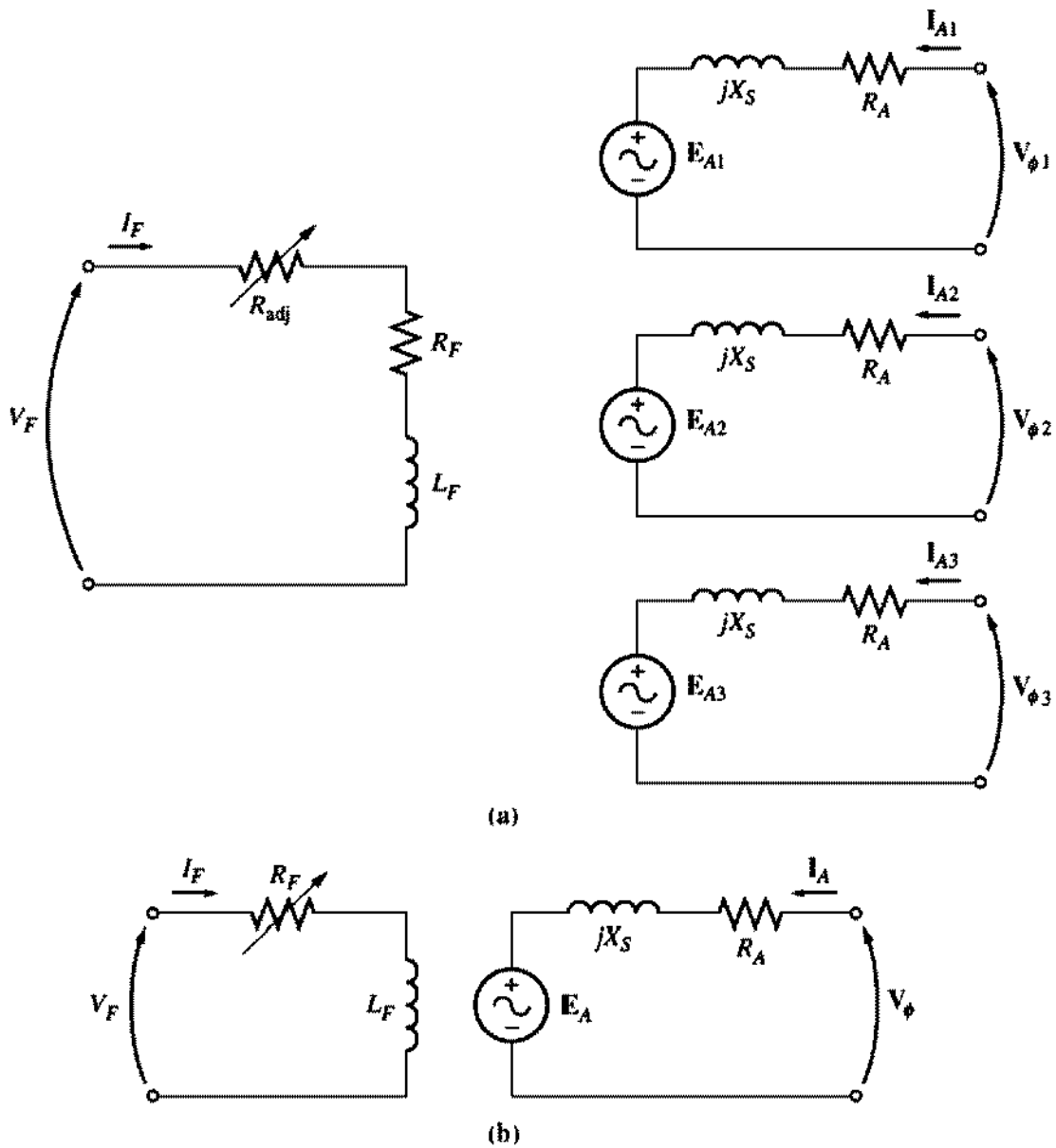


FIGURE 6-2
 (a) The full equivalent circuit of a three-phase synchronous motor. (b) The per-phase equivalent circuit.

turning its shaft, causing it to rotate. The direction of the applied torque τ_{app} from the prime mover is in the direction of motion, because the prime mover makes the generator rotate in the first place.

The phasor diagram of the generator operating with a large field current is shown in Figure 6-3a, and the corresponding magnetic field diagram is shown in Figure 6-3b. As described before, \mathbf{B}_R corresponds to (produces) \mathbf{E}_A , \mathbf{B}_{net} corresponds to (produces) \mathbf{V}_ϕ , and \mathbf{B}_S corresponds to $\mathbf{E}_{stat} (= -jX_S \mathbf{I}_A)$. The rotation of both the phasor diagram and magnetic field diagram is counterclockwise in the figure, following the standard mathematical convention of increasing angle.

The induced torque in the generator can be found from the magnetic field diagram. From Equations (4-60) and (4-61) the induced torque is given by

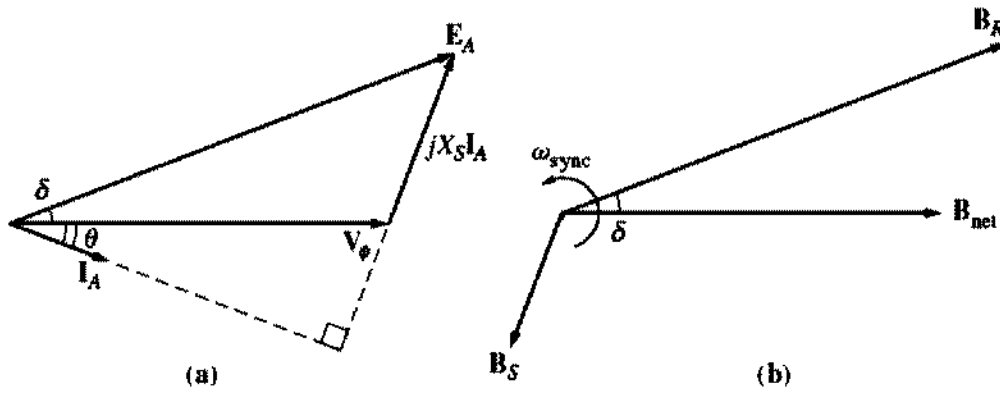


FIGURE 6-3 (a) Phasor diagram of a synchronous generator operating at a lagging power factor. (b) The corresponding magnetic field diagram.

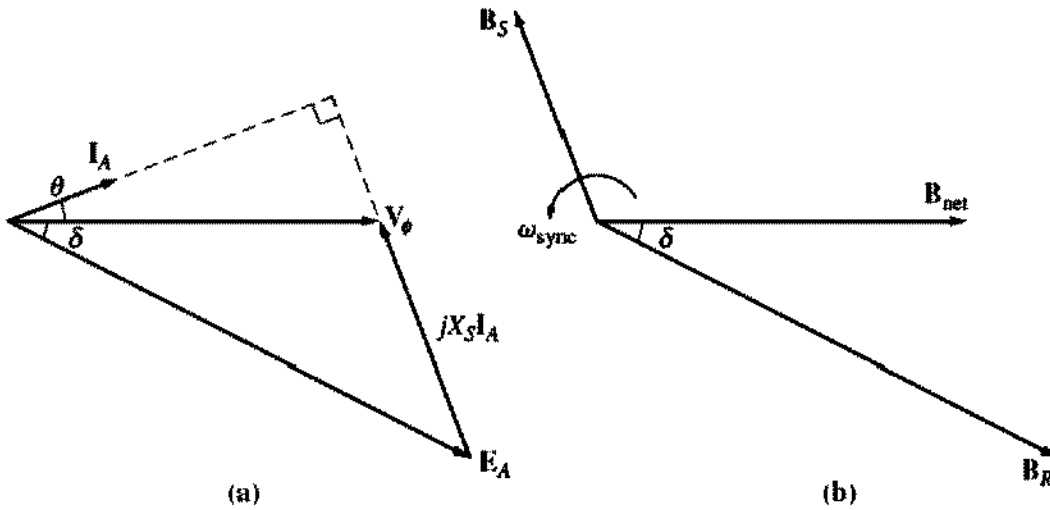


FIGURE 6-4 (a) Phasor diagram of a synchronous motor. (b) The corresponding magnetic field diagram.

$$\tau_{ind} = k B_R \times B_{net} \tag{4-60}$$

or

$$\tau_{ind} = k B_R B_{net} \sin \delta \tag{4-61}$$

Notice that from the magnetic field diagram *the induced torque in this machine is clockwise*, opposing the direction of rotation. In other words, the induced torque in the generator is a countertorque, opposing the rotation caused by the external applied torque τ_{app} .

Suppose that, instead of turning the shaft in the direction of motion, the prime mover suddenly loses power and starts to drag on the machine's shaft. What happens to the machine now? The rotor slows down because of the drag on its shaft and falls behind the net magnetic field in the machine (see Figure 6-4a). As the rotor, and therefore B_R , slows down and falls behind B_{net} the operation of the machine suddenly changes. By Equation (4-60), when B_R is behind B_{net} , the induced

torque's direction reverses and becomes counterclockwise. In other words, the machine's torque is now in the direction of motion, and the machine is acting as a motor. The increasing torque angle δ results in a larger and larger torque in the direction of rotation, until eventually the motor's induced torque equals the load torque on its shaft. At that point, the machine will be operating at steady state and synchronous speed again, but now as a motor.

The phasor diagram corresponding to generator operation is shown in Figure 6-3a, and the phasor diagram corresponding to motor operation is shown in Figure 6-4a. The reason that the quantity $jX_S I_A$ points from V_ϕ to E_A in the generator and from E_A to V_ϕ in the motor is that the reference direction of I_A was reversed in the definition of the motor equivalent circuit. The basic difference between motor and generator operation in synchronous machines can be seen either in the magnetic field diagram or in the phasor diagram. *In a generator*, E_A lies ahead of V_ϕ , and B_R lies ahead of B_{net} . *In a motor*, E_A lies behind V_ϕ , and B_R lies behind B_{net} . In a motor the induced torque is in the direction of motion, and in a generator the induced torque is a countertorque opposing the direction of motion.

6.2 STEADY-STATE SYNCHRONOUS MOTOR OPERATION

This section explores the behavior of synchronous motors under varying conditions of load and field current as well as the question of power-factor correction with synchronous motors. The following discussions will generally ignore the armature resistance of the motors for simplicity. However, R_A will be considered in some of the worked numerical calculations.

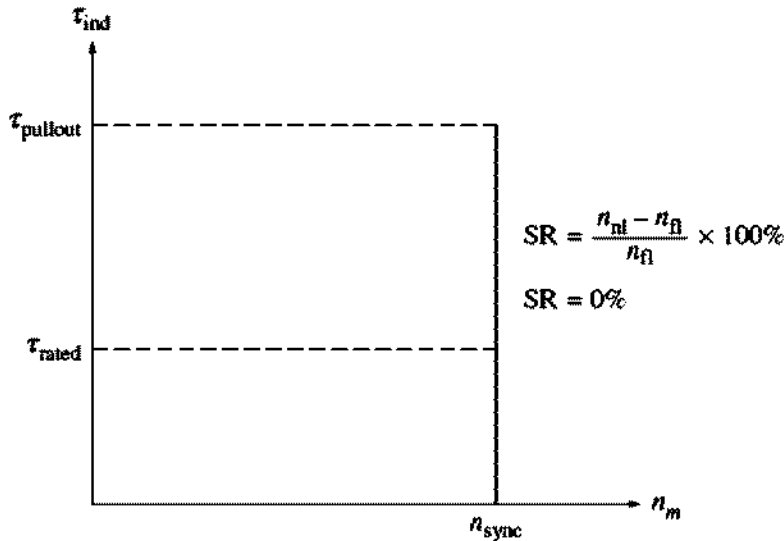
The Synchronous Motor Torque–Speed Characteristic Curve

Synchronous motors supply power to loads that are basically constant-speed devices. They are usually connected to power systems *very* much larger than the individual motors, so the power systems appear as infinite buses to the motors. This means that the terminal voltage and the system frequency will be constant regardless of the amount of power drawn by the motor. The speed of rotation of the motor is locked to the applied electrical frequency, so the speed of the motor will be constant regardless of the load. The resulting torque–speed characteristic curve is shown in Figure 6-5. The steady-state speed of the motor is constant from no load all the way up to the maximum torque that the motor can supply (called the *pull-out torque*), so the speed regulation of this motor [Equation (4-68)] is 0 percent. The torque equation is

$$\tau_{ind} = kB_R B_{net} \sin \delta \quad (4-61)$$

or

$$\tau_{ind} = \frac{3V_\phi E_A \sin \delta}{\omega_m X_S} \quad (5-22)$$


FIGURE 6-5

The torque–speed characteristic of a synchronous motor. Since the speed of the motor is constant, its speed regulation is zero.

The maximum or pullout torque occurs when $\delta = 90^\circ$. Normal full-load torques are much less than that, however. In fact, the pullout torque may typically be 3 times the full-load torque of the machine.

When the torque on the shaft of a synchronous motor exceeds the pullout torque, the rotor can no longer remain locked to the stator and net magnetic fields. Instead, the rotor starts to slip behind them. As the rotor slows down, the stator magnetic field “laps” it repeatedly, and the direction of the induced torque in the rotor reverses with each pass. The resulting huge torque surges, first one way and then the other way, cause the whole motor to vibrate severely. The loss of synchronization after the pullout torque is exceeded is known as *slipping poles*.

The maximum or pullout torque of the motor is given by

$$\tau_{\max} = kB_R B_{\text{net}} \quad (6-3)$$

or

$$\tau_{\max} = \frac{3V_\phi E_A}{\omega_m X_S} \quad (6-4)$$

These equations indicate that the larger the field current (and hence E_A), the greater the maximum torque of the motor. There is therefore a stability advantage in operating the motor with a large field current or a large E_A .

The Effect of Load Changes on a Synchronous Motor

If a load is attached to the shaft of a synchronous motor, the motor will develop enough torque to keep the motor and its load turning at a synchronous speed. What happens when the load is changed on a synchronous motor?

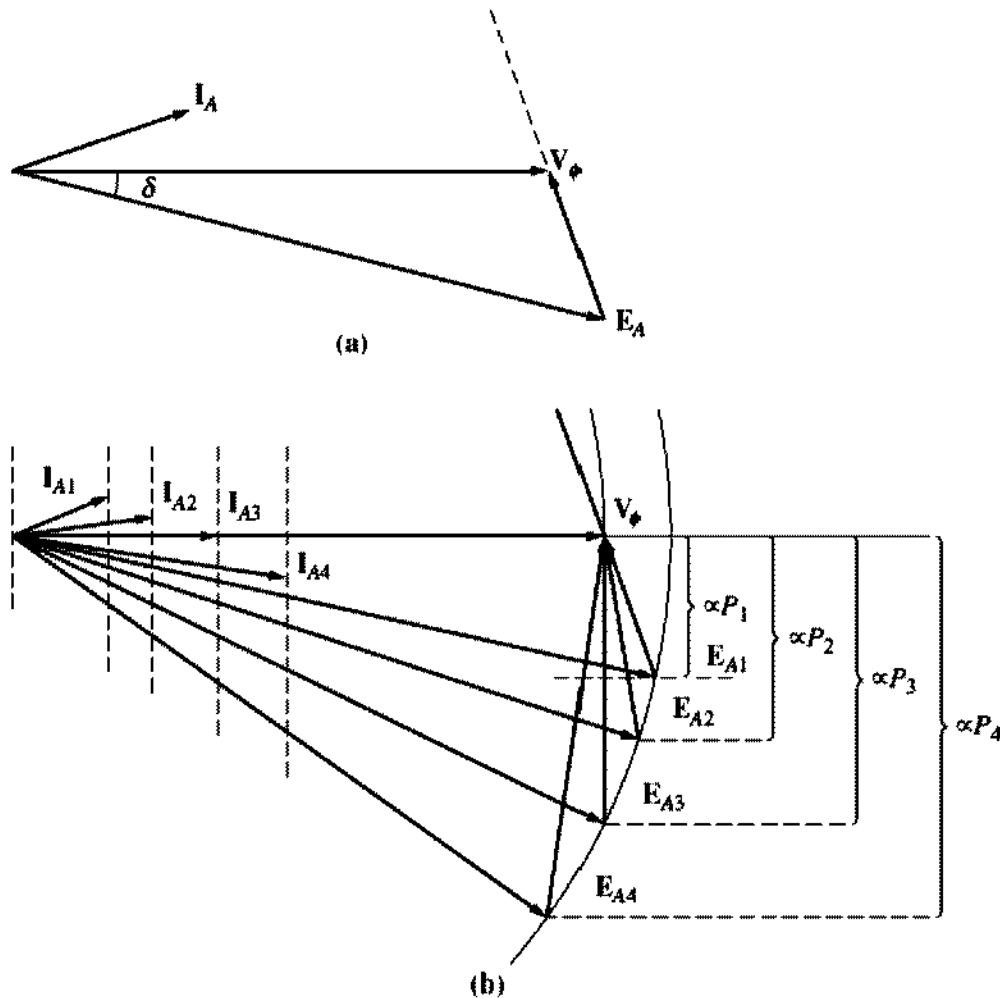


FIGURE 6-6

(a) Phasor diagram of a motor operating at a leading power factor. (b) The effect of an increase in load on the operation of a synchronous motor.

To find out, examine a synchronous motor operating initially with a leading power factor, as shown in Figure 6-6. If the load on the shaft of the motor is increased, the rotor will initially slow down. As it does, the torque angle δ becomes larger, and the induced torque increases. The increase in induced torque eventually speeds the rotor back up, and the motor again turns at synchronous speed but with a larger torque angle δ .

What does the phasor diagram look like during this process? To find out, examine the constraints on the machine during a load change. Figure 6-6a shows the motor's phasor diagram before the loads are increased. The internal generated voltage E_A is equal to $K\phi\omega$ and so depends on *only* the field current in the machine and the speed of the machine. The speed is constrained to be constant by the input power supply, and since no one has touched the field circuit, the field current is constant as well. Therefore, $|E_A|$ *must be constant as the load changes*. The distances proportional to power ($E_A \sin \delta$ and $I_A \cos \theta$) will increase, but the magnitude of E_A must remain constant. As the load increases, E_A swings down in the manner shown in Figure 6-6b. As E_A swings down further and further, the quantity

$jX_s I_A$ has to increase to reach from the tip of E_A to V_ϕ , and therefore the armature current I_A also increases. Notice that the power-factor angle θ changes too, becoming less and less leading and then more and more lagging.

Example 6-1. A 208-V, 45-kVA, 0.8-PF-leading, Δ -connected, 60-Hz synchronous machine has a synchronous reactance of 2.5Ω and a negligible armature resistance. Its friction and windage losses are 1.5 kW, and its core losses are 1.0 kW. Initially, the shaft is supplying a 15-hp load, and the motor's power factor is 0.80 leading.

- Sketch the phasor diagram of this motor, and find the values of I_A , I_L , and E_A .
- Assume that the shaft load is now increased to 30 hp. Sketch the behavior of the phasor diagram in response to this change.
- Find I_A , I_L , and E_A after the load change. What is the new motor power factor?

Solution

- Initially, the motor's output power is 15 hp. This corresponds to an output of

$$P_{\text{out}} = (15 \text{ hp})(0.746 \text{ kW/hp}) = 11.19 \text{ kW}$$

Therefore, the electric power supplied to the machine is

$$\begin{aligned} P_{\text{in}} &= P_{\text{out}} + P_{\text{mech loss}} + P_{\text{core loss}} + P_{\text{elec loss}} \\ &= 11.19 \text{ kW} + 1.5 \text{ kW} + 1.0 \text{ kW} + 0 \text{ kW} = 13.69 \text{ kW} \end{aligned}$$

Since the motor's power factor is 0.80 leading, the resulting line current flow is

$$\begin{aligned} I_L &= \frac{P_{\text{in}}}{\sqrt{3} V_T \cos \theta} \\ &= \frac{13.69 \text{ kW}}{\sqrt{3}(208 \text{ V})(0.80)} = 47.5 \text{ A} \end{aligned}$$

and the armature current is $I_L/\sqrt{3}$, with 0.8 leading power factor, which gives the result

$$I_A = 27.4 \angle 36.87^\circ \text{ A}$$

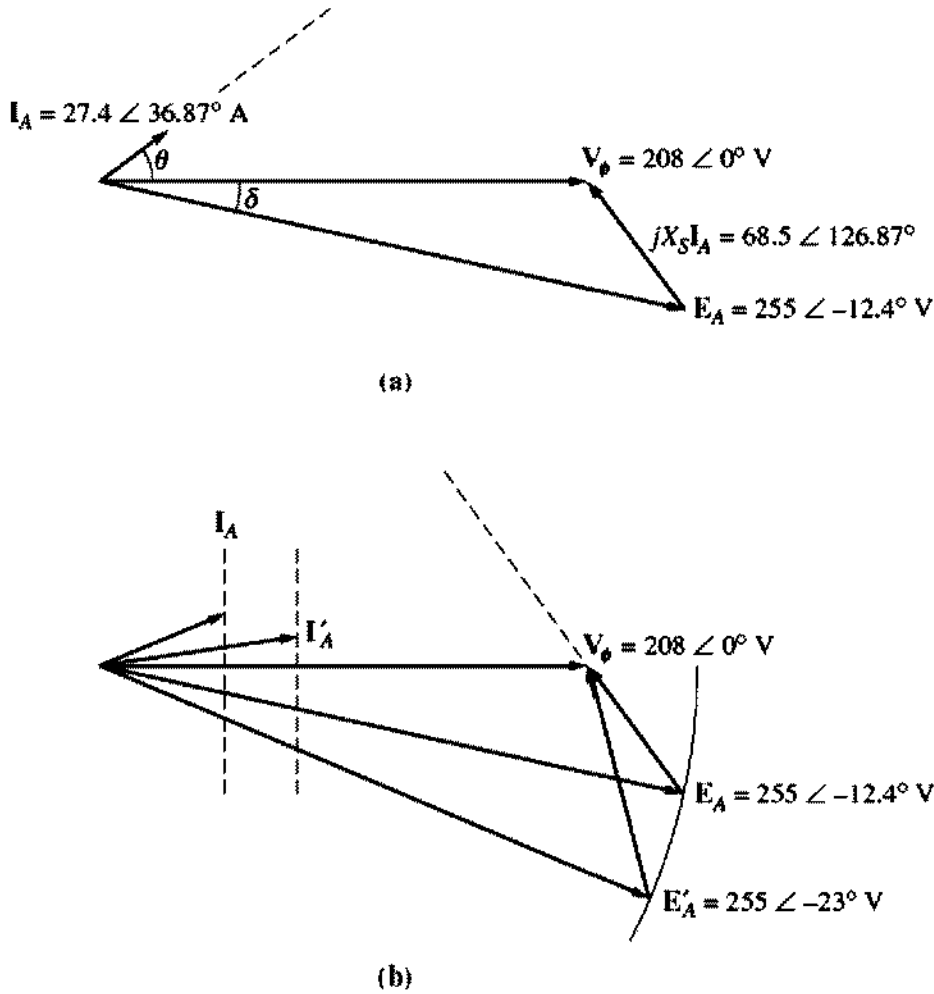
To find E_A , apply Kirchhoff's voltage law [Equation (6-2)]:

$$\begin{aligned} E_A &= V_\phi - jX_s I_A \\ &= 208 \angle 0^\circ \text{ V} - (j2.5 \Omega)(27.4 \angle 36.87^\circ \text{ A}) \\ &= 208 \angle 0^\circ \text{ V} - 68.5 \angle 126.87^\circ \text{ V} \\ &= 249.1 - j54.8 \text{ V} = 255 \angle -12.4^\circ \text{ V} \end{aligned}$$

The resulting phasor diagram is shown in Figure 6-7a.

- As the power on the shaft is increased to 30 hp, the shaft slows momentarily, and the internal generated voltage E_A swings out to a larger angle δ while maintaining a constant magnitude. The resulting phasor diagram is shown in Figure 6-7b.
- After the load changes, the electric input power of the machine becomes

$$\begin{aligned} P_{\text{in}} &= P_{\text{out}} + P_{\text{mech loss}} + P_{\text{core loss}} + P_{\text{elec loss}} \\ &= (30 \text{ hp})(0.746 \text{ kW/hp}) + 1.5 \text{ kW} + 1.0 \text{ kW} + 0 \text{ kW} \\ &= 24.88 \text{ kW} \end{aligned}$$


FIGURE 6-7

(a) The motor phasor diagram for Example 6-1a. (b) The motor phasor diagram for Example 6-1b.

From the equation for power in terms of torque angle [Equation (5-20)], it is possible to find the magnitude of the angle δ (remember that the magnitude of E_A is constant):

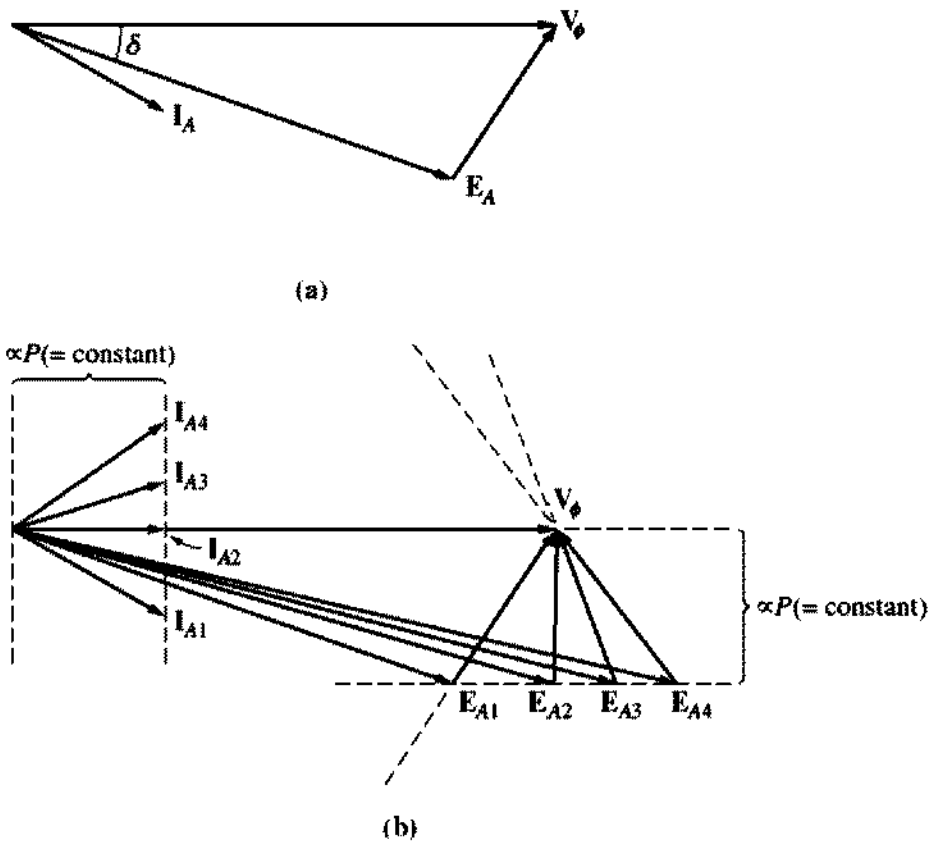
$$P = \frac{3V_\phi E_A \sin \delta}{X_S} \quad (5-20)$$

so

$$\begin{aligned} \delta &= \sin^{-1} \frac{X_S P}{3V_\phi E_A} \\ &= \sin^{-1} \frac{(2.5 \Omega)(24.88 \text{ kW})}{3(208 \text{ V})(255 \text{ V})} \\ &= \sin^{-1} 0.391 = 23^\circ \end{aligned}$$

The internal generated voltage thus becomes $E_A = 355 \angle -23^\circ \text{ V}$. Therefore, I_A will be given by

$$\begin{aligned} I_A &= \frac{V_\phi - E_A}{jX_S} \\ &= \frac{208 \angle 0^\circ \text{ V} - 255 \angle -23^\circ \text{ V}}{j2.5 \Omega} \end{aligned}$$


FIGURE 6-8

(a) A synchronous motor operating at a lagging power factor. (b) The effect of an increase in field current on the operation of this motor.

$$= \frac{103.1 \angle 105^\circ \text{ V}}{j2.5 \Omega} = 41.2 \angle 15^\circ \text{ A}$$

and I_L will become

$$I_L = \sqrt{3}I_A = 71.4 \text{ A}$$

The final power factor will be $\cos(-15^\circ)$ or 0.966 leading.

The Effect of Field Current Changes on a Synchronous Motor

We have seen how a change in shaft load on a synchronous motor affects the motor. There is one other quantity on a synchronous motor that can be readily adjusted—its field current. What effect does a change in field current have on a synchronous motor?

To find out, look at Figure 6-8. Figure 6-8a shows a synchronous motor initially operating at a lagging power factor. Now, increase its field current and see what happens to the motor. Note that *an increase in field current increases the magnitude of E_A but does not affect the real power supplied by the motor*. The power supplied by the motor changes only when the shaft load torque changes. Since a change in I_F does not affect the shaft speed n_m , and since the load attached

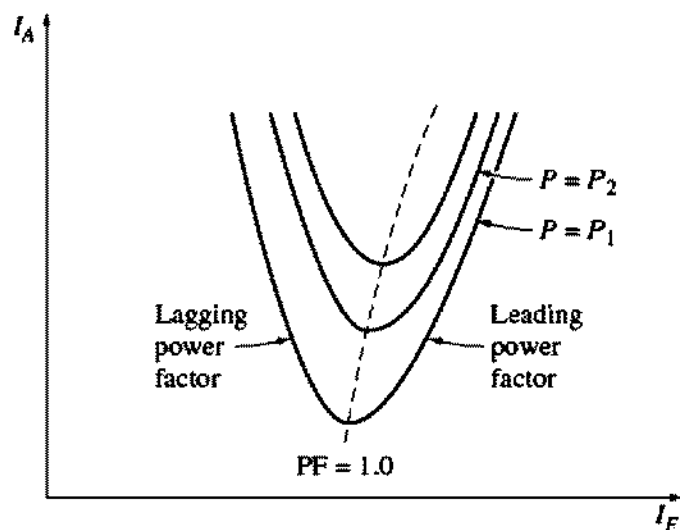


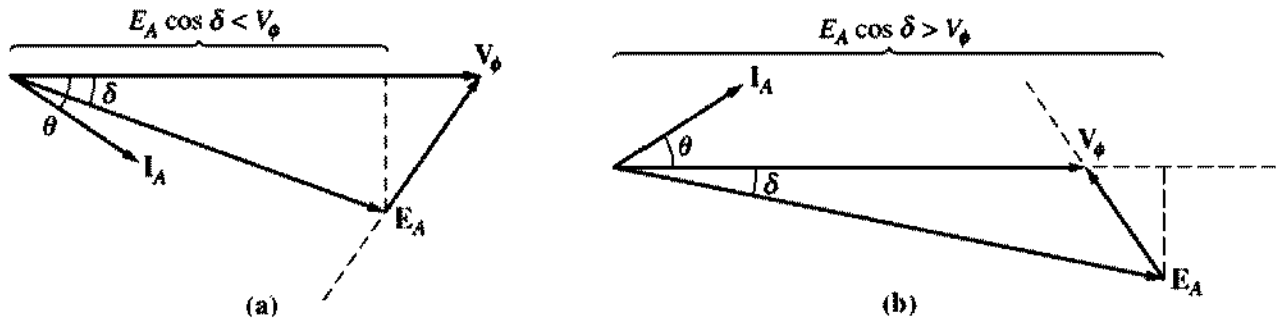
FIGURE 6-9
Synchronous motor V curves.

to the shaft is unchanged, the real power supplied is unchanged. Of course, V_T is also constant, since it is kept constant by the power source supplying the motor. The distances proportional to power on the phasor diagram ($E_A \sin \delta$ and $I_A \cos \theta$) must therefore be constant. When the field current is increased, E_A must increase, but it can only do so by sliding out along the line of constant power. This effect is shown in Figure 6-8b.

Notice that as the value of E_A increases, the magnitude of the armature current I_A first decreases and then increases again. At low E_A , the armature current is lagging, and the motor is an inductive load. It is acting like an inductor-resistor combination, consuming reactive power Q . As the field current is increased, the armature current eventually lines up with V_ϕ , and the motor looks purely resistive. As the field current is increased further, the armature current becomes leading, and the motor becomes a capacitive load. It is now acting like a capacitor-resistor combination, consuming negative reactive power $-Q$ or, alternatively, supplying reactive power Q to the system.

A plot of I_A versus I_F for a synchronous motor is shown in Figure 6-9. Such a plot is called a *synchronous motor V curve*, for the obvious reason that it is shaped like the letter V. There are several V curves drawn, corresponding to different real power levels. For each curve, the minimum armature current occurs at unity power factor, when only real power is being supplied to the motor. At any other point on the curve, some reactive power is being supplied to or by the motor as well. For field currents *less* than the value giving minimum I_A , the armature current is lagging, consuming Q . For field currents *greater* than the value giving the minimum I_A , the armature current is leading, supplying Q to the power system as a capacitor would. Therefore, by controlling the field current of a synchronous motor, the *reactive power* supplied to or consumed by the power system can be controlled.

When the projection of the phasor E_A onto V_ϕ ($E_A \cos \delta$) is *shorter* than V_ϕ itself, a synchronous motor has a lagging current and consumes Q . Since the field current is small in this situation, the motor is said to be *underexcited*. On the other hand, when the projection of E_A onto V_ϕ is *longer* than V_ϕ itself, a synchronous


FIGURE 6-10

(a) The phasor diagram of an *underexcited* synchronous motor. (b) The phasor diagram of an *overexcited* synchronous motor.

motor has a leading current and supplies Q to the power system. Since the field current is large in this situation, the motor is said to be *overexcited*. Phasor diagrams illustrating these concepts are shown in Figure 6-10.

Example 6-2. The 208-V, 45-kVA, 0.8-PF-leading, Δ -connected, 60-Hz synchronous motor of the previous example is supplying a 15-hp load with an initial power factor of 0.85 PF lagging. The field current I_F at these conditions is 4.0 A.

- Sketch the initial phasor diagram of this motor, and find the values I_A and E_A .
- If the motor's flux is increased by 25 percent, sketch the new phasor diagram of the motor. What are E_A , I_A , and the power factor of the motor now?
- Assume that the flux in the motor varies linearly with the field current I_F . Make a plot of I_A versus I_F for the synchronous motor with a 15-hp load.

Solution

- From the previous example, the electric input power with all the losses included is $P_{in} = 13.69$ kW. Since the motor's power factor is 0.85 lagging, the resulting armature current flow is

$$\begin{aligned} I_A &= \frac{P_{in}}{3V_\phi \cos \theta} \\ &= \frac{13.69 \text{ kW}}{3(208 \text{ V})(0.85)} = 25.8 \text{ A} \end{aligned}$$

The angle θ is $\cos^{-1} 0.85 = 31.8^\circ$, so the phasor current I_A is equal to

$$I_A = 25.8 \angle -31.8^\circ \text{ A}$$

To find E_A , apply Kirchhoff's voltage law [Equation (6-2)]:

$$\begin{aligned} E_A &= V_\phi - jX_S I_A \\ &= 208 \angle 0^\circ \text{ V} - (j2.5 \Omega)(25.8 \angle -31.8^\circ \text{ A}) \\ &= 208 \angle 0^\circ \text{ V} - 64.5 \angle 58.2^\circ \text{ V} \\ &= 182 \angle -17.5^\circ \text{ V} \end{aligned}$$

The resulting phasor diagram is shown in Figure 6-11, together with the results for part *b*.

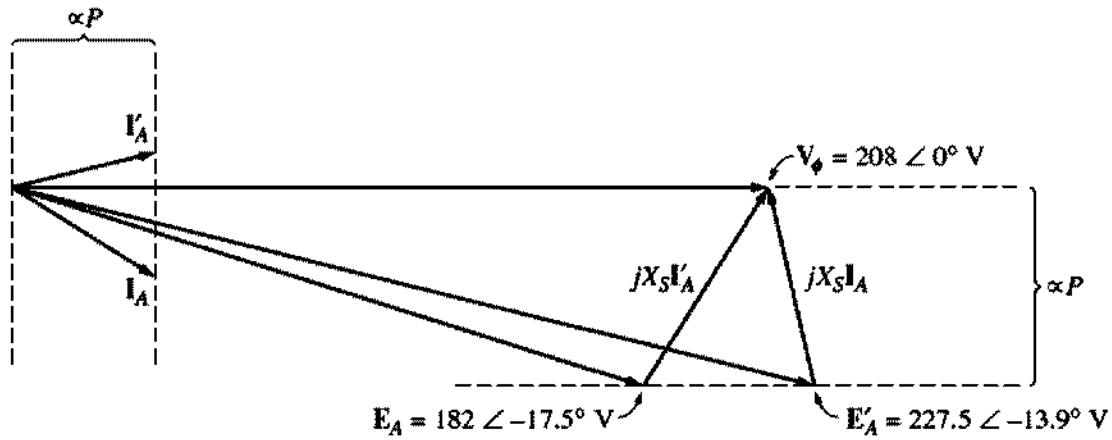


FIGURE 6-11

The phasor diagram of the motor in Example 6-2.

- (b) If the flux ϕ is increased by 25 percent, then $E_A = K\phi\omega$ will increase by 25 percent too:

$$E_{A2} = 1.25 E_{A1} = 1.25(182 \text{ V}) = 227.5 \text{ V}$$

However, the power supplied to the load must remain constant. Since the distance $E_A \sin \delta$ is proportional to the power, that distance on the phasor diagram must be constant from the original flux level to the new flux level. Therefore,

$$E_{A1} \sin \delta_1 = E_{A2} \sin \delta_2$$

$$\begin{aligned} \delta_2 &= \sin^{-1} \left(\frac{E_{A1}}{E_{A2}} \sin \delta_1 \right) \\ &= \sin^{-1} \left[\frac{182 \text{ V}}{227.5 \text{ V}} \sin (-17.5^\circ) \right] = -13.9^\circ \end{aligned}$$

The armature current can now be found from Kirchhoff's voltage law:

$$\begin{aligned} I_{A2} &= \frac{V_\phi - E_{A2}}{jX_S} \\ I_A &= \frac{208 \angle 0^\circ \text{ V} - 227.5 \angle -13.9^\circ \text{ V}}{j2.5 \Omega} \\ &= \frac{56.2 \angle 103.2^\circ \text{ V}}{j2.5 \Omega} = 22.5 \angle 13.2^\circ \text{ A} \end{aligned}$$

Finally, the motor's power factor is now

$$\text{PF} = \cos (13.2^\circ) = 0.974 \quad \text{leading}$$

The resulting phasor diagram is also shown in Figure 6-11.

- (c) Because the flux is assumed to vary linearly with field current, E_A will also vary linearly with field current. We know that E_A is 182 V for a field current of 4.0 A, so E_A for any given field current can be found from the ratio

$$\frac{E_{A2}}{182 \text{ V}} = \frac{I_{F2}}{4.0 \text{ A}}$$

or

$$E_{A2} = 45.5 I_{F2} \quad (6-5)$$

The torque angle δ for any given field current can be found from the fact that the power supplied to the load must remain constant:

$$E_{A1} \sin \delta_1 = E_{A2} \sin \delta_2$$

$$\text{so} \quad \delta_2 = \sin^{-1} \left(\frac{E_{A1}}{E_{A2}} \sin \delta_1 \right) \quad (6-6)$$

These two pieces of information give us the phasor voltage E_A . Once E_A is available, the new armature current can be calculated from Kirchhoff's voltage law:

$$I_{A2} = \frac{V_\phi - E_{A2}}{jX_S} \quad (6-7)$$

A MATLAB M-file to calculate and plot I_A versus I_F using Equations (6-5) through (6-7) is shown below:

```
% M-file: v_curve.m
% M-file create a plot of armature current versus field
% current for the synchronous motor of Example 6-2.

% First, initialize the field current values (21 values
% in the range 3.8-5.8 A)
i_f = (3.8:1:5.8) / 10;

% Now initialize all other values
i_a = zeros(1,21);           % Pre-allocate i_a array
x_s = 2.5;                   % Synchronous reactance
v_phase = 208;               % Phase voltage at 0 degrees
delta1 = -17.5 * pi/180;     % delta 1 in radians
e_a1 = 182 * (cos(delta1) + j * sin(delta1));

% Calculate the armature current for each value
for ii = 1:21
    % Calculate magnitude of e_a2
    e_a2 = 45.5 * i_f(ii);

    % Calculate delta2
    delta2 = asin ( abs(e_a1) / abs(e_a2) * sin(delta1) );

    % Calculate the phasor e_a2
    e_a2 = e_a2 * (cos(delta2) + j * sin(delta2));

    % Calculate i_a
    i_a(ii) = ( v_phase - e_a2 ) / ( j * x_s );
end

% Plot the v-curve
plot(i_f,abs(i_a),'Color','k','Linewidth',2.0);
xlabel('Field Current (A)','Fontweight','Bold');
ylabel('Armature Current (A)','Fontweight','Bold');
title ('Synchronous Motor V-Curve','Fontweight','Bold');
grid on;
```

The plot produced by this M-file is shown in Figure 6-12. Note that for a field current of 4.0 A, the armature current is 25.8 A. This result agrees with part *a* of this example.

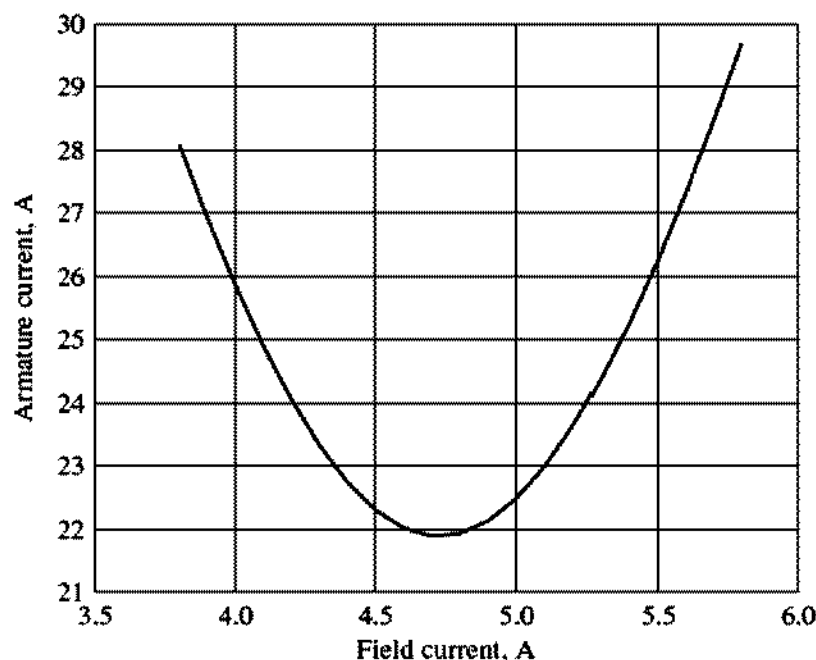


FIGURE 6-12
V curve for the synchronous motor of Example 6-2.

The Synchronous Motor and Power-Factor Correction

Figure 6-13 shows an infinite bus whose output is connected through a transmission line to an industrial plant at a distant point. The industrial plant shown consists of three loads. Two of the loads are induction motors with lagging power factors, and the third load is a synchronous motor with a variable power factor.

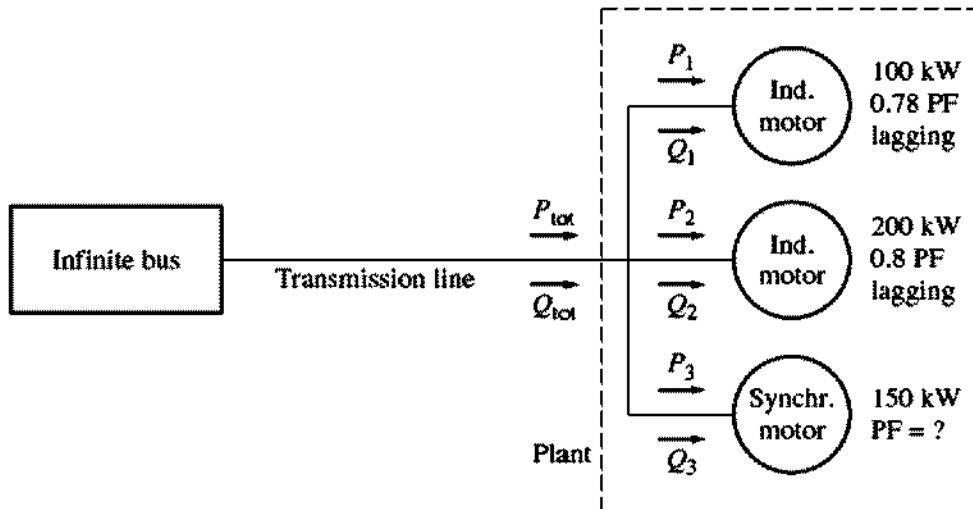
What does the ability to set the power factor of one of the loads do for the power system? To find out, examine the following example problem. (*Note:* A review of the three-phase power equations and their uses is given in Appendix A. Some readers may wish to consult it when studying this problem.)

Example 6-3. The infinite bus in Figure 6-13 operates at 480 V. Load 1 is an induction motor consuming 100 kW at 0.78 PF lagging, and load 2 is an induction motor consuming 200 kW at 0.8 PF lagging. Load 3 is a synchronous motor whose real power consumption is 150 kW.

- If the synchronous motor is adjusted to operate at 0.85 PF lagging, what is the transmission line current in this system?
- If the synchronous motor is adjusted to operate at 0.85 PF leading, what is the transmission line current in this system?
- Assume that the transmission line losses are given by

$$P_{LL} = 3I_L^2 R_L \quad \text{line loss}$$

where LL stands for line losses. How do the transmission losses compare in the two cases?


FIGURE 6-13

A simple power system consisting of an infinite bus supplying an industrial plant through a transmission line.

Solution

(a) In the first case, the real power of load 1 is 100 kW, and the reactive power of load 1 is

$$\begin{aligned} Q_1 &= P_1 \tan \theta \\ &= (100 \text{ kW}) \tan (\cos^{-1} 0.78) = (100 \text{ kW}) \tan 38.7^\circ \\ &= 80.2 \text{ kVAR} \end{aligned}$$

The real power of load 2 is 200 kW, and the reactive power of load 2 is

$$\begin{aligned} Q_2 &= P_2 \tan \theta \\ &= (200 \text{ kW}) \tan (\cos^{-1} 0.80) = (200 \text{ kW}) \tan 36.87^\circ \\ &= 150 \text{ kVAR} \end{aligned}$$

The real power load 3 is 150 kW, and the reactive power of load 3 is

$$\begin{aligned} Q_3 &= P_3 \tan \theta \\ &= (150 \text{ kW}) \tan (\cos^{-1} 0.85) = (150 \text{ kW}) \tan 31.8^\circ \\ &= 93 \text{ kVAR} \end{aligned}$$

Thus, the total real load is

$$\begin{aligned} P_{tot} &= P_1 + P_2 + P_3 \\ &= 100 \text{ kW} + 200 \text{ kW} + 150 \text{ kW} = 450 \text{ kW} \end{aligned}$$

and the total reactive load is

$$\begin{aligned} Q_{tot} &= Q_1 + Q_2 + Q_3 \\ &= 80.2 \text{ kVAR} + 150 \text{ kVAR} + 93 \text{ kVAR} = 323.2 \text{ kVAR} \end{aligned}$$

The equivalent system power factor is thus

$$\begin{aligned} \text{PF} &= \cos \theta = \cos \left(\tan^{-1} \frac{Q}{P} \right) = \cos \left(\tan^{-1} \frac{323.2 \text{ kVAR}}{450 \text{ kW}} \right) \\ &= \cos 35.7^\circ = 0.812 \text{ lagging} \end{aligned}$$

Finally, the line current is given by

$$I_L = \frac{P_{\text{tot}}}{\sqrt{3}V_L \cos \theta} = \frac{450 \text{ kW}}{\sqrt{3}(480 \text{ V})(0.812)} = 667 \text{ A}$$

(b) The real and reactive powers of loads 1 and 2 are unchanged, as is the real power of load 3. The reactive power of load 3 is

$$\begin{aligned} Q_3 &= P_3 \tan \theta \\ &= (150 \text{ kW}) \tan (-\cos^{-1} 0.85) = (150 \text{ kW}) \tan (-31.8^\circ) \\ &= -93 \text{ kVAR} \end{aligned}$$

Thus, the total real load is

$$\begin{aligned} P_{\text{tot}} &= P_1 + P_2 + P_3 \\ &= 100 \text{ kW} + 200 \text{ kW} + 150 \text{ kW} = 450 \text{ kW} \end{aligned}$$

and the total reactive load is

$$\begin{aligned} Q_{\text{tot}} &= Q_1 + Q_2 + Q_3 \\ &= 80.2 \text{ kVAR} + 150 \text{ kVAR} - 93 \text{ kVAR} = 137.2 \text{ kVAR} \end{aligned}$$

The equivalent system power factor is thus

$$\begin{aligned} \text{PF} &= \cos \theta = \cos \left(\tan^{-1} \frac{Q}{P} \right) = \cos \left(\tan^{-1} \frac{137.2 \text{ kVAR}}{450 \text{ kW}} \right) \\ &= \cos 16.96^\circ = 0.957 \text{ lagging} \end{aligned}$$

Finally, the line current is given by

$$I_L = \frac{P_{\text{tot}}}{\sqrt{3}V_L \cos \theta} = \frac{450 \text{ kW}}{\sqrt{3}(480 \text{ V})(0.957)} = 566 \text{ A}$$

(c) The transmission losses in the first case are

$$P_{\text{LL}} = 3I_L^2 R_L = 3(667 \text{ A})^2 R_L = 1,344,700 R_L$$

The transmission losses in the second case are

$$P_{\text{LL}} = 3I_L^2 R_L = 3(566 \text{ A})^2 R_L = 961,070 R_L$$

Notice that in the second case the transmission power losses are 28 percent less than in the first case, while the power supplied to the loads is the same.

As seen in the preceding example, the ability to adjust the power factor of one or more loads in a power system can significantly affect the operating efficiency of the power system. The lower the power factor of a system, the greater the losses in the power lines feeding it. Most loads on a typical power system are induction motors, so power systems are almost invariably lagging in power factor. Having one or more leading loads (overexcited synchronous motors) on the system can be useful for the following reasons:

1. A leading load can supply some reactive power Q for nearby lagging loads, instead of it coming from the generator. Since the reactive power does not have to travel over the long and fairly high-resistance transmission lines, the

transmission line current is reduced and the power system losses are much lower. (This was shown by the previous example.)

2. Since the transmission lines carry less current, they can be smaller for a given rated power flow. A lower equipment current rating reduces the cost of a power system significantly.
3. In addition, requiring a synchronous motor to operate with a leading power factor means that the motor must be run *overexcited*. This mode of operation increases the motor's maximum torque and reduces the chance of accidentally exceeding the pullout torque.

The use of synchronous motors or other equipment to increase the overall power factor of a power system is called *power-factor correction*. Since a synchronous motor can provide power-factor correction and lower power system costs, many loads that can accept a constant-speed motor (even though they do not necessarily *need* one) are driven by synchronous motors. Even though a synchronous motor may cost more than an induction motor on an individual basis, the ability to operate a synchronous motor at leading power factors for power-factor correction saves money for industrial plants. This results in the purchase and use of synchronous motors.

Any synchronous motor that exists in a plant is run overexcited as a matter of course to achieve power-factor correction and to increase its pullout torque. However, running a synchronous motor overexcited requires a high field current and flux, which causes significant rotor heating. An operator must be careful not to overheat the field windings by exceeding the rated field current.

The Synchronous Capacitor or Synchronous Condenser

A synchronous motor purchased to drive a load can be operated overexcited to supply reactive power Q for a power system. In fact, at some times in the past a synchronous motor was purchased and run without a load, *simply for power-factor correction*. The phasor diagram of a synchronous motor operating overexcited at no load is shown in Figure 6-14.

Since there is no power being drawn from the motor, the distances proportional to power ($E_A \sin \delta$ and $I_A \cos \theta$) are zero. Since the Kirchhoff's voltage law equation for a synchronous motor is

$$\mathbf{V}_\phi = \mathbf{E}_A + jX_S \mathbf{I}_A \quad (6-1)$$

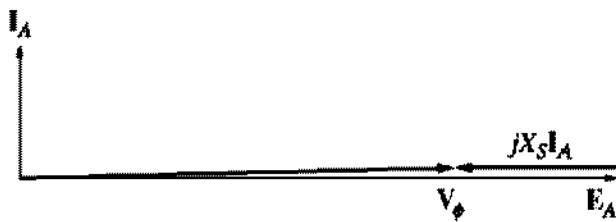


FIGURE 6-14
The phasor diagram of a *synchronous capacitor* or *synchronous condenser*.

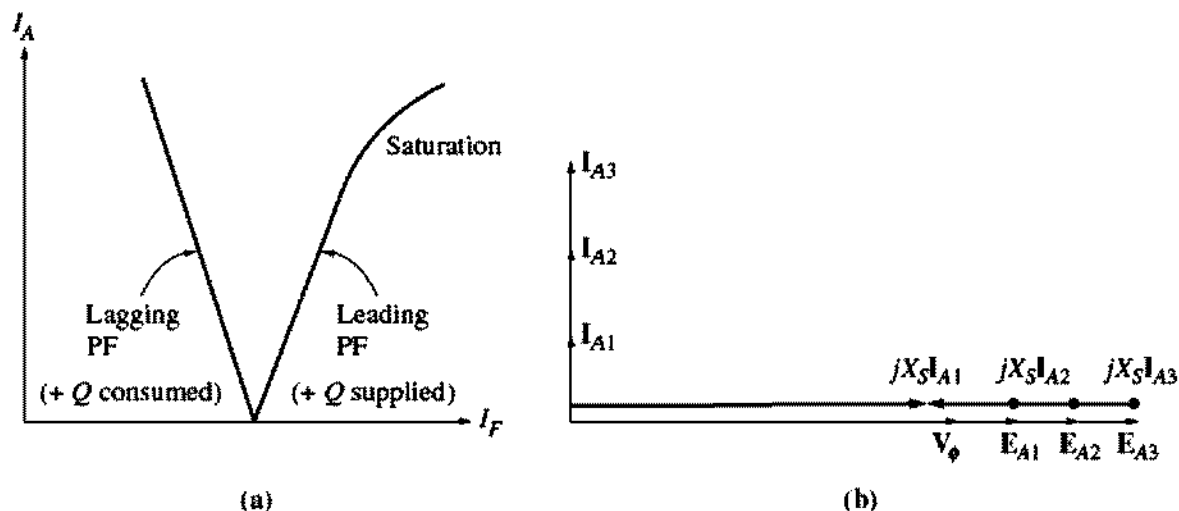


FIGURE 6-15
 (a) The V curve of a synchronous capacitor. (b) The corresponding machine phasor diagram.

the quantity $jX_S I_A$ points to the left, and therefore the armature current I_A points straight up. If V_ϕ and I_A are examined, the voltage-current relationship between them looks like that of a capacitor. An overexcited synchronous motor at no load looks just like a large capacitor to the power system.

Some synchronous motors used to be sold specifically for power-factor correction. These machines had shafts that did not even come through the frame of the motor—no load could be connected to them even if one wanted to do so. Such special-purpose synchronous motors were often called *synchronous condensers* or *synchronous capacitors*. (*Condenser* is an old name for capacitor.)

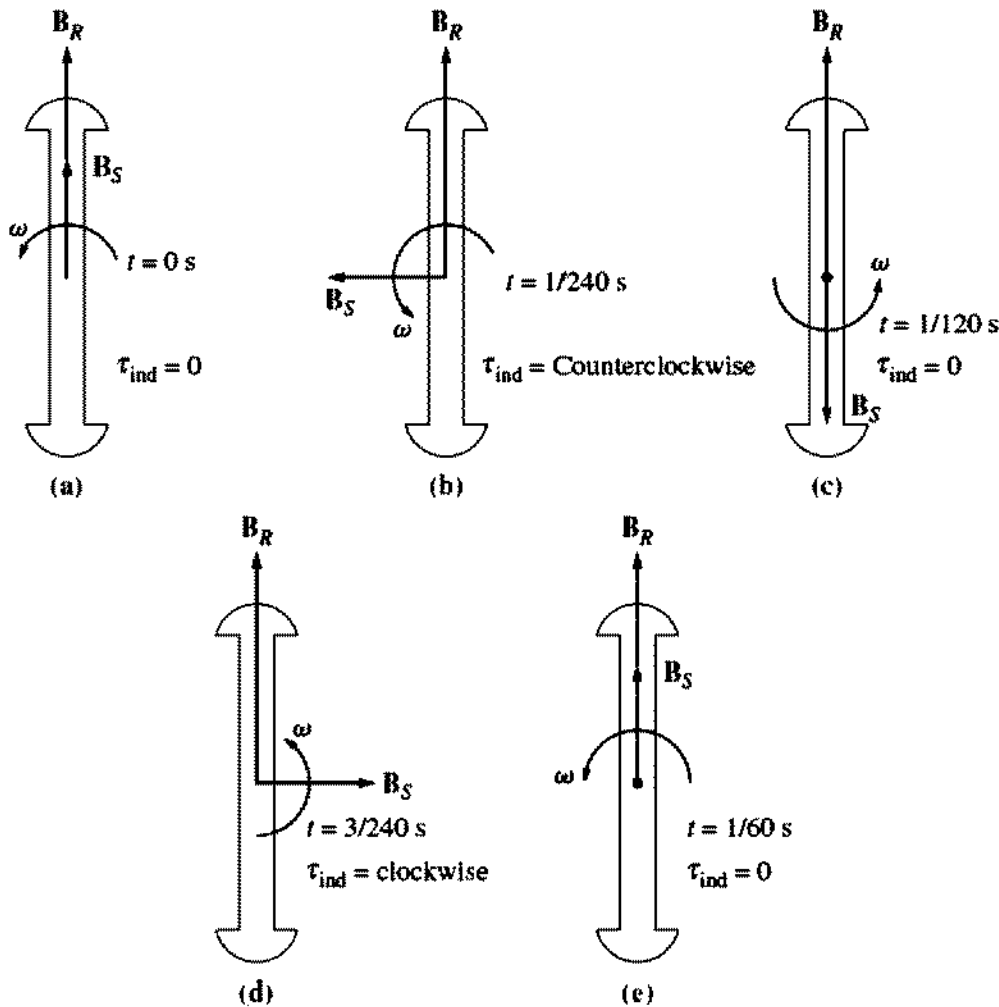
The V curve for a synchronous capacitor is shown in Figure 6-15a. Since the real power supplied to the machine is zero (except for losses), at unity power factor the current $I_A = 0$. As the field current is increased above that point, the line current (and the reactive power supplied by the motor) increases in a nearly linear fashion until saturation is reached. Figure 6-15b shows the effect of increasing the field current on the motor's phasor diagram.

Today, conventional static capacitors are more economical to buy and use than synchronous capacitors. However, some synchronous capacitors may still be in use in older industrial plants.

6.3 STARTING SYNCHRONOUS MOTORS

Section 6.2 explained the behavior of a synchronous motor under steady-state conditions. In that section, the motor was always assumed to be initially turning at *synchronous speed*. What has not yet been considered is the question: How did the motor get to synchronous speed in the first place?

To understand the nature of the starting problem, refer to Figure 6-16. This figure shows a 60-Hz synchronous motor at the moment power is applied to its stator windings. The rotor of the motor is stationary, and therefore the magnetic


FIGURE 6-16

Starting problems in a synchronous motor—the torque alternates rapidly in magnitude and direction, so that the net starting torque is zero.

field \mathbf{B}_R is stationary. The stator magnetic field \mathbf{B}_S is starting to sweep around the motor at synchronous speed.

Figure 6-16a shows the machine at time $t = 0$ s, when \mathbf{B}_R and \mathbf{B}_S are exactly lined up. By the induced-torque equation

$$\tau_{\text{ind}} = k\mathbf{B}_R \times \mathbf{B}_S \quad (4-58)$$

the induced torque on the shaft of the rotor is zero. Figure 6-16b shows the situation at time $t = 1/240$ s. In such a short time, the rotor has barely moved, but the stator magnetic field now points to the left. By the induced-torque equation, the torque on the shaft of the rotor is now *counterclockwise*. Figure 6-16c shows the situation at time $t = 1/120$ s. At that point \mathbf{B}_R and \mathbf{B}_S point in opposite directions, and τ_{ind} again equals zero. At $t = 1/60$ s, the stator magnetic field now points to the right, and the resulting torque is *clockwise*.

Finally, at $t = 1/60$ s, the stator magnetic field is again lined up with the rotor magnetic field, and $\tau_{\text{ind}} = 0$. During one electrical cycle, the torque was first counterclockwise and then clockwise, and the average torque over the complete

cycle was zero. What happens to the motor is that it vibrates heavily with each electrical cycle and finally overheats.

Such an approach to synchronous motor starting is hardly satisfactory—managers tend to frown on employees who burn up their expensive equipment. So just how *can* a synchronous motor be started?

Three basic approaches can be used to safely start a synchronous motor:

1. *Reduce the speed of the stator magnetic field* to a low enough value that the rotor can accelerate and lock in with it during one half-cycle of the magnetic field's rotation. This can be done by reducing the frequency of the applied electric power.
2. *Use an external prime mover* to accelerate the synchronous motor up to synchronous speed, go through the paralleling procedure, and bring the machine on the line as a generator. Then, turning off or disconnecting the prime mover will make the synchronous machine a motor.
3. *Use damper windings or amortisseur windings*. The function of damper windings and their use in motor starting will be explained below.

Each of these approaches to synchronous motor starting will be described in turn.

Motor Starting by Reducing Electrical Frequency

If the stator magnetic fields in a synchronous motor rotate at a low enough speed, there will be no problem for the rotor to accelerate and to lock in with the stator magnetic field. The speed of the stator magnetic fields can then be increased to operating speed by gradually increasing f_e up to its normal 50- or 60-Hz value.

This approach to starting synchronous motors makes a lot of sense, but it does have one big problem: Where does the variable electrical frequency come from? Regular power systems are very carefully regulated at 50 or 60 Hz, so until recently any variable-frequency voltage source had to come from a dedicated generator. Such a situation was obviously impractical except for very unusual circumstances.

Today, things are different. Chapter 3 described the rectifier-inverter and the cycloconverter, which can be used to convert a constant input frequency to any desired output frequency. With the development of such modern solid-state variable-frequency drive packages, it is perfectly possible to continuously control the electrical frequency applied to the motor all the way from a fraction of a hertz up to and above full rated frequency. If such a variable-frequency drive unit is included in a motor-control circuit to achieve speed control, then starting the synchronous motor is very easy—simply adjust the frequency to a very low value for starting, and then raise it up to the desired operating frequency for normal running.

When a synchronous motor is operated at a speed lower than the rated speed, its internal generated voltage $E_A = K\phi\omega$ will be smaller than normal. If E_A is reduced in magnitude, then the terminal voltage applied to the motor must be

reduced as well in order to keep the stator current at safe levels. The voltage in any variable-frequency drive or variable-frequency starter circuit must vary roughly linearly with the applied frequency.

To learn more about such solid-state motor-drive units, refer to Chapter 3 and Reference 9.

Motor Starting with an External Prime Mover

The second approach to starting a synchronous motor is to attach an external starting motor to it and bring the synchronous machine up to full speed with the external motor. Then the synchronous machine can be paralleled with its power system as a generator, and the starting motor can be detached from the shaft of the machine. Once the starting motor is turned off, the shaft of the machine slows down, the rotor magnetic field \mathbf{B}_R falls behind \mathbf{B}_{net} , and the synchronous machine starts to act as a motor. Once paralleling is completed, the synchronous motor can be loaded down in an ordinary fashion.

This whole procedure is not as preposterous as it sounds, since many synchronous motors are parts of motor-generator sets, and the synchronous machine in the motor-generator set may be started with the other machine serving as the starting motor. Also, the starting motor only needs to overcome the inertia of the synchronous machine without a load—no load is attached until the motor is paralleled to the power system. Since only the motor's inertia must be overcome, the starting motor can have a *much* smaller rating than the synchronous motor it starts.

Since most large synchronous motors have brushless excitation systems mounted on their shafts, it is often possible to use these exciters as starting motors.

For many medium-size to large synchronous motors, an external starting motor or starting by using the exciter may be the only possible solution, because the power systems they are tied to may not be able to handle the starting currents needed to use the amortisseur winding approach described next.

Motor Starting by Using Amortisseur Windings

By far the most popular way to start a synchronous motor is to employ *amortisseur* or *damper* windings. Amortisseur windings are special bars laid into notches carved in the face of a synchronous motor's rotor and then shorted out on each end by a large *shorting ring*. A pole face with a set of amortisseur windings is shown in Figure 6-17, and amortisseur windings are visible in Figures 5-2 and 5-4.

To understand what a set of amortisseur windings does in a synchronous motor, examine the stylized salient two-pole rotor shown in Figure 6-18. This rotor shows an amortisseur winding with the shorting bars on the ends of the two rotor pole faces connected by wires. (This is not quite the way normal machines are constructed, but it will serve beautifully to illustrate the point of the windings.)

Assume initially that *the main rotor field winding is disconnected* and that a three-phase set of voltages is applied to the stator of this machine. When the

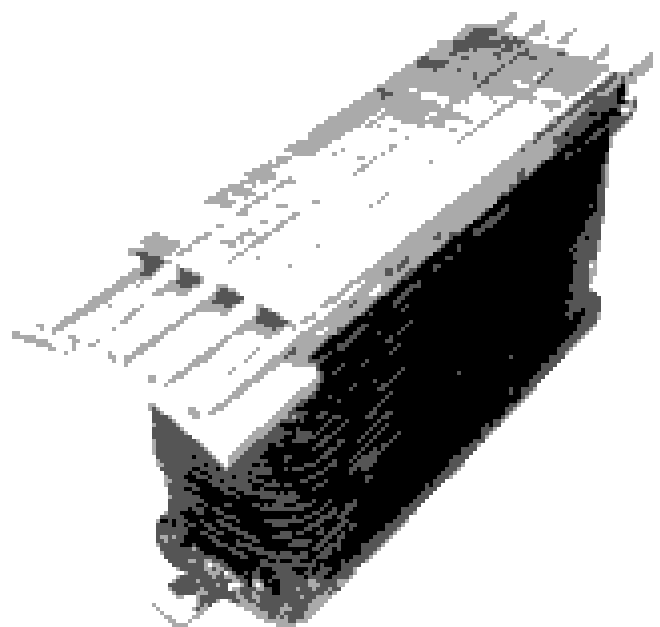


FIGURE 6-17
A rotor field pole for a synchronous machine showing amortisseur windings in the pole face. (Courtesy of General Electric Company.)

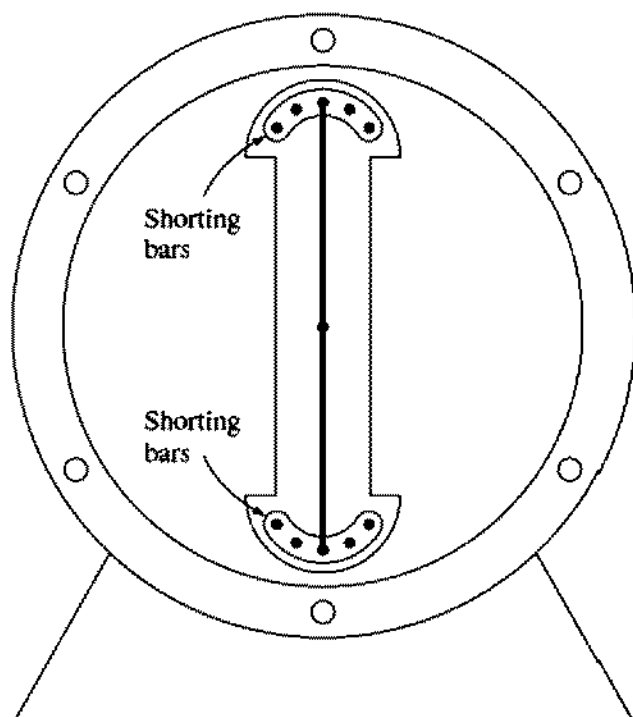


FIGURE 6-18
A simplified diagram of a salient two-pole machine showing amortisseur windings.

power is first applied at time $t = 0$ s, assume that the magnetic field \mathbf{B}_S is vertical, as shown in Figure 6-19a. As the magnetic field \mathbf{B}_S sweeps along in a counter-clockwise direction, it induces a voltage in the bars of the amortisseur winding given by Equation (1-45):

$$e_{\text{ind}} = (\mathbf{v} \times \mathbf{B}) \cdot \mathbf{l} \tag{1-45}$$

- where
- \mathbf{v} = velocity of the bar *relative to the magnetic field*
 - \mathbf{B} = magnetic flux density vector
 - \mathbf{l} = length of conductor in the magnetic field

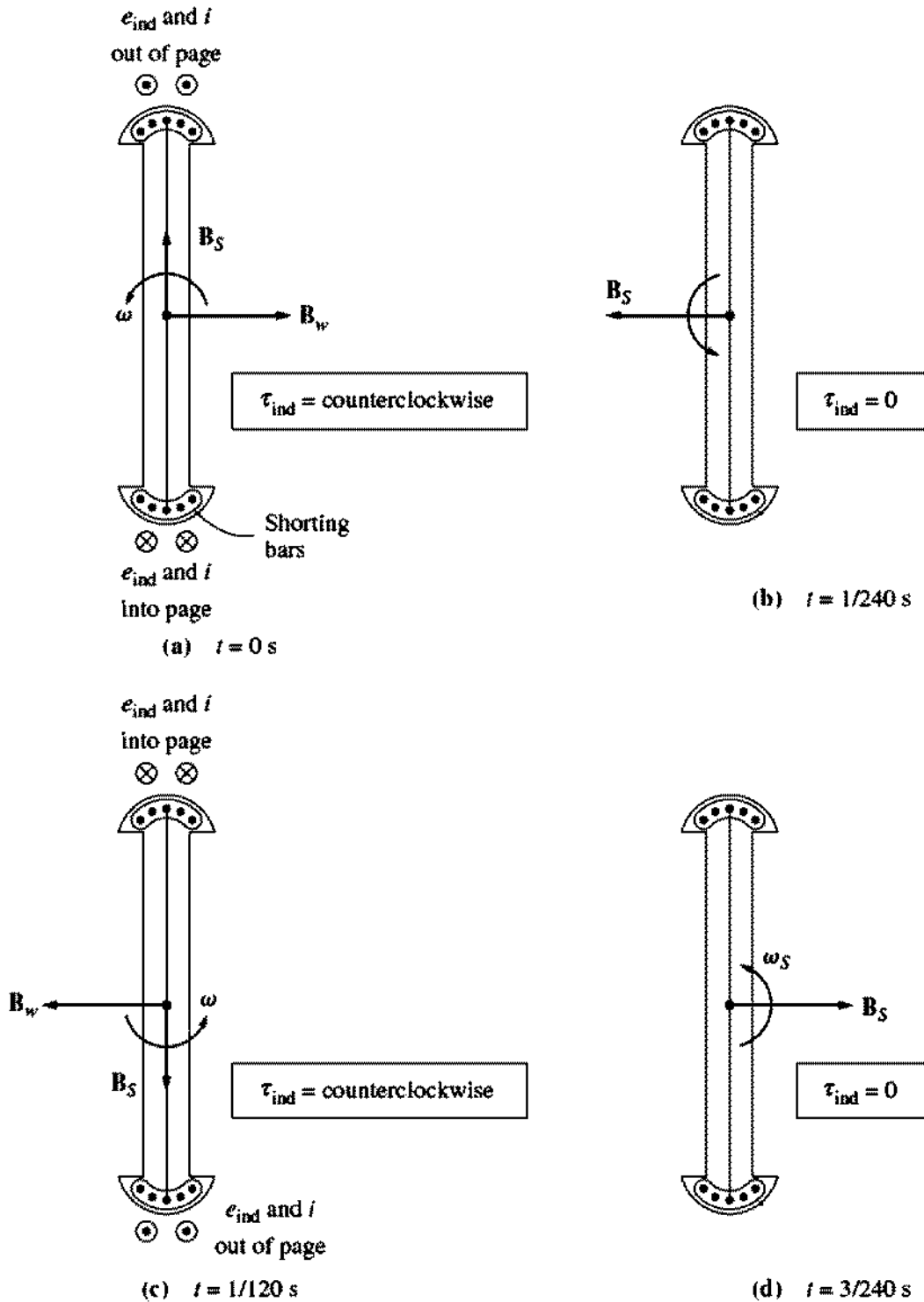


FIGURE 6-19
 The development of a unidirectional torque with synchronous motor amortisseur windings.

The bars at the top of the rotor are moving to the right *relative to the magnetic field*, so the resulting direction of the induced voltage is out of the page. Similarly, the induced voltage is into the page in the bottom bars. These voltages produce a current flow out of the top bars and into the bottom bars, resulting in a winding magnetic field B_w pointing to the right. By the induced-torque equation

$$\tau_{\text{ind}} = k\mathbf{B}_w \times \mathbf{B}_s$$

the resulting torque on the bars (and the rotor) is *counterclockwise*.

Figure 6–19b shows the situation at $t = 1/240$ s. Here, the stator magnetic field has rotated 90° while the rotor has barely moved (it simply cannot speed up in so short a time). At this point, the voltage induced in the amortisseur windings is zero, because \mathbf{v} is parallel to \mathbf{B} . With no induced voltage, there is no current in the windings, and the induced torque is zero.

Figure 6–19c shows the situation at $t = 1/120$ s. Now the stator magnetic field has rotated 90° , and the rotor still has not moved yet. The induced voltage [given by Equation (1–45)] in the amortisseur windings is out of the page in the bottom bars and into the page in the top bars. The resulting current flow is out of the page in the bottom bars and into the page in the top bars, causing a magnetic field \mathbf{B}_w to point to the left. The resulting induced torque, given by

$$\tau_{\text{ind}} = k\mathbf{B}_w \times \mathbf{B}_s$$

is counterclockwise.

Finally, Figure 6–19d shows the situation at time $t = 3/240$ s. Here, as at $t = 1/240$ s, the induced torque is zero.

Notice that sometimes the torque is counterclockwise and sometimes it is essentially zero, but it is *always unidirectional*. Since there is a net torque in a single direction, the motor's rotor speeds up. (This is entirely different from starting a synchronous motor with its normal field current, since in that case torque is first clockwise and then counterclockwise, averaging out to zero. In this case, torque is *always* in the same direction, so there is a nonzero average torque.)

Although the motor's rotor will speed up, it can never quite reach synchronous speed. This is easy to understand. Suppose that a rotor is turning at synchronous speed. Then the speed of the stator magnetic field \mathbf{B}_s is the same as the rotor's speed, and there is *no relative motion* between \mathbf{B}_s and the rotor. If there is no relative motion, the induced voltage in the windings will be zero, the resulting current flow will be zero, and the winding magnetic field will be zero. Therefore, there will be no torque on the rotor to keep it turning. Even though a rotor cannot speed up all the way to synchronous speed, it can get close. It gets close enough to n_{sync} that the regular field current can be turned on, and the rotor will pull into step with the stator magnetic fields.

In a real machine, the field windings are not open-circuited during the starting procedure. If the field windings were open-circuited, then very high voltages would be produced in them during starting. If the field winding is short-circuited during starting, no dangerous voltages are produced, and the induced field current actually contributes extra starting torque to the motor.

To summarize, if a machine has amortisseur windings, it can be started by the following procedure:

1. Disconnect the field windings from their dc power source and short them out.

2. Apply a three-phase voltage to the stator of the motor, and let the rotor accelerate up to near-synchronous speed. The motor should have no load on its shaft, so that its speed can approach n_{sync} as closely as possible.
3. Connect the dc field circuit to its power source. After this is done, the motor will lock into step at synchronous speed, and loads may then be added to its shaft.

The Effect of Amortisseur Windings on Motor Stability

If amortisseur windings are added to a synchronous machine for starting, we get a free bonus—an increase in machine stability. The stator magnetic field rotates at a constant speed n_{sync} , which varies only when the system frequency varies. If the rotor turns at n_{sync} , then the amortisseur windings have no induced voltage at all. If the rotor turns *slower* than n_{sync} , then there will be relative motion between the rotor and the stator magnetic field and a voltage will be induced in the windings. This voltage produces a current flow, and the current flow produces a magnetic field. The interaction of the two magnetic fields produces a torque that tends to speed the machine up again. On the other hand, if the rotor turns *faster* than the stator magnetic field, a torque will be produced that tries to slow the rotor down. Thus, *the torque produced by the amortisseur windings speeds up slow machines and slows down fast machines.*

These windings therefore tend to dampen out the load or other transients on the machine. It is for this reason that amortisseur windings are also called *dampers windings*. Amortisseur windings are also used on synchronous generators, where they serve a similar stabilizing function when a generator is operating in parallel with other generators on an infinite bus. If a variation in shaft torque occurs on the generator, its rotor will momentarily speed up or slow down, and these changes will be opposed by the amortisseur windings. Amortisseur windings improve the overall stability of power systems by reducing the magnitude of power and torque transients.

Amortisseur windings are responsible for most of the subtransient current in a faulted synchronous machine. A short circuit at the terminals of a generator is just another form of transient, and the amortisseur windings respond very quickly to it.

6.4 SYNCHRONOUS GENERATORS AND SYNCHRONOUS MOTORS

A synchronous generator is a synchronous machine that converts mechanical power to electric power, while a synchronous motor is a synchronous machine that converts electric power to mechanical power. In fact, they are both the same physical machine.

	Supply reactive power Q	Consume reactive power Q
	$E_A \cos \delta > V_\phi$	$E_A \cos \delta < V_\phi$
Supply power P Generator E_A leads V_ϕ		
Consume power P Motor E_A lags V_ϕ		

FIGURE 6–20 Phasor diagrams showing the generation and consumption of real power P and reactive power Q by synchronous generators and motors.

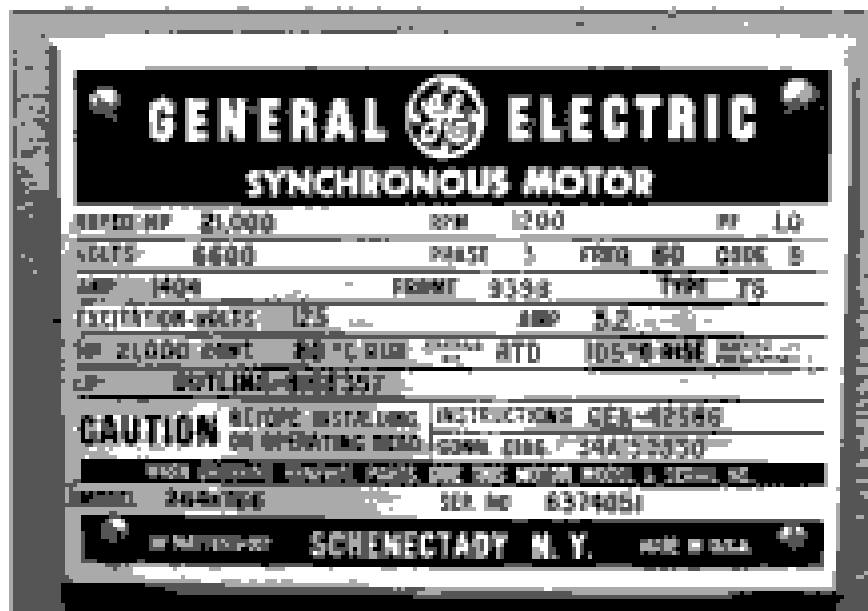
A synchronous machine can supply real power to or consume real power from a power system and can supply reactive power to or consume reactive power from a power system. All four combinations of real and reactive power flows are possible, and Figure 6–20 shows the phasor diagrams for these conditions.

Notice from the figure that

1. The distinguishing characteristic of a synchronous generator (supplying P) is that E_A lies ahead of V_ϕ while for a motor E_A lies behind V_ϕ .
2. The distinguishing characteristic of a machine supplying reactive power Q is that $E_A \cos \delta > V_\phi$ regardless of whether the machine is acting as a generator or as a motor. A machine that is consuming reactive power Q has $E_A \cos \delta < V_\phi$.

6.5 SYNCHRONOUS MOTOR RATINGS

Since synchronous motors are the same physical machines as synchronous generators, the basic machine ratings are the same. The one major difference is that a large


FIGURE 6-21

A typical nameplate for a large synchronous motor. (Courtesy of General Electric Company.)

E_A gives a *leading* power factor instead of a lagging one, and therefore the effect of the maximum field current limit is expressed as a rating at a *leading* power factor. Also, since the output of a synchronous motor is mechanical power, a synchronous motor's power rating is usually given in horsepower rather than kilowatts.

The nameplate of a large synchronous motor is shown in Figure 6-21. In addition to the information shown in the figure, a smaller synchronous motor would have a service factor on its nameplate.

In general, synchronous motors are more adaptable to low-speed, high-power applications than induction motors (see Chapter 7). They are therefore commonly used for low-speed, high-power loads.

6.6 SUMMARY

A synchronous motor is the same physical machine as a synchronous generator, except that the direction of real power flow is reversed. Since synchronous motors are usually connected to power systems containing generators much larger than the motors, the frequency and terminal voltage of a synchronous motor are fixed (i.e., the power system looks like an infinite bus to the motor).

The speed of a synchronous motor is constant from no load to the maximum possible load on the motor. The speed of rotation is

$$n_m = n_{\text{sync}} = \frac{120 f_e}{P}$$

The maximum possible power a machine can produce is

$$P_{\text{max}} = \frac{3V_\phi E_A}{X_S} \quad (5-21)$$

If this value is exceeded, the rotor will not be able to stay locked in with the stator magnetic fields, and the motor will *slip poles*.

If the field current of a synchronous motor is varied while its shaft load remains constant, then the reactive power supplied or consumed by the motor will vary. If $E_A \cos \delta > V_\phi$, the motor will supply reactive power, while if $E_A \cos \delta < V_\phi$, the motor will consume reactive power.

A synchronous motor has no net starting torque and so cannot start by itself. There are three main ways to start a synchronous motor:

1. Reduce the stator frequency to a safe starting level.
2. Use an external prime mover.
3. Put amortisseur or damper windings on the motor to accelerate it to near-synchronous speed before a direct current is applied to the field windings.

If damper windings are present on a motor, they will also increase the stability of the motor during load transients.

QUESTIONS

- 6-1. What is the difference between a synchronous motor and a synchronous generator?
- 6-2. What is the speed regulation of a synchronous motor?
- 6-3. When would a synchronous motor be used even though its constant-speed characteristic was not needed?
- 6-4. Why can't a synchronous motor start by itself?
- 6-5. What techniques are available to start a synchronous motor?
- 6-6. What are amortisseur windings? Why is the torque produced by them unidirectional at starting, while the torque produced by the main field winding alternates direction?
- 6-7. What is a synchronous capacitor? Why would one be used?
- 6-8. Explain, using phasor diagrams, what happens to a synchronous motor as its field current is varied. Derive a synchronous motor V curve from the phasor diagram.
- 6-9. Is a synchronous motor's field circuit in more danger of overheating when it is operating at a leading or at a lagging power factor? Explain, using phasor diagrams.
- 6-10. A synchronous motor is operating at a fixed real load, and its field current is increased. If the armature current falls, was the motor initially operating at a lagging or a leading power factor?
- 6-11. Why must the voltage applied to a synchronous motor be derated for operation at frequencies lower than the rated value?

PROBLEMS

- 6-1. A 480-V, 60 Hz four-pole synchronous motor draws 50 A from the line at unity power factor and full load. Assuming that the motor is lossless, answer the following questions:
 - (a) What is the output torque of this motor? Express the answer both in newton-meters and in pound-feet.

- (b) What must be done to change the power factor to 0.8 leading? Explain your answer, using phasor diagrams.
- (c) What will the magnitude of the line current be if the power factor is adjusted to 0.8 leading?
- 6-2. A 480-V, 60 Hz 400-hp, 0.8-PF-leading, six-pole, Δ -connected synchronous motor has a synchronous reactance of 1.1Ω and negligible armature resistance. Ignore its friction, windage, and core losses for the purposes of this problem.
- (a) If this motor is initially supplying 400 hp at 0.8 PF lagging, what are the magnitudes and angles of E_A and I_A ?
- (b) How much torque is this motor producing? What is the torque angle δ ? How near is this value to the maximum possible induced torque of the motor for this field current setting?
- (c) If $|E_A|$ is increased by 15 percent, what is the new magnitude of the armature current? What is the motor's new power factor?
- (d) Calculate and plot the motor's V curve for this load condition.
- 6-3. A 2300-V, 1000-hp, 0.8-PF leading, 60-Hz, two-pole, Y-connected synchronous motor has a synchronous reactance of 2.8Ω and an armature resistance of 0.4Ω . At 60 Hz, its friction and windage losses are 24 kW, and its core losses are 18 kW. The field circuit has a dc voltage of 200 V, and the maximum I_F is 10 A. The open-circuit characteristic of this motor is shown in Figure P6-1. Answer the following questions about the motor, assuming that it is being supplied by an infinite bus.
- (a) How much field current would be required to make this machine operate at unity power factor when supplying full load?
- (b) What is the motor's efficiency at full load and unity power factor?
- (c) If the field current were increased by 5 percent, what would the new value of the armature current be? What would the new power factor be? How much reactive power is being consumed or supplied by the motor?
- (d) What is the maximum torque this machine is theoretically capable of supplying at unity power factor? At 0.8 PF leading?
- 6-4. Plot the V curves (I_A versus I_F) for the synchronous motor of Problem 6-3 at no-load, half-load, and full-load conditions. (Note that an electronic version of the open-circuit characteristics in Figure P6-1 is available at the book's website. It may simplify the calculations required by this problem. Also, you may assume that R_A is negligible for this calculation.)
- 6-5. If a 60-Hz synchronous motor is to be operated at 50 Hz, will its synchronous reactance be the same as at 60 Hz, or will it change? (*Hint*: Think about the derivation of X_S .)
- 6-6. A 480-V, 100-kW, 0.85-PF-leading, 50-Hz, six-pole, Y-connected synchronous motor has a synchronous reactance of 1.5Ω and a negligible armature resistance. The rotational losses are also to be ignored. This motor is to be operated over a continuous range of speeds from 300 to 1000 r/min, where the speed changes are to be accomplished by controlling the system frequency with a solid-state drive.
- (a) Over what range must the input frequency be varied to provide this speed control range?
- (b) How large is E_A at the motor's rated conditions?
- (c) What is the maximum power that the motor can produce at rated speed with the E_A calculated in part (b)?
- (d) What is the largest E_A could be at 300 r/min?

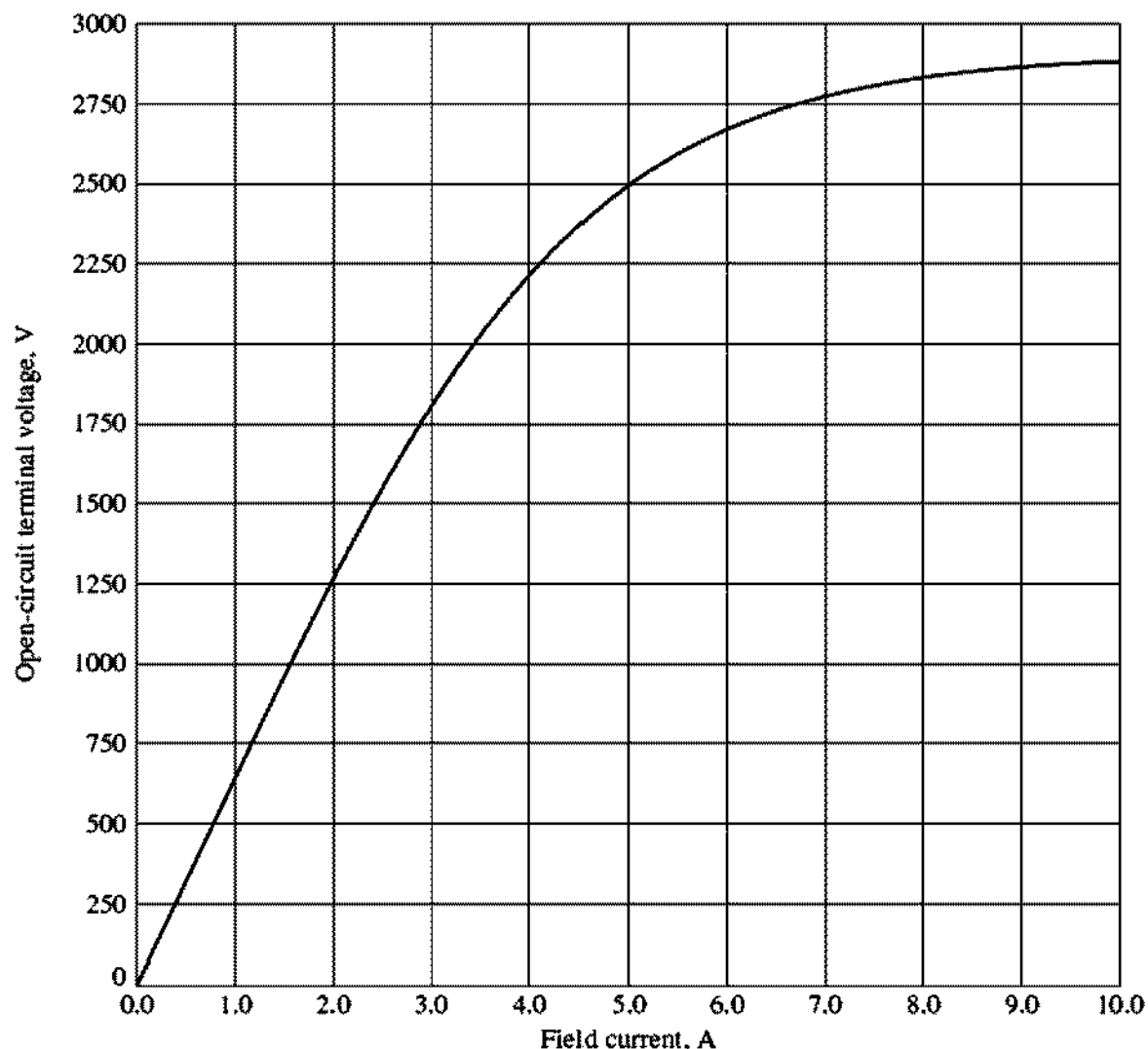


FIGURE P6-1

The open-circuit characteristic for the motor in Problems 6-3 and 6-4.

- (e) Assuming that the applied voltage V_ϕ is derated by the same amount as E_A , what is the maximum power the motor could supply at 300 r/min?
- (f) How does the power capability of a synchronous motor relate to its speed?
- 6-7. A 208-V, Y-connected synchronous motor is drawing 40 A at unity power factor from a 208-V power system. The field current flowing under these conditions is 2.7 A. Its synchronous reactance is 0.8Ω . Assume a linear open-circuit characteristic.
- (a) Find the torque angle δ .
- (b) How much field current would be required to make the motor operate at 0.8 PF leading?
- (c) What is the new torque angle in part *b*?
- 6-8. A synchronous machine has a synchronous reactance of 2.0Ω per phase and an armature resistance of 0.4Ω per phase. If $E_A = 460 \angle -8^\circ$ V and $V_\phi = 480 \angle 0^\circ$ V, is this machine a motor or a generator? How much power P is this machine consuming from or supplying to the electrical system? How much reactive power Q is this machine consuming from or supplying to the electrical system?

- 6-9. Figure P6-2 shows a synchronous motor phasor diagram for a motor operating at a leading power factor with no R_A . For this motor, the torque angle is given by

$$\tan \delta = \frac{X_S I_A \cos \theta}{V_\phi + X_S I_A \sin \theta}$$

$$\delta = \tan^{-1} \left(\frac{X_S I_A \cos \theta}{V_\phi + X_S I_A \sin \theta} \right)$$

Derive an equation for the torque angle of the synchronous motor if the armature resistance is included.

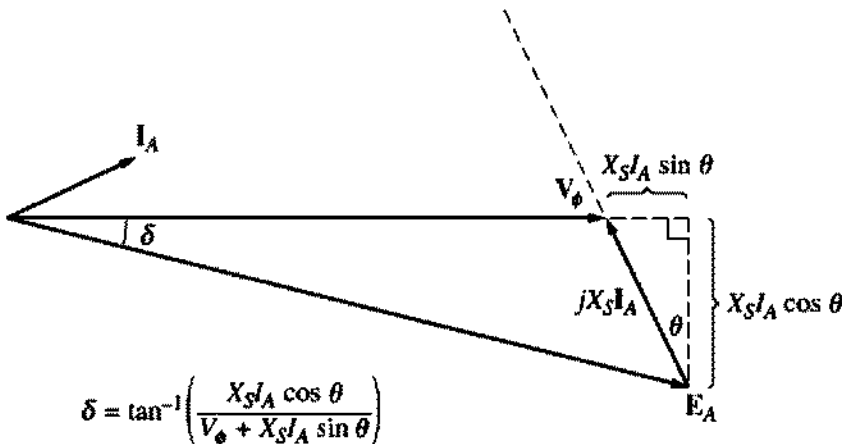


FIGURE P6-2

Phasor diagram of a motor at a leading power factor.

- 6-10. A 480-V, 375-kVA, 0.8-PF-lagging, Y-connected synchronous generator has a synchronous reactance of 0.4Ω and a negligible armature resistance. This generator is supplying power to a 480-V, 80-kW, 0.8-PF-leading, Y-connected synchronous motor with a synchronous reactance of 1.1Ω and a negligible armature resistance. The synchronous generator is adjusted to have a terminal voltage of 480 V when the motor is drawing the rated power at unity power factor.
- Calculate the magnitudes and angles of E_A for both machines.
 - If the flux of the motor is increased by 10 percent, what happens to the terminal voltage of the power system? What is its new value?
 - What is the power factor of the motor after the increase in motor flux?
- 6-11. A 480-V, 100-kW, 50-Hz, four-pole, Y-connected synchronous motor has a rated power factor of 0.85 leading. At full load, the efficiency is 91 percent. The armature resistance is 0.08Ω , and the synchronous reactance is 1.0Ω . Find the following quantities for this machine when it is operating at full load:
- Output torque
 - Input power
 - n_m
 - E_A
 - I_A
 - P_{conv}
 - $P_{\text{mech}} + P_{\text{core}} + P_{\text{stray}}$

- 6-12. The Y-connected synchronous motor whose nameplate is shown in Figure 6-21 has a per-unit synchronous reactance of 0.9 and a per-unit resistance of 0.02.
- What is the rated input power of this motor?
 - What is the magnitude of E_A at rated conditions?
 - If the input power of this motor is 10 MW, what is the maximum reactive power the motor can simultaneously supply? Is it the armature current or the field current that limits the reactive power output?
 - How much power does the field circuit consume at the rated conditions?
 - What is the efficiency of this motor at full load?
 - What is the output torque of the motor at the rated conditions? Express the answer both in newton-meters and in pound-feet.
- 6-13. A 440-V, three-phase, Y-connected synchronous motor has a synchronous reactance of 1.5Ω per phase. The field current has been adjusted so that the torque angle δ is 28° when the power supplied by the generator is 90 kW.
- What is the magnitude of the internal generated voltage E_A in this machine?
 - What are the magnitude and angle of the armature current in the machine? What is the motor's power factor?
 - If the field current remains constant, what is the absolute maximum power this motor could supply?
- 6-14. A 460-V, 200-kVA, 0.80-PF-leading, 400-Hz, six-pole, Y-connected synchronous motor has negligible armature resistance and a synchronous reactance of 0.50 per unit. Ignore all losses.
- What is the speed of rotation of this motor?
 - What is the output torque of this motor at the rated conditions?
 - What is the internal generated voltage of this motor at the rated conditions?
 - With the field current remaining at the value present in the motor in part *c*, what is the maximum possible output power from the machine?
- 6-15. A 100-hp, 440-V, 0.8-PF-leading, Δ -connected synchronous motor has an armature resistance of 0.22Ω and a synchronous reactance of 3.0Ω . Its efficiency at full load is 89 percent.
- What is the input power to the motor at rated conditions?
 - What is the line current of the motor at rated conditions? What is the phase current of the motor at rated conditions?
 - What is the reactive power consumed by or supplied by the motor at rated conditions?
 - What is the internal generated voltage E_A of this motor at rated conditions?
 - What are the stator copper losses in the motor at rated conditions?
 - What is P_{conv} at rated conditions?
 - If E_A is decreased by 10 percent, how much reactive power will be consumed by or supplied by the motor?
- 6-16. Answer the following questions about the machine of Problem 6-15.
- If $E_A = 430 \angle 13.5^\circ \text{ V}$ and $V_\phi = 440 \angle 0^\circ \text{ V}$, is this machine consuming real power from or supplying real power to the power system? Is it consuming reactive power from or supplying reactive power to the power system?
 - Calculate the real power P and reactive power Q supplied or consumed by the machine under the conditions in part *a*. Is the machine operating within its ratings under these circumstances?

- (c) If $E_A = 470 \angle -12^\circ \text{ V}$ and $V_\phi = 440 \angle 0^\circ \text{ V}$, is this machine consuming real power from or supplying real power to the power system? Is it consuming reactive power from or supplying reactive power to the power system?
- (d) Calculate the real power P and reactive power Q supplied or consumed by the machine under the conditions in part c. Is the machine operating within its ratings under these circumstances?

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