CHAPTER 7

INDUCTION MOTORS

In the last chapter, we saw how amortisseur windings on a synchronous motor could develop a starting torque without the necessity of supplying an external field current to them. In fact, amortisseur windings work so well that a motor could be built without the synchronous motor's main dc field circuit at all. A machine with only amortisseur windings is called an *induction machine*. Such machines are called induction machines because the rotor voltage (which produces the rotor current and the rotor magnetic field) is *induced* in the rotor windings rather than being physically connected by wires. The distinguishing feature of an induction motor is that *no dc field current is required* to run the machine.

Although it is possible to use an induction machine as either a motor or a generator, it has many disadvantages as a generator and so is rarely used in that manner. For this reason, induction machines are usually referred to as induction motors.

7.1 INDUCTION MOTOR CONSTRUCTION

An induction motor has the same physical stator as a synchronous machine, with a different rotor construction. A typical two-pole stator is shown in Figure 7–1. It looks (and is) the same as a synchronous machine stator. There are two different types of induction motor rotors which can be placed inside the stator. One is called a *cage rotor*, while the other is called a *wound rotor*.

Figures 7–2 and 7–3 show cage induction motor rotors. A cage induction motor rotor consists of a series of conducting bars laid into slots carved in the face of the rotor and shorted at either end by large *shorting rings*. This design is referred to as a cage rotor because the conductors, if examined by themselves, would look like one of the exercise wheels that squirrels or hamsters run on.

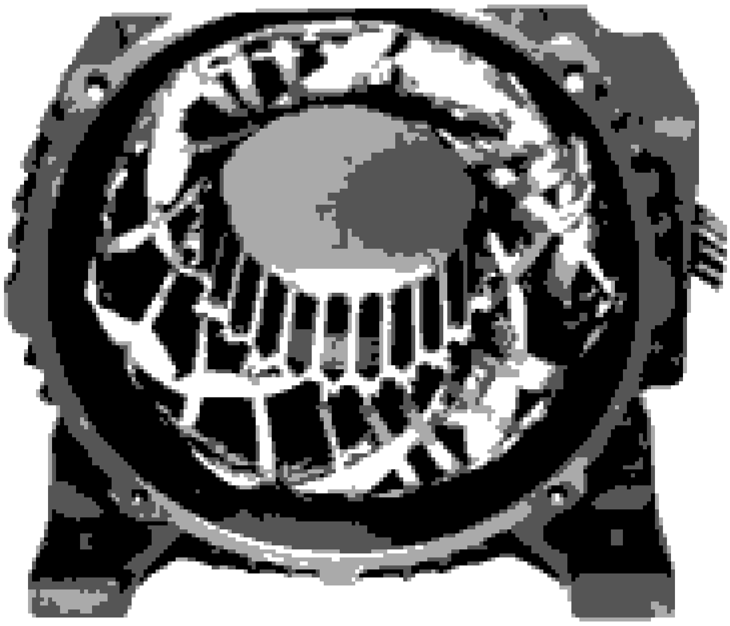
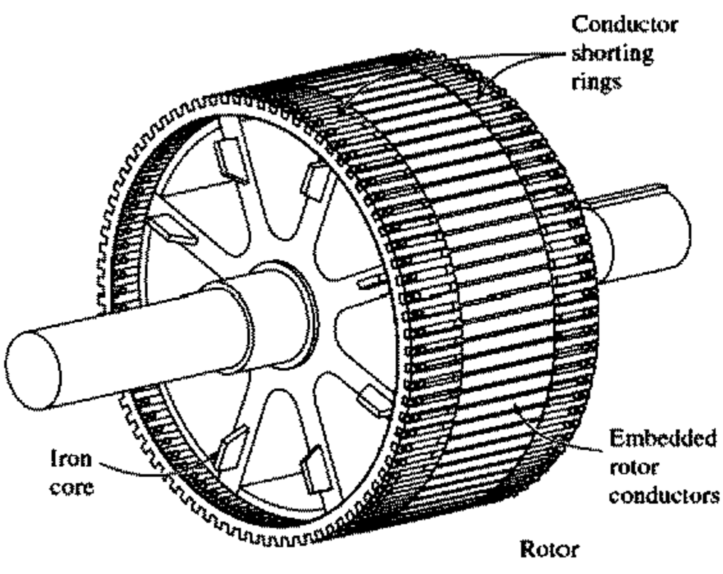
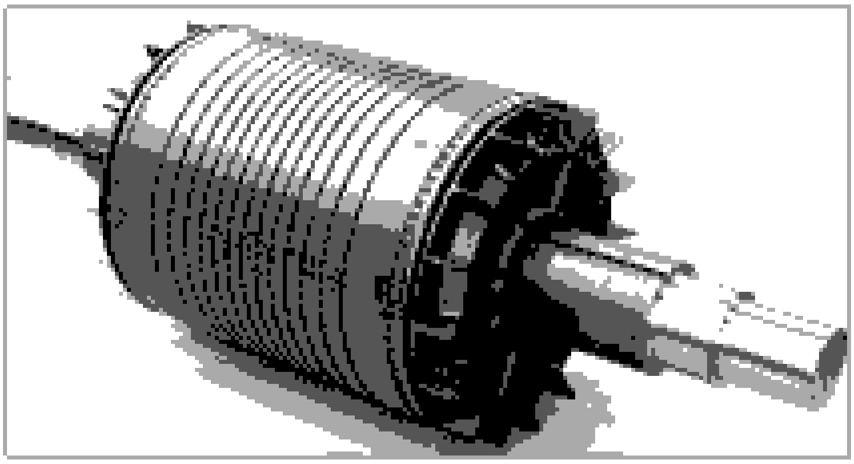


FIGURE 7–1
The stator of a typical induction motor, showing the stator windings. (*Courtesy of MagneTek, Inc.*)

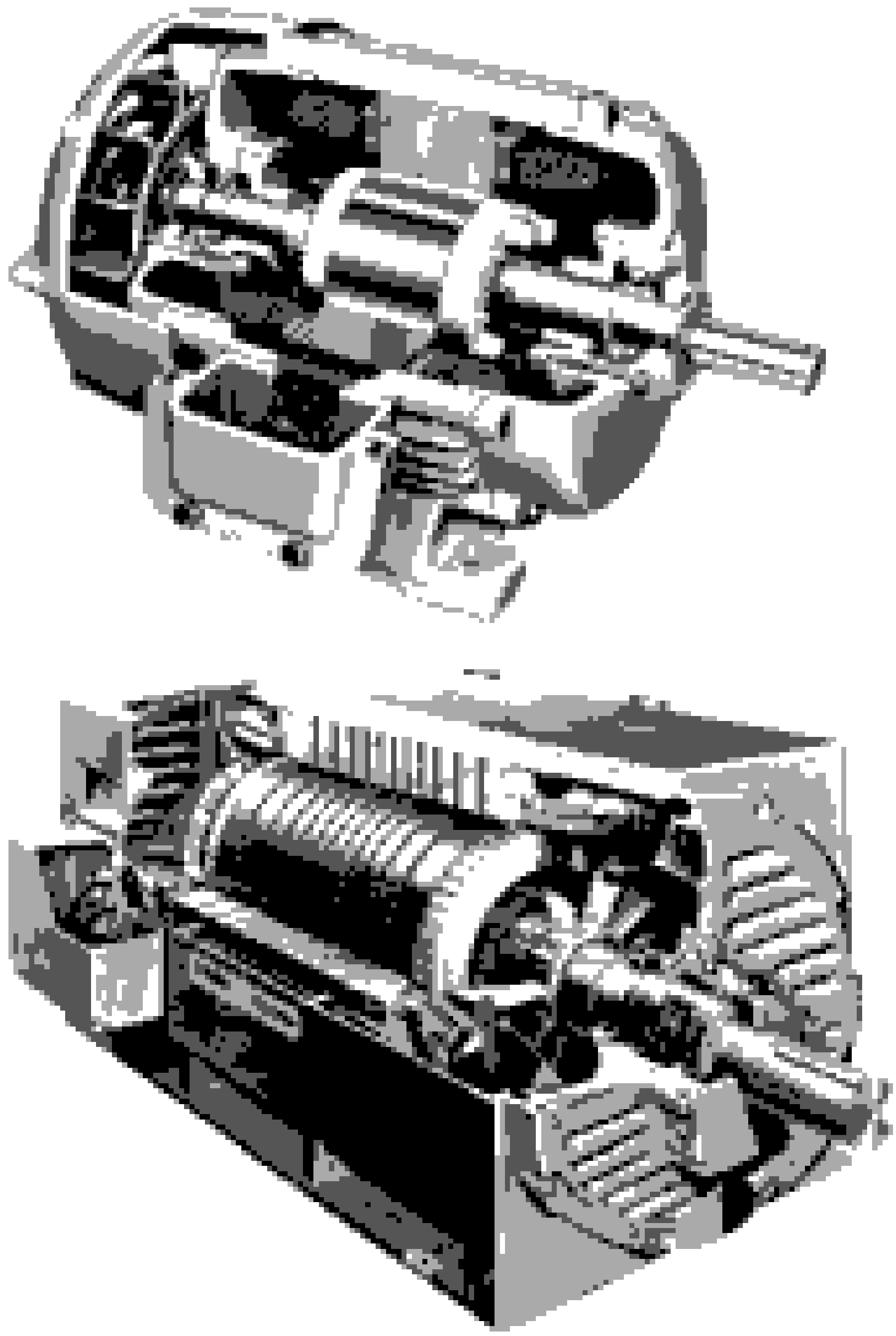


(a)



(b)

FIGURE 7–2
(a) Sketch of cage rotor. (b) A typical cage rotor. (*Courtesy of General Electric Company.*)

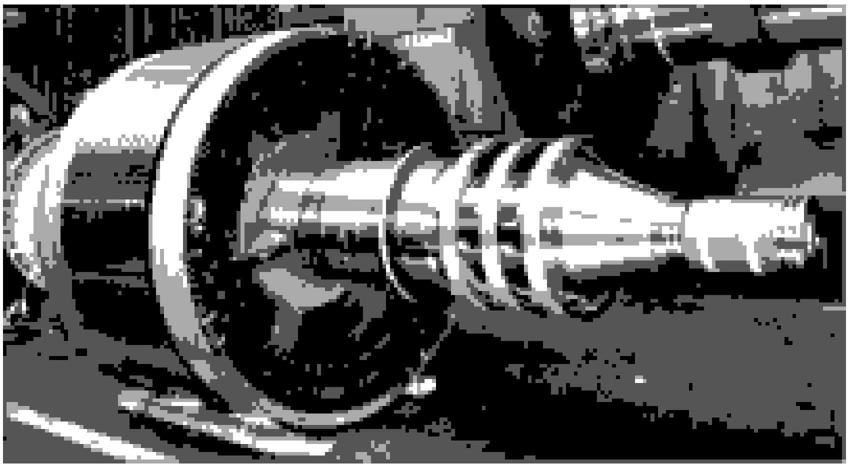


(a)

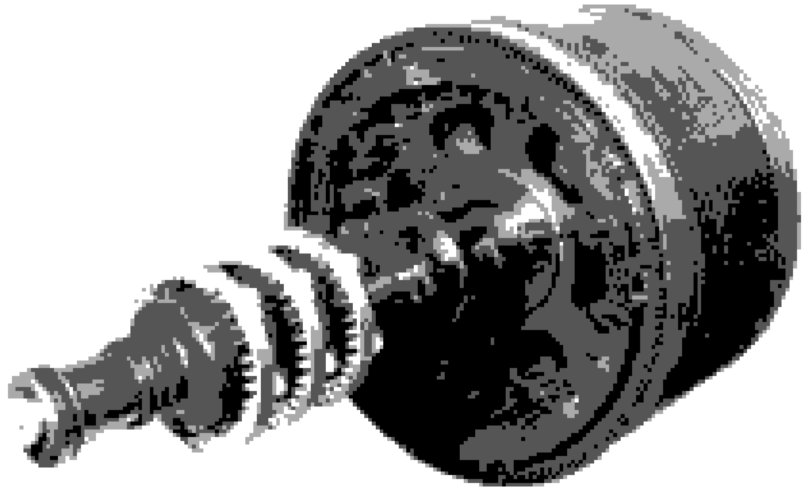
(b)

FIGURE 7–3
(a) Cutaway diagram of a typical small cage rotor induction motor. (*Courtesy of MagneTek, Inc.*)
(b) Cutaway diagram of a typical large cage rotor induction motor. (*Courtesy of General Electric Company.*)

The other type of rotor is a wound rotor. A ***wound rotor*** has a complete set of three-phase windings that are mirror images of the windings on the stator. The three phases of the rotor windings are usually Y-connected, and the ends of the three rotor wires are tied to slip rings on the rotor's shaft. The rotor windings are shorted through brushes riding on the slip rings. Wound-rotor induction motors therefore have their rotor currents accessible at the stator brushes, where they can be examined and where extra resistance can be inserted into the rotor circuit. It is possible to take advantage of this feature to modify the torque–speed characteristic of the motor. Two wound rotors are shown in Figure 7–4, and a complete wound-rotor induction motor is shown in Figure 7–5.



(a)



(b)

FIGURE 7–4
Typical wound rotors for induction motors. Notice the slip rings and the bars connecting the rotor windings to the slip rings. (*Courtesy of General Electric Company.*)

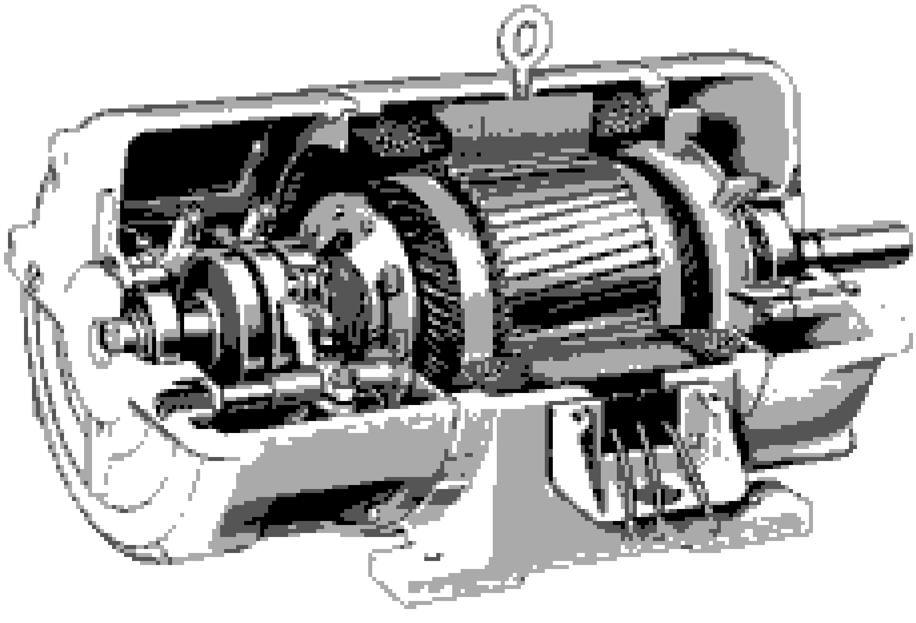


FIGURE 7–5
Cutaway diagram of a wound-rotor induction motor. Notice the brushes and slip rings. Also notice that the rotor windings are skewed to eliminate slot harmonics. (*Courtesy of MagneTek, Inc.*)

Wound-rotor induction motors are more expensive than cage induction motors, and they require much more maintenance because of the wear associated with their brushes and slip rings. As a result, wound-rotor induction motors are rarely used.

7.2 BASIC INDUCTION MOTOR CONCEPTS

Induction motor operation is basically the same as that of amortisseur windings on synchronous motors. That basic operation will now be reviewed, and some important induction motor terms will be defined.

The Development of Induced Torque in an Induction Motor

Figure 7–6 shows a cage rotor induction motor. A three-phase set of voltages has been applied to the stator, and a three-phase set of stator currents is flowing. These currents produce a magnetic field $\mathbf{B}_S$, which is rotating in a counterclockwise direction. The speed of the magnetic field's rotation is given by

$$\boxed{n_{sync} = \frac{120 f_e}{P}} \tag{7–1}$$

where f_e is the system frequency in hertz and P is the number of poles in the machine. This rotating magnetic field $\mathbf{B}_S$ passes over the rotor bars and induces a voltage in them.

The voltage induced in a given rotor bar is given by the equation

$$e_{\text{ind}} = (\mathbf{v} \times \mathbf{B}) \cdot \mathbf{l} \tag{1–45}$$

where $\mathbf{v}$ = velocity of the bar *relative to the magnetic field*

$\mathbf{B}$ = magnetic flux density vector

$\mathbf{l}$ = length of conductor in the magnetic field

It is the *relative* motion of the rotor compared to the stator magnetic field that produces induced voltage in a rotor bar. The velocity of the upper rotor bars relative to the magnetic field is to the right, so the induced voltage in the upper bars is out of the page, while the induced voltage in the lower bars is into the page. This results in a current flow out of the upper bars and into the lower bars. However, since the rotor assembly is inductive, the peak rotor current lags behind the peak rotor voltage (see Figure 7–6b). The rotor current flow produces a rotor magnetic field $\mathbf{B}_R$.

Finally, since the induced torque in the machine is given by

$$\tau_{\text{ind}} = k\mathbf{B}_R \times \mathbf{B}_S \tag{4–58}$$

the resulting torque is counterclockwise. Since the rotor induced torque is counterclockwise, the rotor accelerates in that direction.

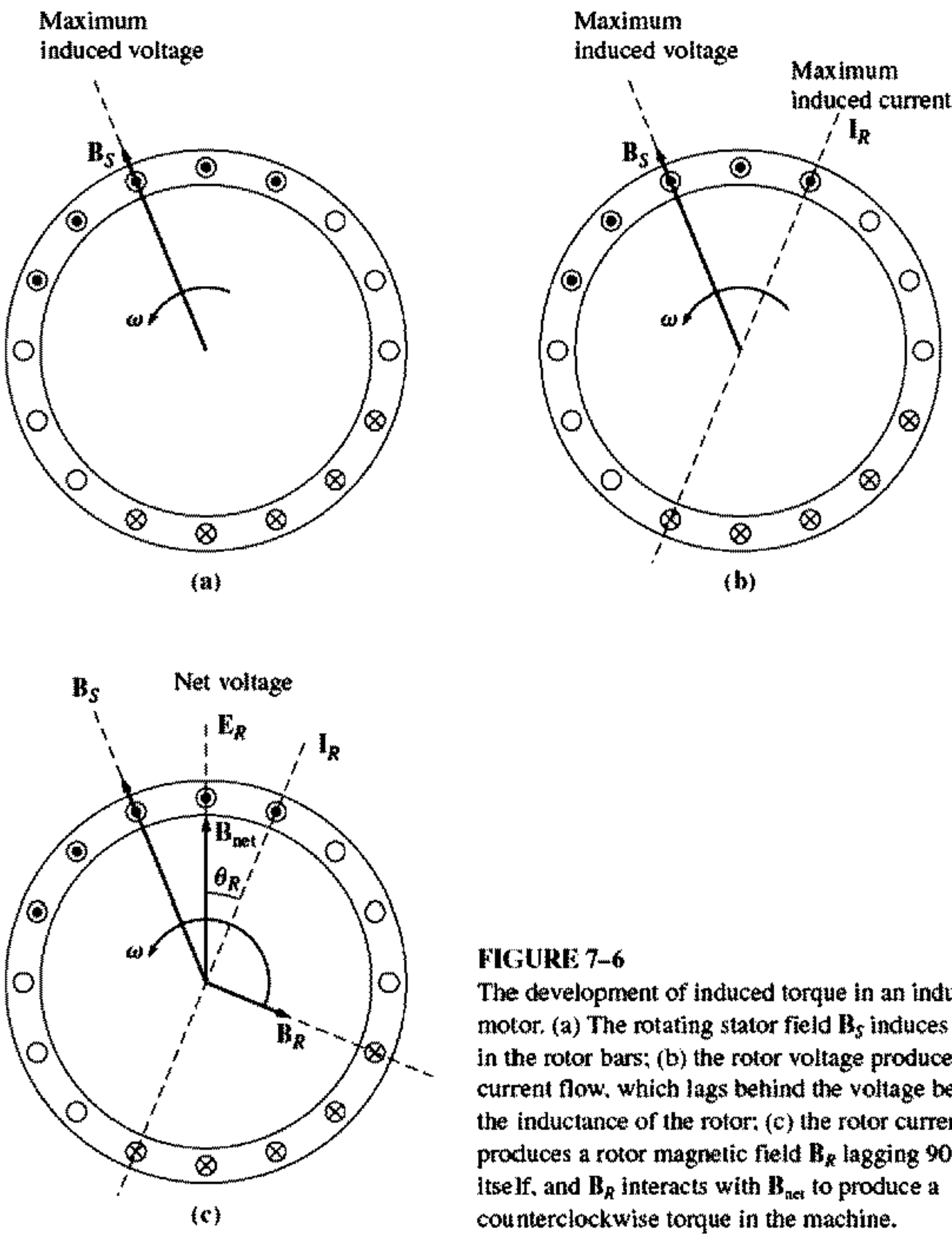


FIGURE 7–6
The development of induced torque in an induction motor. (a) The rotating stator field $\mathbf{B}_S$ induces a voltage in the rotor bars; (b) the rotor voltage produces a rotor current flow, which lags behind the voltage because of the inductance of the rotor; (c) the rotor current produces a rotor magnetic field $\mathbf{B}_R$ lagging 90° behind itself, and $\mathbf{B}_R$ interacts with $\mathbf{B}_{\text{net}}$ to produce a counterclockwise torque in the machine.

There is a finite upper limit to the motor's speed, however. If the induction motor's rotor were turning at *synchronous speed*, then the rotor bars would be stationary *relative to the magnetic field* and there would be no induced voltage. If e_{ind} were equal to 0, then there would be no rotor current and no rotor magnetic field. With no rotor magnetic field, the induced torque would be zero, and the rotor would slow down as a result of friction losses. An induction motor can thus speed up to near-synchronous speed, but it can never exactly reach synchronous speed.

Note that in normal operation *both the rotor and stator magnetic fields* $\mathbf{B}_R$ *and* $\mathbf{B}_S$ *rotate together at synchronous speed* n_{sync}, *while the rotor itself turns at a slower speed.*

The Concept of Rotor Slip

The voltage induced in a rotor bar of an induction motor depends on the speed of the rotor *relative to the magnetic fields*. Since the behavior of an induction motor depends on the rotor's voltage and current, it is often more logical to talk about this relative speed. Two terms are commonly used to define the relative motion of the rotor and the magnetic fields. One is *slip speed*, defined as the difference between synchronous speed and rotor speed:

$$\boxed{n_{\text{slip}} = n_{\text{sync}} - n_m} \tag{7–2}$$

where n_{slip} = slip speed of the machine

n_{sync} = speed of the magnetic fields

n_m = mechanical shaft speed of motor

The other term used to describe the relative motion is *slip*, which is the relative speed expressed on a per-unit or a percentage basis. That is, slip is defined as

$$s = \frac{n_{\text{slip}}}{n_{\text{sync}}}(\times\ 100\%) \tag{7–3}$$

$$\boxed{s = \frac{n_{\text{sync}} - n_m}{n_{\text{sync}}}(\times\ 100\%)} \tag{7–4}$$

This equation can also be expressed in terms of angular velocity ω (radians per second) as

$$\boxed{s = \frac{\omega_{\text{sync}} - \omega_m}{\omega_{\text{sync}}}(\times\ 100\%)} \tag{7–5}$$

Notice that if the rotor turns at synchronous speed, $s = 0$, while if the rotor is stationary, $s = 1$. All normal motor speeds fall somewhere between those two limits.

It is possible to express the mechanical speed of the rotor shaft in terms of synchronous speed and slip. Solving Equations (7–4) and (7–5) for mechanical speed yields

$$\boxed{n_m = (1 - s)n_{\text{sync}}} \tag{7–6}$$

or

$$\boxed{\omega_m = (1 - s)\omega_{\text{sync}}} \tag{7–7}$$

These equations are useful in the derivation of induction motor torque and power relationships.

The Electrical Frequency on the Rotor

An induction motor works by inducing voltages and currents in the rotor of the machine, and for that reason it has sometimes been called a *rotating transformer*. Like a transformer, the primary (stator) induces a voltage in the secondary (rotor), but *unlike* a transformer, the secondary frequency is not necessarily the same as the primary frequency.

If the rotor of a motor is locked so that it cannot move, then the rotor will have the same frequency as the stator. On the other hand, if the rotor turns at synchronous speed, the frequency on the rotor will be zero. What will the rotor frequency be for any arbitrary rate of rotor rotation?

At $n_m = 0$ r/min, the rotor frequency $f_r = f_e$, and the slip $s = 1$. At $n_m = n_{\text{sync}}$, the rotor frequency $f_r = 0$ Hz, and the slip $s = 0$. For any speed in between, the rotor frequency is directly proportional to the *difference* between the speed of the magnetic field n_{sync} and the speed of the rotor n_m. Since the slip of the rotor is defined as

$$s = \frac{n_{\text{sync}} - n_m}{n_{\text{sync}}} \tag{7–4}$$

the rotor frequency can be expressed as

$$\boxed{f_r = sf_e} \tag{7–8}$$

Several alternative forms of this expression exist that are sometimes useful. One of the more common expressions is derived by substituting Equation (7–4) for the slip into Equation (7–8) and then substituting for n_{sync} in the denominator of the expression:

$$f_r = \frac{n_{\text{sync}} - n_m}{n_{\text{sync}}} f_e$$

But $n_{\text{sync}} = 120 f_e / P$ [from Equation (7–1)], so

$$f_r = (n_{\text{sync}} - n_m) \frac{P}{120 f_e} f_e$$

Therefore,

$$\boxed{f_r = \frac{P}{120}(n_{\text{sync}} - n_m)} \tag{7–9}$$

Example 7–1. A 208-V, 10-hp, four-pole, 60-Hz, Y-connected induction motor has a full-load slip of 5 percent.

(*a*) What is the synchronous speed of this motor?
(*b*) What is the rotor speed of this motor at the rated load?
(*c*) What is the rotor frequency of this motor at the rated load?
(*d*) What is the shaft torque of this motor at the rated load?

Solution

(*a*) The synchronous speed of this motor is

$$n_{\text{sync}} = \frac{120 f_e}{P} \tag{7–1}$$

$$= \frac{120(60 \text{ Hz})}{4 \text{ poles}} = 1800 \text{ r/min}$$

(*b*) The rotor speed of the motor is given by

$$n_m = (1 - s)n_{\text{sync}} \tag{7–6}$$
$$= (1 - 0.95)(1800 \text{ r/min}) = 1710 \text{ r/min}$$

(*c*) The rotor frequency of this motor is given by

$$f_r = sf_e = (0.05)(60 \text{ Hz}) = 3 \text{ Hz} \tag{7–8}$$

Alternatively, the frequency can be found from Equation (7–9):

$$f_r = \frac{P}{120}(n_{\text{sync}} - n_m) \tag{7–9}$$
$$= \frac{4}{120}(1800 \text{ r/min} - 1710 \text{ r/min}) = 3 \text{ Hz}$$

(*d*) The shaft load torque is given by

$$\tau_{\text{load}} = \frac{P_{\text{out}}}{\omega_m}$$
$$= \frac{(10 \text{ hp})(746 \text{ W/hp})}{(1710 \text{ r/min})(2\pi \text{ rad/r})(1 \text{ min}/60 \text{ s})} = 41.7 \text{ N} \bullet \text{m}$$

The shaft load torque in English units is given by Equation (1–17):

$$\tau_{\text{load}} = \frac{5252P}{n}$$

where τ is in pound-feet, P is in horsepower, and n_m is in revolutions per minute. Therefore,

$$\tau_{\text{load}} = \frac{5252(10 \text{ hp})}{1710 \text{ r/min}} = 30.7 \text{ lb} \bullet \text{ft}$$

7.3 THE EQUIVALENT CIRCUIT OF AN INDUCTION MOTOR

An induction motor relies for its operation on the induction of voltages and currents in its rotor circuit from the stator circuit (transformer action). Because the induction of voltages and currents in the rotor circuit of an induction motor is essentially a transformer operation, the equivalent circuit of an induction motor will turn out to be very similar to the equivalent circuit of a transformer. An induction motor is called a *singly excited* machine (as opposed to a *doubly excited* synchronous machine), since power is supplied to only the stator circuit. Because an induction motor does not have an independent field circuit, its model will not contain an internal voltage source such as the internal generated voltage $\mathbf{E}_A$ in a synchronous machine.

It is possible to derive the equivalent circuit of an induction motor from a knowledge of transformers and from what we already know about the variation of rotor frequency with speed in induction motors. The induction motor model will be

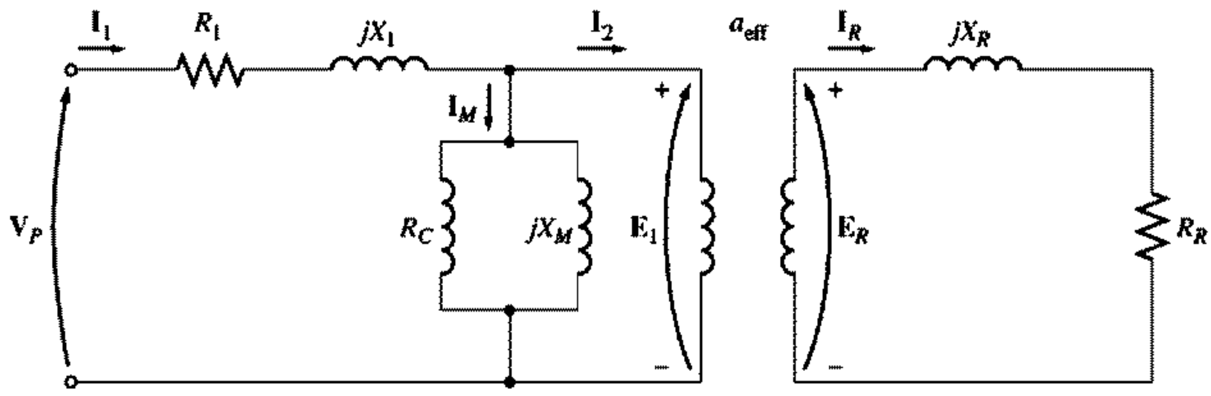


FIGURE 7–7
The transformer model or an induction motor, with rotor and stator connected by an ideal transformer of turns ratio a_{eff}.

developed by starting with the transformer model in Chapter 2 and then deciding how to take the variable rotor frequency and other similar induction motor effects into account.

The Transformer Model of an Induction Motor

A transformer per-phase equivalent circuit, representing the operation of an induction motor, is shown in Figure 7–7. As in any transformer, there is a certain resistance and self-inductance in the primary (stator) windings, which must be represented in the equivalent circuit of the machine. The stator resistance will be called R_1, and the stator leakage reactance will be called X_1. These two components appear right at the input to the machine model.

Also, like any transformer with an iron core, the flux in the machine is related to the integral of the applied voltage $\mathbf{E}_1$. The curve of magnetomotive force versus flux (magnetization curve) for this machine is compared to a similar curve for a power transformer in Figure 7–8. Notice that the slope of the induction motor's magnetomotive force-flux curve is much shallower than the curve of a good transformer. This is because there must be an air gap in an induction motor, which greatly increases the reluctance of the flux path and therefore reduces the coupling between primary and secondary windings. The higher reluctance caused by the air gap means that a higher magnetizing current is required to obtain a given flux level. Therefore, the magnetizing reactance X_M in the equivalent circuit will have a much smaller value (or the susceptance B_M will have a much larger value) than it would in an ordinary transformer.

The primary internal stator voltage $\mathbf{E}_1$ is coupled to the secondary $\mathbf{E}_R$ by an ideal transformer with an effective turns ratio a_{eff}. The effective turns ratio a_{eff} is fairly easy to determine for a wound-rotor motor—it is basically the ratio of the conductors per phase on the stator to the conductors per phase on the rotor, modified by any pitch and distribution factor differences. It is rather difficult to see a_{eff}

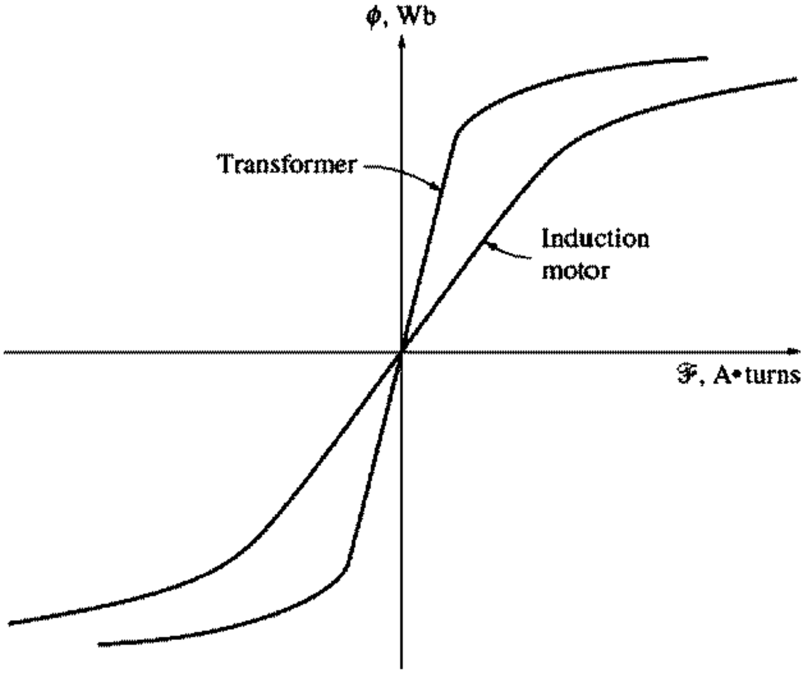


FIGURE 7–8
The magnetization curve of an induction motor compared to that of a transformer.

clearly in the cage of a case rotor motor because there are no distinct windings on the cage rotor. In either case, there *is* an effective turns ratio for the motor.

The voltage $\mathbf{E}_R$ produced in the rotor in turn produces a current flow in the shorted rotor (or secondary) circuit of the machine.

The primary impedances and the magnetization current of the induction motor are very similar to the corresponding components in a transformer equivalent circuit. An induction motor equivalent circuit differs from a transformer equivalent circuit primarily in the effects of varying rotor frequency on the rotor voltage $\mathbf{E}_R$ and the rotor impedances R_R and jX_R.

The Rotor Circuit Model

In an induction motor, when the voltage is applied to the stator windings, a voltage is induced in the rotor windings of the machine. In general, *the greater the relative motion between the rotor and the stator magnetic fields, the greater the resulting rotor voltage and rotor frequency*. The largest relative motion occurs when the rotor is stationary, called the *locked-rotor* or *blocked-rotor* condition, so the largest voltage and rotor frequency are induced in the rotor at that condition. The smallest voltage (0 V) and frequency (0 Hz) occur when the rotor moves at the same speed as the stator magnetic field, resulting in no relative motion. The magnitude and frequency of the voltage induced in the rotor at any speed between these extremes is *directly proportional to the slip of the rotor*. Therefore, if the magnitude of the induced rotor voltage at locked-rotor conditions is called E_{R0}, the magnitude of the induced voltage at any slip will be given by the equation

$$E_R = sE_{R0} \tag{7–10}$$

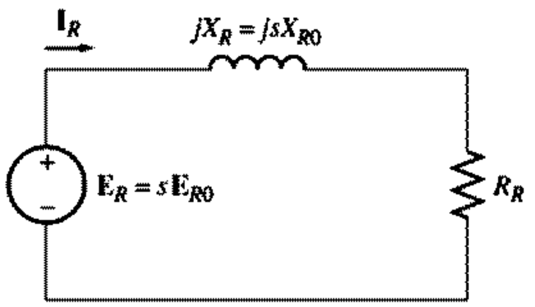


FIGURE 7–9
The rotor circuit model of an induction motor.

and the frequency of the induced voltage at any slip will be given by the equation

$$f_r = sf_e \tag{7–8}$$

This voltage is induced in a rotor containing both resistance and reactance. The rotor resistance R_R is a constant (except for the skin effect), independent of slip, while the rotor reactance is affected in a more complicated way by slip.

The reactance of an induction motor rotor depends on the inductance of the rotor and the frequency of the voltage and current in the rotor. With a rotor inductance of L_R, the rotor reactance is given by

$$X_R = \omega_r L_R = 2\pi f_r L_R$$

By Equation (7–8), $f_r = sf_e$, so

$$\begin{aligned} X_R &= 2\pi s f_e L_R \\ &= s(2\pi f_e L_R) \\ &= sX_{R0} \end{aligned} \tag{7–11}$$

where X_{R0} is the blocked-rotor rotor reactance.

The resulting rotor equivalent circuit is shown in Figure 7–9. The rotor current flow can be found as

$$\mathbf{I}_R = \frac{\mathbf{E}_R}{R_R + jX_R}$$

$$\boxed{\mathbf{I}_R = \frac{\mathbf{E}_R}{R_R + jsX_{R0}}} \tag{7–12}$$

or

$$\boxed{\mathbf{I}_R = \frac{\mathbf{E}_{R0}}{R_R/s + jX_{R0}}} \tag{7–13}$$

Notice from Equation (7–13) that it is possible to treat all of the rotor effects due to varying rotor speed as being caused by a *varying impedance* supplied with power from a constant-voltage source $\mathbf{E}_{R0}$. The equivalent rotor impedance from this point of view is

$$Z_{R,\text{eq}} = R_R/s + jX_{R0} \tag{7–14}$$

and the rotor equivalent circuit using this convention is shown in Figure 7–10. In the equivalent circuit in Figure 7–10, the rotor voltage is a constant $\mathbf{E}_{R0}$ V and the

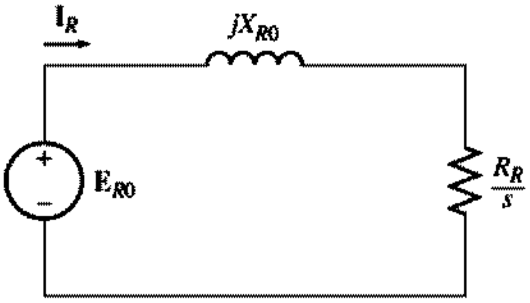


FIGURE 7–10
The rotor circuit model with all the frequency (slip) effects concentrated in resistor R_R.

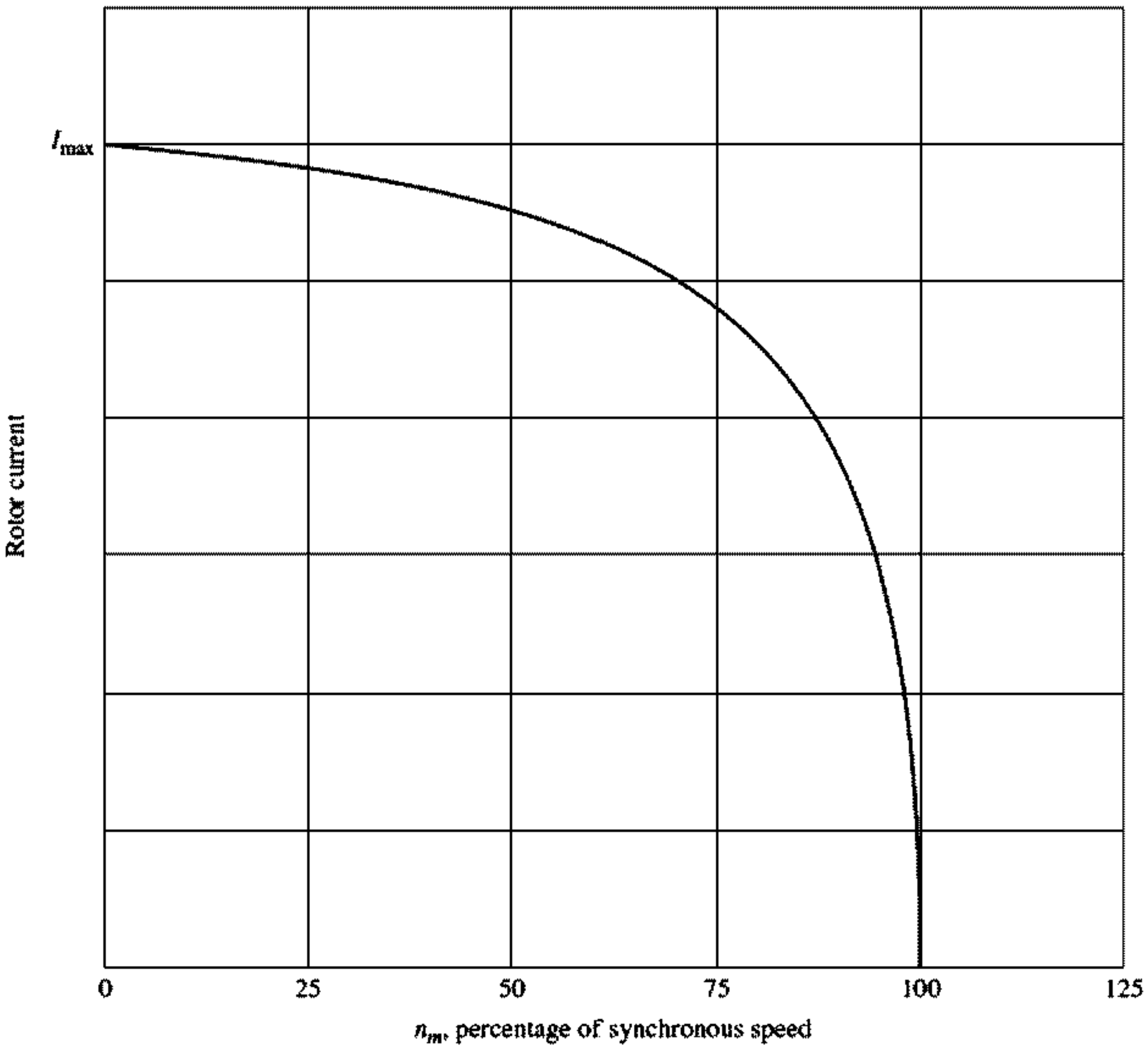


FIGURE 7–11
Rotor current as a function of rotor speed.

rotor impedance $Z_{R,eq}$ contains all the effects of varying rotor slip. A plot of the current flow in the rotor as developed in Equations (7–12) and (7–13) is shown in Figure 7–11.

Notice that at very low slips the resistive term $R_R/s >> X_{R0}$, so the rotor resistance predominates and the rotor current varies *linearly* with slip. At high slips,

X_{R0} is much larger than R_R/s, and the rotor current *approaches a steady-state value* as the slip becomes very large.

The Final Equivalent Circuit

To produce the final per-phase equivalent circuit for an induction motor, it is necessary to refer the rotor part of the model over to the stator side. The rotor circuit model that will be referred to the stator side is the model shown in Figure 7–10, which has all the speed variation effects concentrated in the impedance term.

In an ordinary transformer, the voltages, currents, and impedances on the secondary side of the device can be referred to the primary side by means of the turns ratio of the transformer:

$$\mathbf{V}_P = \mathbf{V}'_S = a\mathbf{V}_S \tag{7–15}$$

$$\mathbf{I}_P = \mathbf{I}'_S = \frac{\mathbf{I}_S}{a} \tag{7–16}$$

and

$$Z'_S = a^2 Z_S \tag{7–17}$$

where the prime refers to the referred values of voltage, current, and impedance.

Exactly the same sort of transformation can be done for the induction motor's rotor circuit. If the effective turns ratio of an induction motor is a_{eff}, then the transformed rotor voltage becomes

$$\mathbf{E}_1 = \mathbf{E}'_R = a_{\text{eff}}\mathbf{E}_{R0} \tag{7–18}$$

the rotor current becomes

$$\mathbf{I}_2 = \frac{\mathbf{I}_R}{a_{\text{eff}}} \tag{7–19}$$

and the rotor impedance becomes

$$Z_2 = a_{\text{eff}}^2\left(\frac{R_R}{s} + jX_{R0}\right) \tag{7–20}$$

If we now make the following definitions:

$$R_2 = a_{\text{eff}}^2 R_R \tag{7–21}$$

$$X_2 = a_{\text{eff}}^2 X_{R0} \tag{7–22}$$

then the final per-phase equivalent circuit of the induction motor is as shown in Figure 7–12.

The rotor resistance R_R and the locked-rotor rotor reactance X_{R0} are very difficult or impossible to determine directly on cage rotors, and the effective turns ratio a_{eff} is also difficult to obtain for cage rotors. Fortunately, though, it is possible to make measurements that will directly give the *referred resistance and reactance* R_2 and X_2, even though R_R, X_{R0} and a_{eff} are not known separately. The measurement of induction motor parameters will be taken up in Section 7.7.

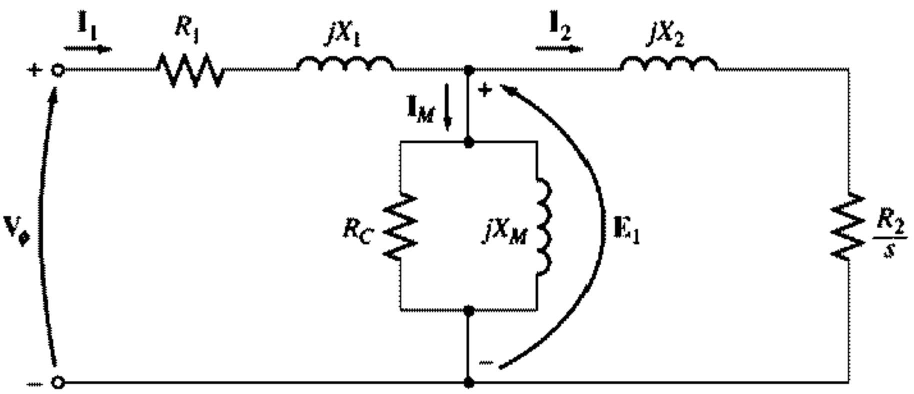


FIGURE 7–12
The per-phase equivalent circuit of an induction motor.

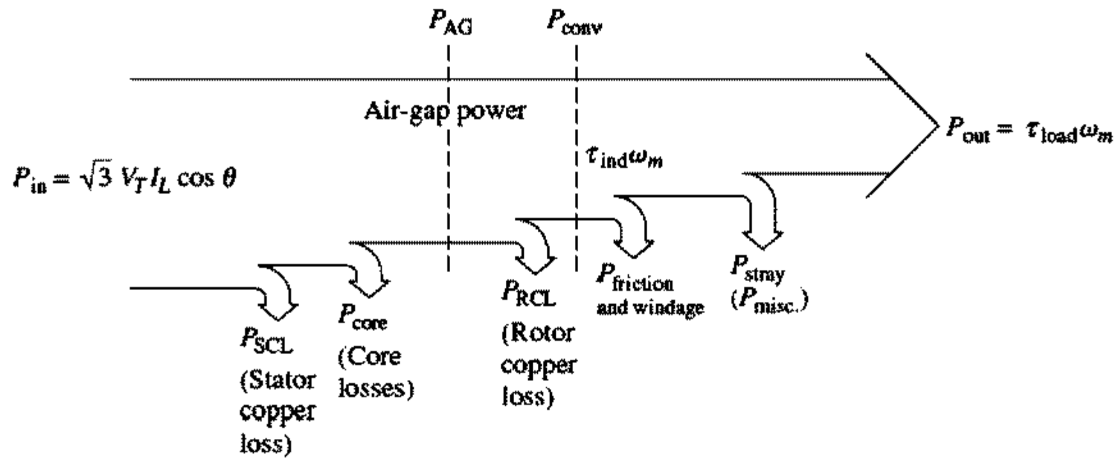


FIGURE 7–13
The power-flow diagram of an induction motor.

7.4 POWER AND TORQUE IN INDUCTION MOTORS

Because induction motors are singly excited machines, their power and torque relationships are considerably different from the relationships in the synchronous machines previously studied. This section reviews the power and torque relationships in induction motors.

Losses and the Power-Flow Diagram

An induction motor can be basically described as a rotating transformer. Its input is a three-phase system of voltages and currents. For an ordinary transformer, the output is electric power from the secondary windings. The secondary windings in an induction motor (the rotor) are shorted out, so no electrical output exists from normal induction motors. Instead, the output is mechanical. The relationship between the input electric power and the output mechanical power of this motor is shown in the power-flow diagram in Figure 7–13.

The input power to an induction motor P_{in} is in the form of three-phase electric voltages and currents. The first losses encountered in the machine are I^2R losses in the stator windings (the *stator copper loss* P_{SCL}). Then some amount of power is lost as hysteresis and eddy currents in the stator (P_{core}). The power remaining at this point is transferred to the rotor of the machine across the air gap between the stator and rotor. This power is called the *air-gap power* P_{AG} of the machine. After the power is transferred to the rotor, some of it is lost as I^2R losses (the *rotor copper loss* P_{RCL}), and the rest is converted from electrical to mechanical form (P_{conv}). Finally, friction and windage losses $P_{F\&W}$ and stray losses P_{misc} are subtracted. The remaining power is the output of the motor P_{out}.

The *core losses* do not always appear in the power-flow diagram at the point shown in Figure 7–13. Because of the nature of core losses, where they are accounted for in the machine is somewhat arbitrary. The core losses of an induction motor come partially from the stator circuit and partially from the rotor circuit. Since an induction motor normally operates at a speed near synchronous speed, the relative motion of the magnetic fields over the rotor surface is quite slow, and the rotor core losses are very tiny compared to the stator core losses. Since the largest fraction of the core losses comes from the stator circuit, all the core losses are lumped together at that point on the diagram. These losses are represented in the induction motor equivalent circuit by the resistor R_C (or the conductance G_C). If core losses are just given by a number (X watts) instead of as a circuit element they are often lumped together with the mechanical losses and subtracted at the point on the diagram where the mechanical losses are located.

The ***higher*** the speed of an induction motor, the ***higher*** its friction, windage, and stray losses. On the other hand, the ***higher*** the speed of the motor (up to n_{sync}), the *lower* its core losses. Therefore, these three categories of losses are sometimes lumped together and called *rotational losses*. The total rotational losses of a motor are often considered to be constant with changing speed, since the component losses change in opposite directions with a change in speed.

Example 7–2. A 480-V, 60-Hz, 50-hp, three-phase induction motor is drawing 60 A at 0.85 PF lagging. The stator copper losses are 2 kW, and the rotor copper losses are 700 W. The friction and windage losses are 600 W, the core losses are 1800 W, and the stray losses are negligible. Find the following quantities:

(a) The air-gap power P_{AG}
(b) The power converted P_{conv}
(c) The output power P_{out}
(d) The efficiency of the motor

Solution
To answer these questions, refer to the power-flow diagram for an induction motor (Figure 7–13).

(a) The air-gap power is just the input power minus the stator I^2R losses. The input power is given by

$$P_{in} = \sqrt{3}V_T I_L \cos\theta$$
$$= \sqrt{3}(480 \text{ V})(60 \text{ A})(0.85) = 42.4 \text{ kW}$$

From the power-flow diagram, the air-gap power is given by

$$P_{AG} = P_{in} - P_{SCL} - P_{core}$$
$$= 42.4 \text{ kW} - 2 \text{ kW} - 1.8 \text{ kW} = 38.6 \text{ kW}$$

(b) From the power-flow diagram, the power converted from electrical to mechanical form is

$$P_{conv} = P_{AG} - P_{RCL}$$
$$= 38.6 \text{ kW} - 700 \text{ W} = 37.9 \text{ kW}$$

(c) From the power-flow diagram, the output power is given by

$$P_{out} = P_{conv} - P_{F\&W} - P_{misc}$$
$$= 37.9 \text{ kW} - 600 \text{ W} - 0 \text{ W} = 37.3 \text{ kW}$$

or, in horsepower,

$$P_{out} = (37.3 \text{ kW}) \frac{1 \text{ hp}}{0.746 \text{ kW}} = 50 \text{ hp}$$

(d) Therefore, the induction motor's efficiency is

$$\eta = \frac{P_{out}}{P_{in}} \times 100\%$$
$$= \frac{37.3 \text{ kW}}{42.4 \text{ kW}} \times 100\% = 88\%$$

Power and Torque in an Induction Motor

Figure 7–12 shows the per-phase equivalent circuit of an induction motor. If the equivalent circuit is examined closely, it can be used to derive the power and torque equations governing the operation of the motor.

The input current to a phase of the motor can be found by dividing the input voltage by the total equivalent impedance:

$$\mathbf{I}_1 = \frac{\mathbf{V}_\phi}{Z_{eq}} \tag{7–23}$$

where
$$Z_{eq} = R_1 + jX_1 + \cfrac{1}{G_C - jB_M + \cfrac{1}{V_2/s + jX_2}} \tag{7–24}$$

Therefore, the stator copper losses, the core losses, and the rotor copper losses can be found. The stator copper losses in the three phases are given by

$$\boxed{P_{SCL} = 3I_1^2 R_1} \tag{7–25}$$

The core losses are given by

$$\boxed{P_{core} = 3E_1^2 G_C} \tag{7–26}$$

so the air-gap power can be found as

$$\boxed{P_{AG} = P_{in} - P_{SCL} - P_{core}} \tag{7–27}$$

Look closely at the equivalent circuit of the rotor. The *only* element in the equivalent circuit where the air-gap power can be consumed is in the resistor R_2/s. Therefore, the *air-gap power* can also be given by

$$\boxed{P_{AG} = 3I_2^2 \frac{R_2}{s}} \tag{7–28}$$

The actual resistive losses in the rotor circuit are given by the equation

$$P_{RCL} = 3I_R^2 R_R \tag{7–29}$$

Since power is unchanged when referred across an ideal transformer, the rotor copper losses can also be expressed as

$$\boxed{P_{RCL} = 3I_2^2 R_2} \tag{7–30}$$

After stator copper losses, core losses, and rotor copper losses are subtracted from the input power to the motor, the remaining power is converted from electrical to mechanical form. This power converted, which is sometimes called *developed mechanical power*, is given by

$$\begin{aligned} P_{conv} &= P_{AG} - P_{RCL} \\ &= 3I_2^2 \frac{R_2}{s} - 3I_2^2 R_2 \\ &= 3I_2^2 R_2\left(\frac{1}{s} - 1\right) \end{aligned}$$

$$\boxed{P_{conv} = 3I_2^2 R_2\left(\frac{1-s}{s}\right)} \tag{7–31}$$

Notice from Equations (7–28) and (7–30) that the rotor copper losses are equal to the air-gap power times the slip:

$$P_{RCL} = sP_{AG} \tag{7–32}$$

Therefore, the lower the slip of the motor, the lower the rotor losses in the machine. Note also that if the rotor is not turning, the slip $s = 1$ and the *air-gap power is entirely consumed in the rotor*. This is logical, since if the rotor is not turning, the output power P_{out} $(= \tau_{load}\,\omega_m)$ must be zero. Since $P_{conv} = P_{AG} - P_{RCL}$, this also gives another relationship between the air-gap power and the power converted from electrical to mechanical form:

$$\begin{aligned} P_{conv} &= P_{AG} - P_{RCL} \\ &= P_{AG} - sP_{AG} \end{aligned}$$

$$\boxed{P_{conv} = (1 - s)P_{AG}} \tag{7–33}$$

Finally, if the friction and windage losses and the stray losses are known, the output power can be found as

$$\boxed{P_{\text{out}} = P_{\text{conv}} - P_{\text{F\&W}} - P_{\text{misc}}} \tag{7–34}$$

The *induced torque* τ_{ind} in a machine was defined as the torque generated by the internal electric-to-mechanical power conversion. This torque differs from the torque actually available at the terminals of the motor by an amount equal to the friction and windage torques in the machine. The induced torque is given by the equation

$$\tau_{\text{ind}} = \frac{P_{\text{conv}}}{\omega_m} \tag{7–35}$$

This torque is also called the ***developed torque*** of the machine.

The induced torque of an induction motor can be expressed in a different form as well. Equation (7–7) expresses actual speed in terms of synchronous speed and slip, while Equation (7–33) expresses P_{conv} in terms of P_{AG} and slip. Substituting these two equations into Equation (7–35) yields

$$\tau_{\text{ind}} = \frac{(1 - s)P_{\text{AG}}}{(1 - s)\omega_{\text{sync}}}$$

$$\boxed{\tau_{\text{ind}} = \frac{P_{\text{AG}}}{\omega_{\text{sync}}}} \tag{7–36}$$

The last equation is especially useful because it expresses induced torque directly in terms of air-gap power and *synchronous speed*, which does not vary. A knowledge of P_{AG} thus directly yields τ_{ind}.

Separating the Rotor Copper Losses and the Power Converted in an Induction Motor's Equivalent Circuit

Part of the power coming across the air gap in an induction motor is consumed in the rotor copper losses, and part of it is converted to mechanical power to drive the motor's shaft. It is possible to separate the two uses of the air-gap power and to indicate them separately on the motor equivalent circuit.

Equation (7–28) gives an expression for the total air-gap power in an induction motor, while Equation (7–30) gives the actual rotor losses in the motor. The air-gap power is the power which would be consumed in a resistor of value R_2/s, while the rotor copper losses are the power which would be consumed in a resistor of value R_2. The difference between them is P_{conv}, which must therefore be the power consumed in a resistor of value

$$R_{\text{conv}} = \frac{R_2}{s} - R_2 = R_2\left(\frac{1}{s} - 1\right)$$

$$\boxed{R_{\text{conv}} = R_2\left(\frac{1 - s}{s}\right)} \tag{7–37}$$

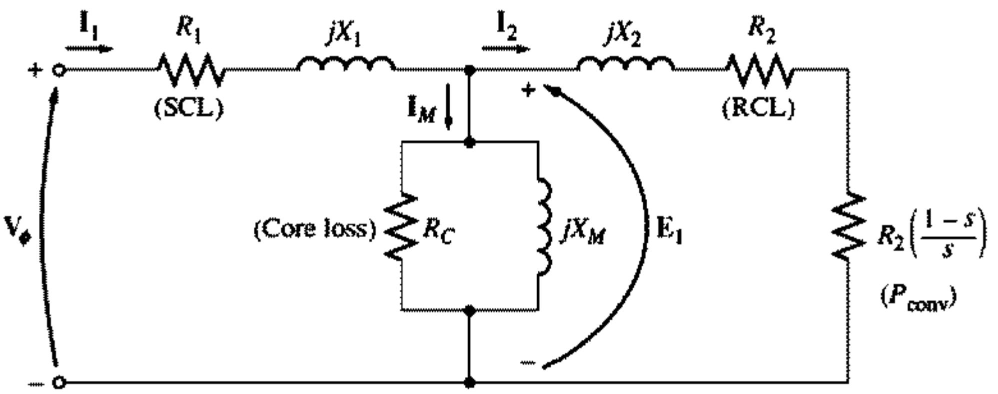


FIGURE 7–14
The per-phase equivalent circuit with rotor losses and P_{core} separated.

Per-phase equivalent circuit with the rotor copper losses and the power converted to mechanical form separated into distinct elements is shown in Figure 7–14.

Example 7–3. A 460-V, 25-hp, 60-Hz, four-pole, Y-connected induction motor has the following impedances in ohms per phase referred to the stator circuit:

$$R_1 = 0.641\ \Omega \qquad R_2 = 0.332\ \Omega$$
$$X_1 = 1.106\ \Omega \qquad X_2 = 0.464\ \Omega \qquad X_M = 26.3\ \Omega$$

The total rotational losses are 1100 W and are assumed to be constant. The core loss is lumped in with the rotational losses. For a rotor slip of 2.2 percent at the rated voltage and rated frequency, find the motor's

(*a*) Speed
(*b*) Stator current
(*c*) Power factor
(*d*) P_{conv} and P_{out}
(*e*) τ_{ind} and τ_{load}
(*f*) Efficiency

Solution

The per-phase equivalent circuit of this motor is shown in Figure 7–12, and the power-flow diagram is shown in Figure 7–13. Since the core losses are lumped together with the friction and windage losses and the stray losses, they will be treated like the mechanical losses and be subtracted after P_{conv} in the power-flow diagram.

(*a*) The synchronous speed is

$$n_{sync} = \frac{120 f_e}{P} = \frac{120(60\text{ Hz})}{4\text{ poles}} = 1800\text{ r/min}$$

or $$\omega_{sync} = (1800\text{ r/min})\left(\frac{2\pi\text{ rad}}{1\text{ r}}\right)\left(\frac{1\text{ min}}{60\text{ s}}\right) = 188.5\text{ rad/s}$$

The rotor's mechanical shaft speed is

$$\begin{aligned} n_m &= (1 - s)n_{sync} \\ &= (1 - 0.022)(1800\text{ r/min}) = 1760\text{ r/min} \end{aligned}$$

or $$\omega_m = (1 - s)\omega_{sync}$$
$$= (1 - 0.022)(188.5 \text{ rad/s}) = 184.4 \text{ rad/s}$$

(b) To find the stator current, get the equivalent impedance of the circuit. The first step is to combine the referred rotor impedance in parallel with the magnetization branch, and then to add the stator impedance to that combination in series. The referred rotor impedance is

$$Z_2 = \frac{R_2}{s} + jX_2$$
$$= \frac{0.332}{0.022} + j0.464$$
$$= 15.09 + j0.464\ \Omega = 15.10\angle 1.76^\circ\ \Omega$$

The combined magnetization plus rotor impedance is given by

$$Z_f = \frac{1}{1/jX_M + 1/Z_2}$$
$$= \frac{1}{-j0.038 + 0.0662\angle -1.76^\circ}$$
$$= \frac{1}{0.0773\angle -31.1^\circ} = 12.94\angle 31.1^\circ\ \Omega$$

Therefore, the total impedance is

$$Z_{tot} = Z_{stat} + Z_f$$
$$= 0.641 + j1.106 + 12.94\angle 31.1^\circ\ \Omega$$
$$= 11.72 + j7.79 = 14.07\angle 33.6^\circ\ \Omega$$

The resulting stator current is

$$\mathbf{I}_1 = \frac{\mathbf{V}_\phi}{Z_{tot}}$$
$$= \frac{266\angle 0^\circ \text{ V}}{14.07\angle 33.6^\circ\ \Omega} = 18.88\angle -33.6^\circ \text{ A}$$

(c) The power motor power factor is

$$\text{PF} = \cos 33.6^\circ = 0.833 \qquad \text{lagging}$$

(d) The input power to this motor is

$$P_{in} = \sqrt{3}V_T I_L \cos\theta$$
$$= \sqrt{3}(460 \text{ V})(18.88 \text{ A})(0.833) = 12{,}530 \text{ W}$$

The stator copper losses in this machine are

$$P_{SCL} = 3I_1^2 R_1 \qquad (7\text{–}25)$$
$$= 3(18.88 \text{ A})^2(0.641\ \Omega) = 685 \text{ W}$$

The air-gap power is given by

$$P_{AG} = P_{in} - P_{SCL} = 12{,}530 \text{ W} - 685 \text{ W} = 11{,}845 \text{ W}$$

Therefore, the power converted is

$$P_{\text{conv}} = (1 - s)P_{\text{AG}} = (1 - 0.022)(11{,}845 \text{ W}) = 11{,}585 \text{ W}$$

The power P_{out} is given by

$$P_{\text{out}} = P_{\text{conv}} - P_{\text{rot}} = 11{,}585 \text{ W} - 1100 \text{ W} = 10{,}485 \text{ W}$$

$$= 10{,}485 \text{ W}\left(\frac{1 \text{ hp}}{746 \text{ W}}\right) = 14.1 \text{ hp}$$

(e) The induced torque is given by

$$\tau_{\text{ind}} = \frac{P_{\text{AG}}}{\omega_{\text{sync}}}$$

$$= \frac{11{,}845 \text{ W}}{188.5 \text{ rad/s}} = 62.8 \text{ N} \cdot \text{m}$$

and the output torque is given by

$$\tau_{\text{load}} = \frac{P_{\text{out}}}{\omega_m}$$

$$= \frac{10{,}485 \text{ W}}{184.4 \text{ rad/s}} = 56.9 \text{ N} \cdot \text{m}$$

(In English units, these torques are 46.3 and 41.9 lb-ft, respectively.)

(f) The motor's efficiency at this operating condition is

$$\eta = \frac{P_{\text{out}}}{P_{\text{in}}} \times 100\%$$

$$= \frac{10{,}485 \text{ W}}{12{,}530 \text{ W}} \times 100\% = 83.7\%$$

7.5 INDUCTION MOTOR TORQUE–SPEED CHARACTERISTICS

How does the torque of an induction motor change as the load changes? How much torque can an induction motor supply at starting conditions? How much does the speed of an induction motor drop as its shaft load increases? To find out the answers to these and similar questions, it is necessary to clearly understand the relationships among the motor's torque, speed, and power.

In the following material, the torque–speed relationship will be examined first from the physical viewpoint of the motor's magnetic field behavior. Then, a general equation for torque as a function of slip will be derived from the induction motor equivalent circuit (Figure 7–12).

Induced Torque from a Physical Standpoint

Figure 7–15a shows a cage rotor induction motor that is initially operating at no load and therefore very nearly at synchronous speed. The net magnetic field $\mathbf{B}_{\text{net}}$ in this machine is produced by the magnetization current $\mathbf{I}_M$ flowing in the motor's equivalent circuit (see Figure 7–12). The magnitude of the magnetization current and hence of $\mathbf{B}_{\text{net}}$ is directly proportional to the voltage $\mathbf{E}_1$. If $\mathbf{E}_1$ is constant, then the

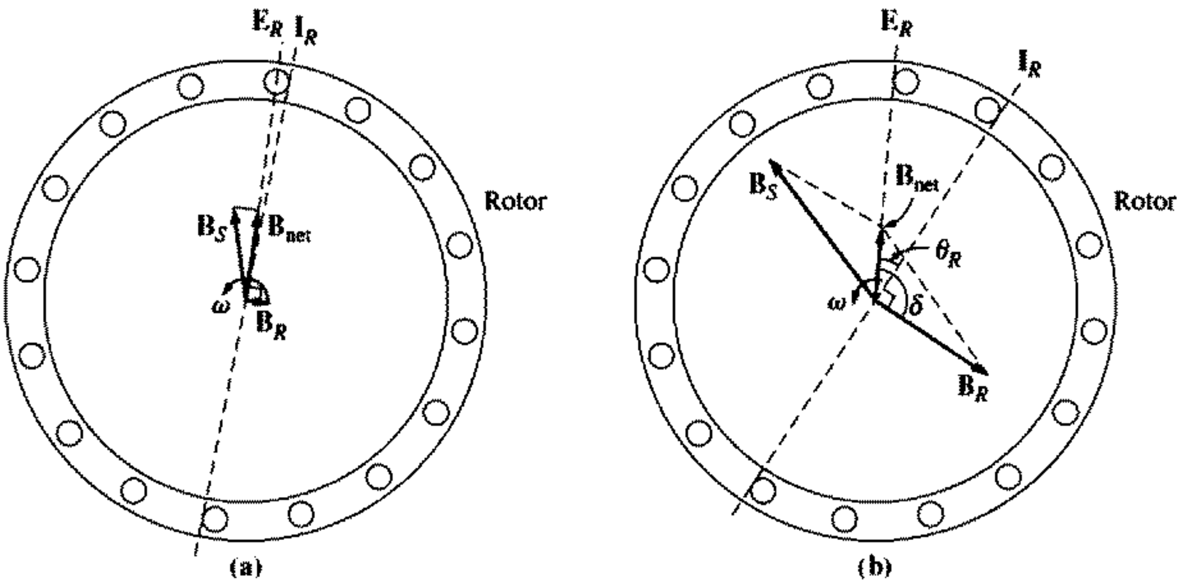


FIGURE 7–15
(a) The magnetic fields in an induction motor under light loads. (b) The magnetic fields in an induction motor under heavy loads.

net magnetic field in the motor is constant. In an actual machine, $\mathbf{E}_1$ varies as the load changes, because the stator impedances R_1 and X_1 cause varying voltage drops with varying load. However, these drops in the stator windings are relatively small, so $\mathbf{E}_1$ (and hence $\mathbf{I}_M$ and $\mathbf{B}_{net}$) is approximately constant with changes in load.

Figure 7–15a shows the induction motor at no load. At no load, the rotor slip is very small, and so the relative motion between the rotor and the magnetic fields is very small and the rotor frequency is also very small. Since the relative motion is small, the voltage $\mathbf{E}_R$ induced in the bars of the rotor is very small, and the resulting current flow $\mathbf{I}_R$ is small. Also, because the rotor frequency is so very small, the reactance of the rotor is nearly zero, and the maximum rotor current $\mathbf{I}_R$ is almost in phase with the rotor voltage $\mathbf{E}_R$. The rotor current thus produces a small magnetic field $\mathbf{B}_R$ at an angle just slightly greater than 90° behind the net magnetic field $\mathbf{B}_{net}$. Notice that the stator current must be quite large even at no load, since it must supply most of $\mathbf{B}_{net}$. (This is why induction motors have large no-load currents compared to other types of machines.)

The induced torque, which keeps the rotor turning, is given by the equation

$$\tau_{ind} = k\mathbf{B}_R \times \mathbf{B}_{net} \qquad (4\text{–}60)$$

Its magnitude is given by

$$\tau_{ind} = kB_R B_{net} \sin \delta \qquad (4\text{–}61)$$

Since the rotor magnetic field is very small, the induced torque is also quite small—just large enough to overcome the motor's rotational losses.

Now suppose the induction motor is loaded down (Figure 7–15b). As the motor's load increases, its slip increases, and the rotor speed falls. Since the rotor speed is slower, there is now *more relative motion* between the rotor and the stator magnetic fields in the machine. Greater relative motion produces a stronger rotor voltage $\mathbf{E}_R$ which in turn produces a larger rotor current $\mathbf{I}_R$. With a larger rotor current, the rotor magnetic field $\mathbf{B}_R$ also increases. However, the angle of the rotor current and $\mathbf{B}_R$ changes as well. Since the rotor slip is larger, the rotor frequency rises ($f_r = sf_e$), and the rotor's reactance increases (ωL_R). Therefore, the rotor current now lags further behind the rotor voltage, and the rotor magnetic field shifts with the current. Figure 7–15b shows the induction motor operating at a fairly high load. Notice that the rotor current has increased and that the angle δ has increased. The increase in B_R tends to increase the torque, while the increase in angle δ tends to decrease the torque (τ_{ind} is proportional to sin δ, and $\delta > 90°$). Since the first effect is larger than the second one, the overall induced torque increases to supply the motor's increased load.

When does an induction motor reach pullout torque? This happens when the point is reached where, as the load on the shaft is increased, the sin δ term decreases more than the B_R term increases. At that point, a further increase in load decreases τ_{ind}, and the motor stops.

It is possible to use a knowledge of the machine's magnetic fields to approximately derive the output torque-versus-speed characteristic of an induction motor. Remember that the magnitude of the induced torque in the machine is given by

$$\tau_{\text{ind}} = kB_R B_{\text{net}} \sin \delta \tag{4–61}$$

Each term in this expression can be considered separately to derive the overall machine behavior. The individual terms are

1. B_R. The rotor magnetic field is directly proportional to the current flowing in the rotor, as long as the rotor is unsaturated. The current flow in the rotor increases with increasing slip (decreasing speed) according to Equation (7–13). This current flow was plotted in Figure 7–11 and is shown again in Figure 7–16a.
2. B_{net}. The net magnetic field in the motor is proportional to E_1 and therefore is approximately constant (E_1 actually decreases with increasing current flow, but this effect is small compared to the other two, and it will be ignored in this graphical development). The curve for B_{net} versus speed is shown in Figure 7–16b.
3. sin δ. The angle δ between the net and rotor magnetic fields can be expressed in a very useful way. Look at Figure 7–15b. In this figure, it is clear that *the angle δ is just equal to the power-factor angle of the rotor plus* 90°:

$$\delta = \theta_R + 90° \tag{7–38}$$

Therefore, $\sin \delta = \sin (\theta_R + 90°) = \cos \theta_R$. This term is the power factor of the rotor. The rotor power-factor angle can be calculated from the equation

$$\theta_R = \tan^{-1} \frac{X_R}{R_R} = \tan^{-1} \frac{sX_{R0}}{R_R} \tag{7–39}$$

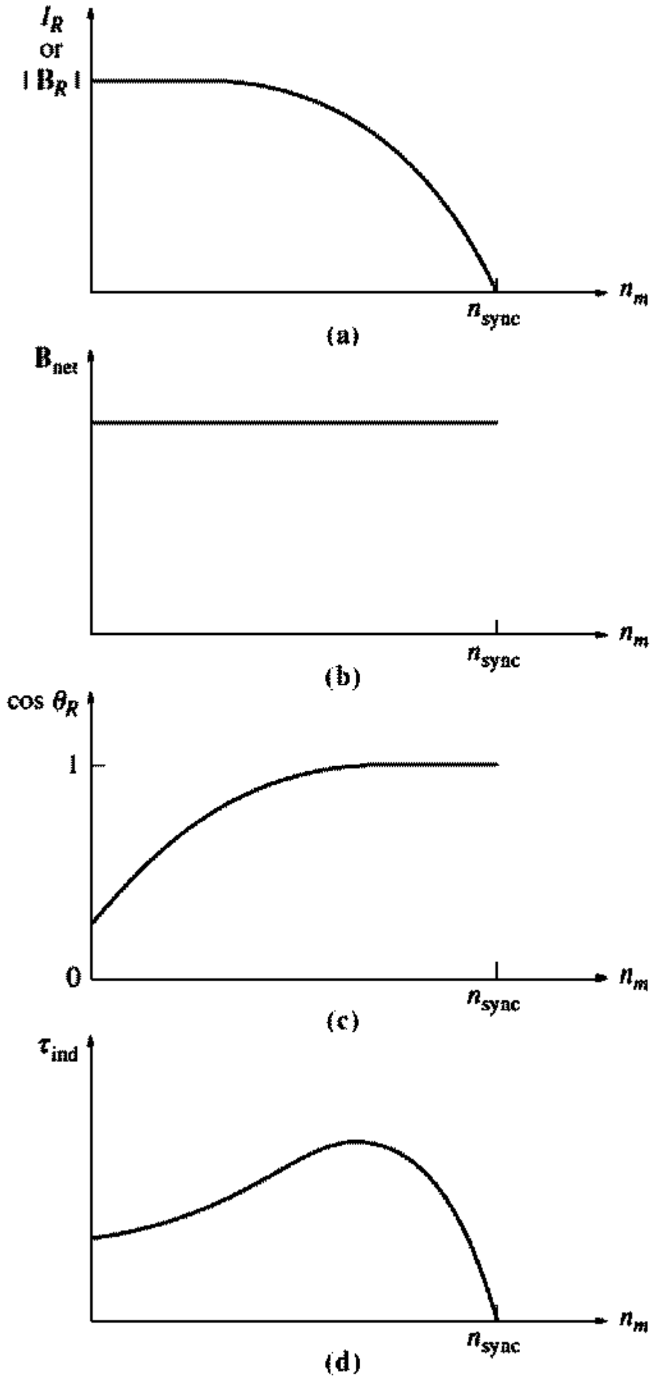


FIGURE 7–16
Graphical development of an induction motor torque–speed characteristic. (a) Plot of rotor current (and thus $|\mathbf{B}_R|$) versus speed for an induction motor; (b) plot of net magnetic field versus speed for the motor; (c) plot of rotor power factor versus speed for the motor; (d) the resulting torque–speed characteristic.

The resulting rotor power factor is given by

$$\mathrm{PF}_R = \cos\theta_R$$

$$\boxed{\mathrm{PF}_R = \cos\left(\tan^{-1}\frac{sX_{R0}}{R_R}\right)} \tag{7–40}$$

A plot of rotor power factor versus speed is shown in Figure 7–16c.

Since the induced torque is proportional to the product of these three terms, the torque–speed characteristic of an induction motor can be constructed from the graphical multiplication of the previous three plots (Figure 7–16a to c). The torque–speed characteristic of an induction motor derived in this fashion is shown in Figure 7–16d.

This characteristic curve can be divided roughly into three regions. The first region is the *low-slip region* of the curve. In the low-slip region, the motor slip increases approximately linearly with increased load, and the rotor mechanical speed decreases approximately linearly with load. In this region of operation, the rotor reactance is negligible, so the rotor power factor is approximately unity, while the rotor current increases linearly with slip. *The entire normal steady-state operating range of an induction motor is included in this linear low-slip region.* Thus in normal operation, an induction motor has a linear speed droop.

The second region on the induction motor's curve can be called the *moderate-slip region*. In the moderate-slip region, the rotor frequency is higher than before, and the rotor reactance is on the same order of magnitude as the rotor resistance. In this region, the rotor current no longer increases as rapidly as before, and the power factor starts to drop. The peak torque (the *pullout torque*) of the motor occurs at the point where, for an incremental increase in load, the increase in the rotor current is exactly balanced by the decrease in the rotor power factor.

The third region on the induction motor's curve is called the *high-slip region*. In the high-slip region, the induced torque actually decreases with increased load, since the increase in rotor current is completely overshadowed by the decrease in rotor power factor.

For a typical induction motor, the pullout torque on the curve will be 200 to 250 percent of the rated full-load torque of the machine, and the *starting torque* (the torque at zero speed) will be 150 percent or so of the full-load torque. Unlike a synchronous motor, the induction motor can start with a full load attached to its shaft.

The Derivation of the Induction Motor Induced-Torque Equation

It is possible to use the equivalent circuit of an induction motor and the power-flow diagram for the motor to derive a general expression for induced torque as a function of speed. The induced torque in an induction motor is given by Equation (7–35) or (7–36):

$$\tau_{\text{ind}} = \frac{P_{\text{conv}}}{\omega_m} \tag{7–35}$$

$$\tau_{\text{ind}} = \frac{P_{\text{AG}}}{\omega_{\text{sync}}} \tag{7–36}$$

The latter equation is especially useful, since the synchronous speed is a constant for a given frequency and number of poles. Since ω_{sync} is constant, a knowledge of the air-gap power gives the induced torque of the motor.

The air-gap power is the power crossing the gap from the stator circuit to the rotor circuit. It is equal to the power absorbed in the resistance R_2/s. How can this power be found?

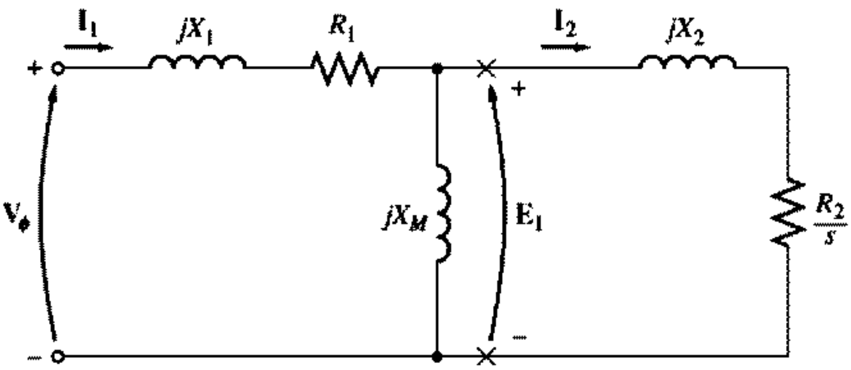


FIGURE 7–17
Per-phase equivalent circuit of an induction motor.

Refer to the equivalent circuit given in Figure 7–17. In this figure, the air-gap power supplied to one phase of the motor can be seen to be

$$P_{AG,1\phi} = I_2^2 \frac{R_2}{s}$$

Therefore, the total air-gap power is

$$P_{AG} = 3I_2^2 \frac{R_2}{s}$$

If I_2 can be determined, then the air-gap power and the induced torque will be known.

Although there are several ways to solve the circuit in Figure 7–17 for the current I_2, perhaps the easiest one is to determine the Thevenin equivalent of the portion of the circuit to the left of the *X*'s in the figure. Thevenin's theorem states that any linear circuit that can be separated by two terminals from the rest of the system can be replaced by a single voltage source in series with an equivalent impedance. If this were done to the induction motor equivalent circuit, the resulting circuit would be a simple series combination of elements as shown in Figure 7–18c.

To calculate the Thevenin equivalent of the input side of the induction motor equivalent circuit, first open-circuit the terminals at the *X*'s and find the resulting open-circuit voltage present there. Then, to find the Thevenin impedance, kill (short-circuit) the phase voltage and find the Z_{eq} seen "looking" into the terminals.

Figure 7–18a shows the open terminals used to find the Thevenin voltage. By the voltage divider rule,

$$\begin{aligned}\mathbf{V}_{TH} &= \mathbf{V}_\phi \frac{Z_M}{Z_M + Z_1} \\ &= \mathbf{V}_\phi \frac{jX_M}{R_1 + jX_1 + jX_M}\end{aligned}$$

The magnitude of the Thevenin voltage $\mathbf{V}_{TH}$ is

$$\boxed{V_{TH} = V_\phi \frac{X_M}{\sqrt{R_1^2 + (X_1 + X_M)^2}}} \qquad (7\text{–}41a)$$

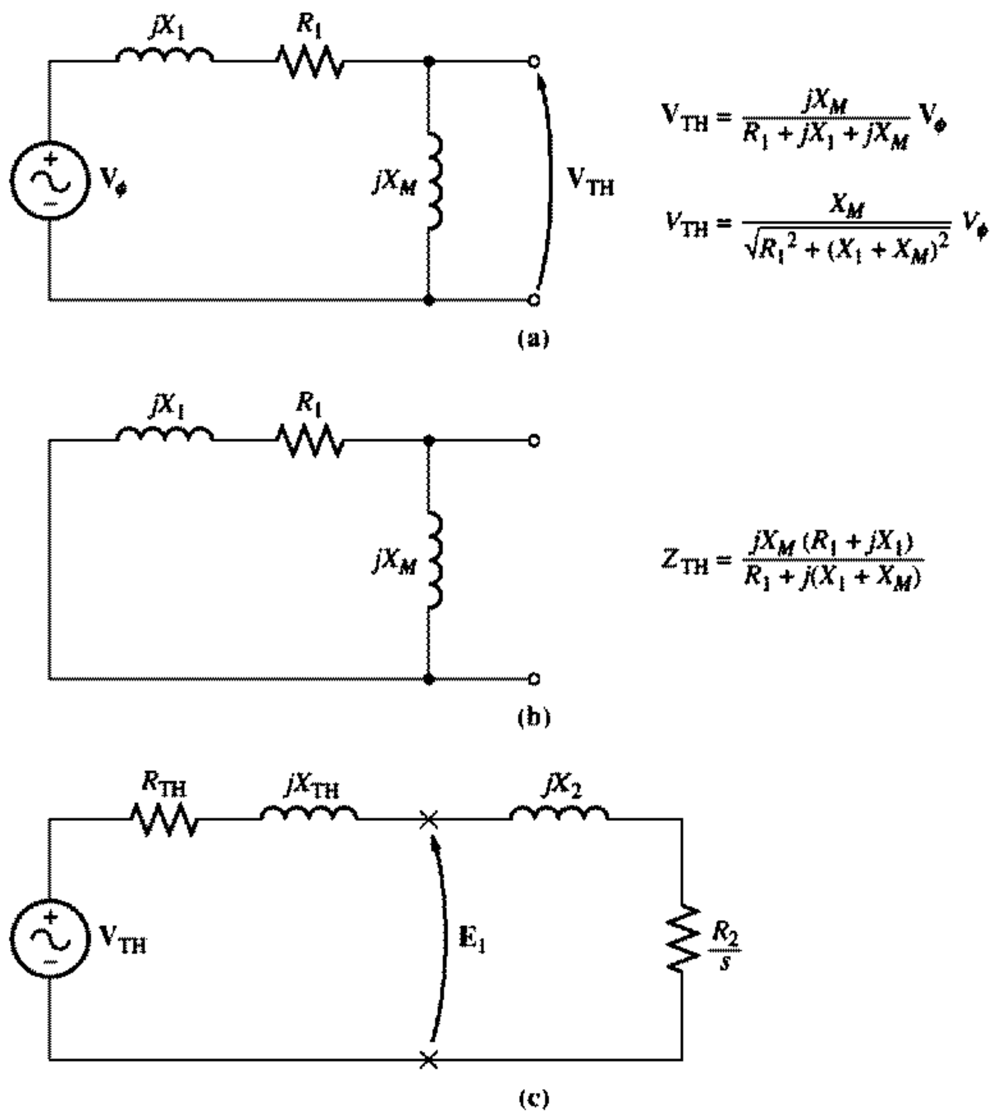


FIGURE 7–18
(a) The Thevenin equivalent voltage of an induction motor input circuit. (b) The Thevenin equivalent impedance of the input circuit. (c) The resulting simplified equivalent circuit of an induction motor.

Since the magnetization reactance $X_M >> X_1$ and $X_M >> R_1$, the magnitude of the Thevenin voltage is approximately

$$\boxed{V_{TH} \approx V_\phi \frac{X_M}{X_1 + X_M}} \tag{7–41b}$$

to quite good accuracy.

Figure 7–18b shows the input circuit with the input voltage source killed. The two impedances are in parallel, and the Thevenin impedance is given by

$$Z_{TH} = \frac{Z_1 Z_M}{Z_1 + Z_M} \tag{7–42}$$

This impedance reduces to

$$Z_{\mathrm{TH}} = R_{\mathrm{TH}} + jX_{\mathrm{TH}} = \frac{jX_M(R_1 + jX_1)}{R_1 + j(X_1 + X_M)} \tag{7–43}$$

Because $X_M >> X_1$ and $X_M + X_1 >> R_1$, the Thevenin resistance and reactance are approximately given by

$$\boxed{R_{\mathrm{TH}} \approx R_1\left(\frac{X_M}{X_1 + X_M}\right)^2} \tag{7–44}$$

$$\boxed{X_{\mathrm{TH}} \approx X_1} \tag{7–45}$$

The resulting equivalent circuit is shown in Figure 7–18c. From this circuit, the current $\mathbf{I}_2$ is given by

$$\mathbf{I}_2 = \frac{\mathbf{V}_{\mathrm{TH}}}{Z_{\mathrm{TH}} + Z_2} \tag{7–46}$$

$$= \frac{\mathbf{V}_{\mathrm{TH}}}{R_{\mathrm{TH}} + R_2/s + jX_{TH} + jX_2} \tag{7–47}$$

The magnitude of this current is

$$I_2 = \frac{V_{\mathrm{TH}}}{\sqrt{(R_{\mathrm{TH}} + R_2/s)^2 + (X_{\mathrm{TH}} + X_2)^2}} \tag{7–48}$$

The air-gap power is therefore given by

$$P_{\mathrm{AG}} = 3I_2^2\frac{R_2}{s}$$

$$= \frac{3V_{\mathrm{TH}}^2 R_2/s}{(R_{\mathrm{TH}} + R_2/s)^2 + (X_{\mathrm{TH}} + X_2)^2} \tag{7–49}$$

and the rotor-induced torque is given by

$$\tau_{\mathrm{ind}} = \frac{P_{\mathrm{AG}}}{\omega_{\mathrm{sync}}}$$

$$\boxed{\tau_{\mathrm{ind}} = \frac{3V_{\mathrm{TH}}^2 R_2/s}{\omega_{\mathrm{sync}}[(R_{\mathrm{TH}} + R_2/s)^2 + (X_{\mathrm{TH}} + X_2)^2]}} \tag{7–50}$$

A plot of induction motor torque as a function of speed (and slip) is shown in Figure 7–19, and a plot showing speeds both above and below the normal motor range is shown in Figure 7–20.

Comments on the Induction Motor Torque–Speed Curve

The induction motor torque–speed characteristic curve plotted in Figures 7–19 and 7–20 provides several important pieces of information about the operation of induction motors. This information is summarized as follows:

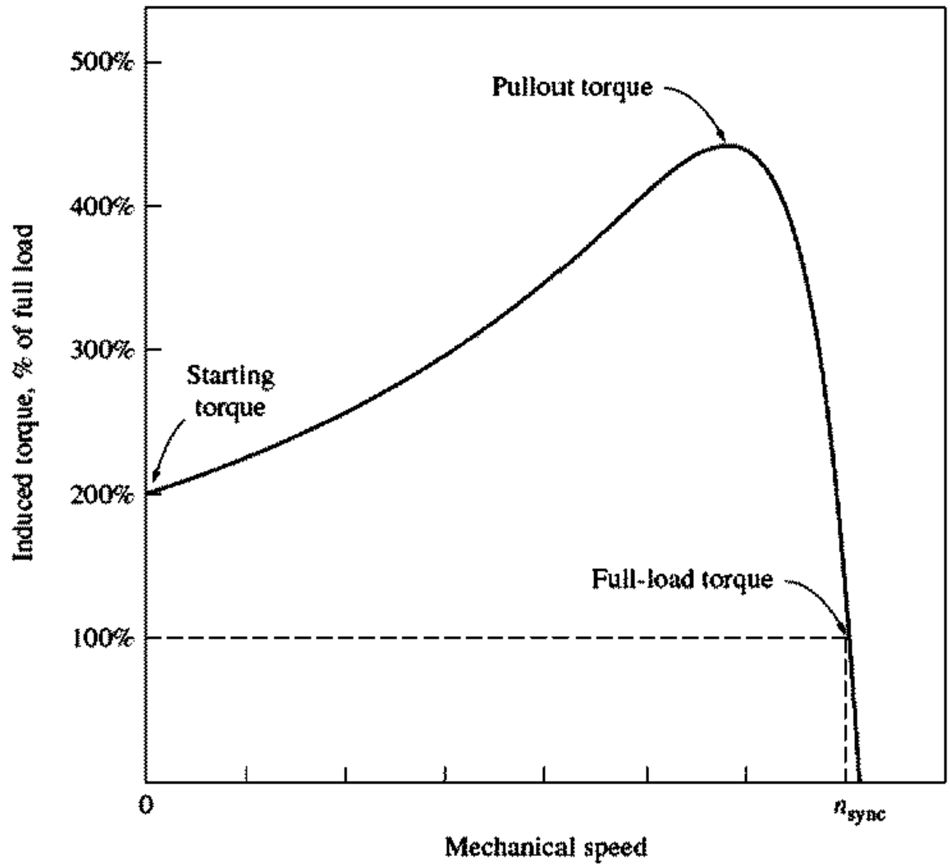


FIGURE 7–19
A typical induction motor torque–speed characteristic curve.

1. The induced torque of the motor is zero at synchronous speed. This fact has been discussed previously.
2. The torque–speed curve is nearly linear between no load and full load. In this range, the rotor resistance is much larger than the rotor reactance, so the rotor current, the rotor magnetic field, and the induced torque increase linearly with increasing slip.
3. There is a maximum possible torque that cannot be exceeded. This torque, called the *pullout torque* or *breakdown torque*, is 2 to 3 times the rated full-load torque of the motor. The next section of this chapter contains a method for calculating pullout torque.
4. The starting torque on the motor is slightly larger than its full-load torque, so this motor will start carrying any load that it can supply at full power.
5. Notice that the torque on the motor for a given slip varies as the square of the applied voltage. This fact is useful in one form of induction motor speed control that will be described later.
6. If the rotor of the induction motor is driven faster than synchronous speed, then the direction of the induced torque in the machine reverses and the machine becomes a *generator*, converting mechanical power to electric power. The use of induction machines as generators will be described later.

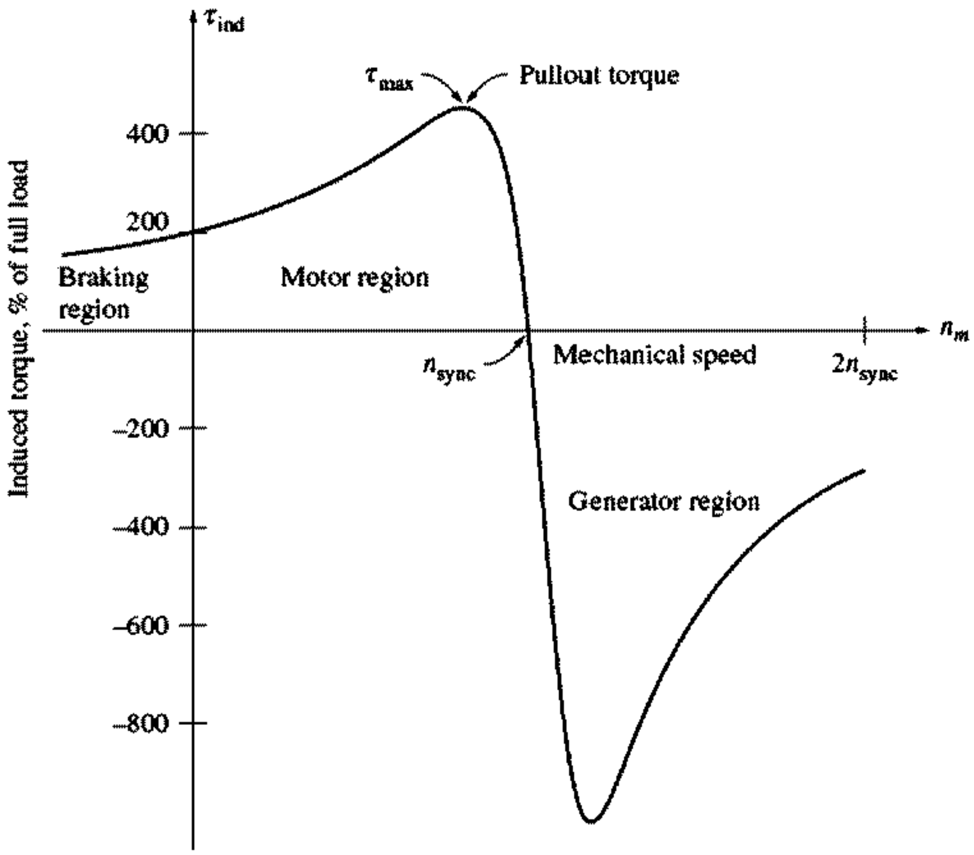


FIGURE 7–20
Induction motor torque–speed characteristic curve, showing the extended operating ranges (braking region and generator region).

7. If the motor is turning backward relative to the direction of the magnetic fields, the induced torque in the machine will stop the machine very rapidly and will try to rotate it in the other direction. Since reversing the direction of magnetic field rotation is simply a matter of switching any two stator phases, this fact can be used as a way to very rapidly stop an induction motor. The act of switching two phases in order to stop the motor very rapidly is called *plugging*.

The power converted to mechanical form in an induction motor is equal to

$$P_{conv} = \tau_{ind}\,\omega_m$$

and is shown plotted in Figure 7–21. Notice that the peak power supplied by the induction motor occurs at a different speed than the maximum torque; and, of course, no power is converted to mechanical form when the rotor is at zero speed.

Maximum (Pullout) Torque in an Induction Motor

Since the induced torque is equal to P_{AG}/ω_{sync}, the maximum possible torque occurs when the air-gap power is maximum. Since the air-gap power is equal to the power consumed in the resistor R_2/s, the *maximum induced torque will occur when the power consumed by that resistor is maximum.*

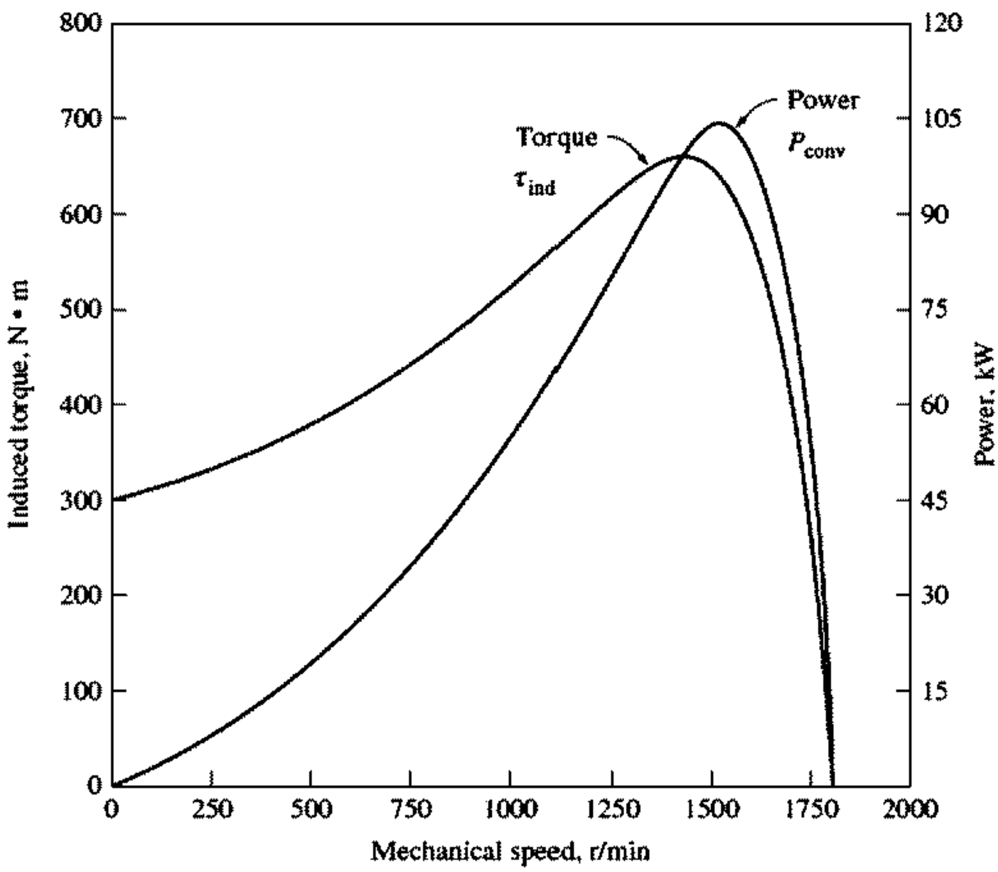


FIGURE 7–21
Induced torque and power converted versus motor speed in revolutions per minute for an example four-pole induction motor.

When is the power supplied to R_2/s at its maximum? Refer to the simplified equivalent circuit in Figure 7–18c. In a situation where the angle of the load impedance is fixed, the maximum power transfer theorem states that maximum power transfer to the load resistor R_2/s will occur when the *magnitude* of that impedance is equal to the *magnitude* of the source impedance. The equivalent source impedance in the circuit is

$$Z_{source} = R_{TH} + jX_{TH} + jX_2 \tag{7–51}$$

so the maximum power transfer occurs when

$$\frac{R_2}{s} = \sqrt{R_{TH}^2 + (X_{TH} + X_2)^2} \tag{7–52}$$

Solving Equation (7–52) for slip, we see that *the slip at pullout torque is given by*

$$\boxed{s_{max} = \frac{R_2}{\sqrt{R_{TH}^2 + (X_{TH} + X_2)^2}}} \tag{7–53}$$

Notice that the referred rotor resistance R_2 appears only in the numerator, so the slip of the rotor at maximum torque is directly proportional to the rotor resistance.

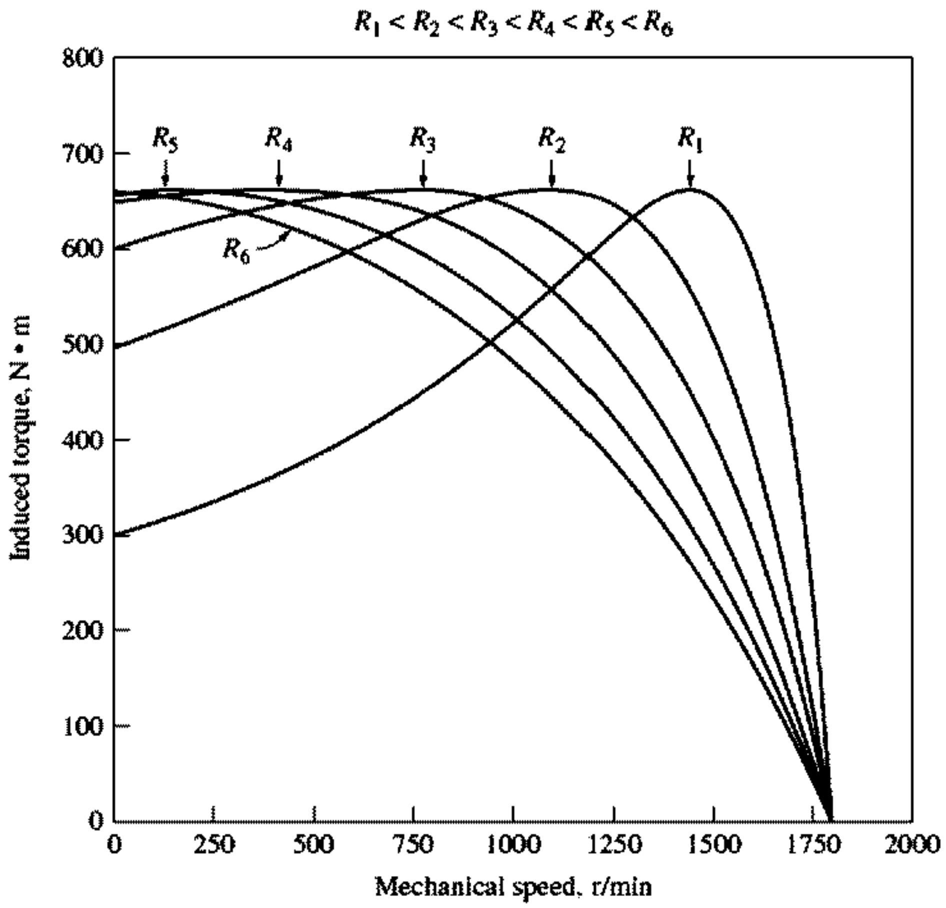


FIGURE 7–22
The effect of varying rotor resistance on the torque–speed characteristic of a wound-rotor induction motor.

The value of the maximum torque can be found by inserting the expression for the slip at maximum torque into the torque equation [Equation (7–50)]. The resulting equation for the maximum or pullout torque is

$$\boxed{\tau_{\max} = \frac{3V_{\mathrm{TH}}^2}{2\omega_{\mathrm{sync}}[R_{\mathrm{TH}} + \sqrt{R_{\mathrm{TH}}^2 + (X_{\mathrm{TH}} + X_2)^2}]}} \qquad (7\text{–}54)$$

This torque is proportional to the square of the supply voltage and is also inversely related to the size of the stator impedances and the rotor reactance. The smaller a machine's reactances, the larger the maximum torque it is capable of achieving. Note that *slip* at which the maximum torque occurs is directly proportional to rotor resistance [Equation (7–53)], but the *value* of the maximum torque is independent of the value of rotor resistance [Equation (7–54)].

The torque–speed characteristic for a wound-rotor induction motor is shown in Figure 7–22. Recall that it is possible to insert resistance into the rotor circuit of a wound rotor because the rotor circuit is brought out to the stator through slip rings. Notice on the figure that as the rotor resistance is increased, the pullout speed of the motor decreases, but the maximum torque remains constant.

It is possible to take advantage of this characteristic of wound-rotor induction motors to start very heavy loads. If a resistance is inserted into the rotor circuit, the maximum torque can be adjusted to occur at starting conditions. Therefore, the maximum possible torque would be available to start heavy loads. On the other hand, once the load is turning, the extra resistance can be removed from the circuit, and the maximum torque will move up to near-synchronous speed for regular operation.

Example 7–4. A two-pole, 50-Hz induction motor supplies 15 kW to a load at a speed of 2950 r/min.

(a) What is the motor's slip?
(b) What is the induced torque in the motor in N • m under these conditions?
(c) What will the operating speed of the motor be if its torque is doubled?
(d) How much power will be supplied by the motor when the torque is doubled?

Solution

(a) The synchronous speed of this motor is

$$n_{\text{sync}} = \frac{120f_e}{P} = \frac{120(50\text{ Hz})}{2\text{ poles}} = 3000\text{ r/min}$$

Therefore, the motor's slip is

$$s = \frac{n_{\text{sync}} - n_{\text{m}}}{n_{\text{sync}}}(\times 100\%) \tag{7–4}$$

$$= \frac{3000\text{ r/min} - 2950\text{ r/min}}{3000\text{ r/min}}(\times 100\%)$$

$$= 0.0167 \text{ or } 1.67\%$$

(b) The induced torque in the motor must be assumed equal to the load torque, and P_{conv} must be assumed equal to P_{load}, since no value was given for mechanical losses. The torque is thus

$$\tau_{\text{ind}} = \frac{P_{\text{conv}}}{\omega_{\text{m}}}$$

$$= \frac{15\text{ kW}}{(2950\text{ r/min})(2\pi\text{ rad/r})(1\text{ min}/60\text{ s})}$$

$$= 48.6\text{ N} \cdot \text{m}$$

(c) In the low-slip region, the torque–speed curve is linear, and the induced torque is directly proportional to slip. Therefore, if the torque doubles, then the new slip will be 3.33 percent. The operating speed of the motor is thus

$$n_m = (1 - s)n_{\text{sync}} = (1 - 0.0333)(3000\text{ r/min}) = 2900\text{ r/min}$$

(d) The power supplied by the motor is given by

$$P_{\text{conv}} = \tau_{\text{ind}}\omega_m$$

$$= (97.2\text{ N} \cdot \text{m})(2900\text{ r/min})(2\pi\text{ rad/r})(1\text{ min}/60\text{ s})$$

$$= 29.5\text{ kW}$$

Example 7–5. A 460-V, 25-hp, 60-Hz, four-pole, Y-connected wound-rotor induction motor has the following impedances in ohms per phase referred to the stator circuit:

$$R_1 = 0.641\ \Omega \qquad R_2 = 0.332\ \Omega$$
$$X_1 = 1.106\ \Omega \qquad X_2 = 0.464\ \Omega \qquad X_M = 26.3\ \Omega$$

(a) What is the maximum torque of this motor? At what speed and slip does it occur?

(b) What is the starting torque of this motor?

(c) When the rotor resistance is doubled, what is the speed at which the maximum torque now occurs? What is the new starting torque of the motor?

(d) Calculate and plot the torque–speed characteristics of this motor both with the original rotor resistance and with the rotor resistance doubled.

Solution

The Thevenin voltage of this motor is

$$V_{TH} = V_\phi \frac{X_M}{\sqrt{R_1^2 + (X_1 + X_M)^2}} \tag{7–41a}$$

$$= \frac{(266\ \text{V})(26.3\ \Omega)}{\sqrt{(0.641\ \Omega)^2 + (1.106\ \Omega + 26.3\ \Omega)^2}} = 255.2\ \text{V}$$

The Thevenin resistance is

$$R_{TH} \approx R_1\left(\frac{X_M}{X_1 + X_M}\right)^2 \tag{7–44}$$

$$\approx (0.641\ \Omega)\left(\frac{26.3\ \Omega}{1.106\ \Omega + 26.3\ \Omega}\right)^2 = 0.590\ \Omega$$

The Thevenin reactance is

$$X_{TH} \approx X_1 = 1.106\ \Omega$$

(a) The slip at which maximum torque occurs is given by Equation (7–53):

$$s_{max} = \frac{R_2}{\sqrt{R_{TH}^2 + (X_{TH} + X_2)^2}} \tag{7–53}$$

$$= \frac{0.332\ \Omega}{\sqrt{(0.590\ \Omega)^2 + (1.106\ \Omega + 0.464\ \Omega)^2}} = 0.198$$

This corresponds to a mechanical speed of

$$n_m = (1 - s)n_{sync} = (1 - 0.198)(1800\ \text{r/min}) = 1444\ \text{r/min}$$

The torque at this speed is

$$\tau_{max} = \frac{3V_{TH}^2}{2\omega_{sync}\left[R_{TH} + \sqrt{R_{TH}^2 + (X_{TH} + X_2)^2}\right]} \tag{7–54}$$

$$= \frac{3(255.2\ \text{V})^2}{2(188.5\ \text{rad/s})[0.590\ \Omega + \sqrt{(0.590\ \Omega)^2 + (1.106\ \Omega + 0.464\ \Omega)^2}]}$$

$$= 229\ \text{N} \cdot \text{m}$$

(*b*) The starting torque of this motor is found by setting $s = 1$ in Equation (7–50):

$$\tau_{\text{start}} = \frac{3V_{\text{TH}}^2 R_2}{\omega_{\text{sync}}[(R_{\text{TH}} + R_2)^2 + (X_{\text{TH}} + X_2)^2]}$$

$$= \frac{3(255.2\text{ V})^2(0.332\ \Omega)}{(188.5\text{ rad/s})[(0.590\ \Omega + 0.332\ \Omega)^2 + (1.106\ \Omega + 0.464\ \Omega)^2]}$$

$$= 104\text{ N}\cdot\text{m}$$

(*c*) If the rotor resistance is doubled, then the slip at maximum torque doubles, too. Therefore,

$$s_{\max} = 0.396$$

and the speed at maximum torque is

$$n_m = (1 - s)n_{\text{sync}} = (1 - 0.396)(1800\text{ r/min}) = 1087\text{ r/min}$$

The maximum torque is still

$$\tau_{\max} = 229\text{ N}\cdot\text{m}$$

The starting torque is now

$$\tau_{\text{start}} = \frac{3(255.2\text{ V})^2(0.664\ \Omega)}{(188.5\text{ rad/s})[(0.590\ \Omega + 0.664\ \Omega)^2 + (1.106\ \Omega + 0.464\ \Omega)^2]}$$

$$= 170\text{ N}\cdot\text{m}$$

(*d*) We will create a MATLAB M-file to calculate and plot the torque–speed characteristic of the motor both with the original rotor resistance and with the doubled rotor resistance. The M-file will calculate the Thevenin impedance using the exact equations for V_{TH} and Z_{TH} [Equations (7–41a) and (7–43)] instead of the approximate equations, because the computer can easily perform the exact calculations. It will then calculate the induced torque using Equation (7–50) and plot the results. The resulting M-file is shown below:

```
% M-file: torque_speed_curve.m
% M-file create a plot of the torque-speed curve of the
%   induction motor of Example 7-5.

% First, initialize the values needed in this program.
r1 = 0.641;                 % Stator resistance
x1 = 1.106;                 % Stator reactance
r2 = 0.332;                 % Rotor resistance
x2 = 0.464;                 % Rotor reactance
xm = 26.3;                  % Magnetization branch reactance
v_phase = 460 / sqrt(3);    % Phase voltage
n_sync = 1800;              % Synchronous speed (r/min)
w_sync = 188.5;             % Synchronous speed (rad/s)

% Calculate the Thevenin voltage and impedance from Equations
% 7-41a and 7-43.
v_th = v_phase * ( xm / sqrt(r1^2 + (x1 + xm)^2) );
z_th = ((j*xm) * (r1 + j*x1)) / (r1 + j*(x1 + xm));
r_th = real(z_th);
x_th = imag(z_th);
```

```
% Now calculate the torque-speed characteristic for many
% slips between 0 and 1.  Note that the first slip value
% is set to 0.001 instead of exactly 0 to avoid divide-
% by-zero problems.
s = (0:1:50) / 50;                    % Slip
s(1) = 0.001;
nm = (1 - s) * n_sync;                % Mechanical speed

% Calculate torque for original rotor resistance
for ii = 1:51
  t_ind1(ii) = (3 * v_th^2 * r2 / s(ii)) / ...
         (w_sync * ((r_th + r2/s(ii))^2 + (x_th + x2)^2) );
end

% Calculate torque for doubled rotor resistance
for ii = 1:51
  t_ind2(ii) = (3 * v_th^2 * (2*r2) / s(ii)) / ...
         (w_sync * ((r_th + (2*r2)/s(ii))^2 + (x_th + x2)^2) );
end

% Plot the torque-speed curve
plot(nm,t_ind1,'Color','k','LineWidth',2.0);
hold on;
plot(nm,t_ind2,'Color','k','LineWidth',2.0,'LineStyle','-.');
xlabel('\itn_{m}','Fontweight','Bold');
ylabel('\tau_{ind}','Fontweight','Bold');
title ('Induction motor torque-speed characteristic',...
   'Fontweight','Bold');
legend ('Original R_{2}','Doubled R_{2}');
grid on;
hold off;
```

The resulting torque–speed characteristics are shown in Figure 7–23. Note that the peak torque and starting torque values on the curves match the calculations of parts (*a*) through (*c*). Also, note that the starting torque of the motor rose as R_2 increased.

7.6 VARIATIONS IN INDUCTION MOTOR TORQUE–SPEED CHARACTERISTICS

Section 7.5 contained the derivation of the torque–speed characteristic for an induction motor. In fact, several characteristic curves were shown, depending on the rotor resistance. Example 7–5 illustrated an induction motor designer's dilemma—if a rotor is designed with high resistance, then the motor's starting torque is quite high, but the slip is also quite high at normal operating conditions. Recall that $P_{\text{conv}} = (1 - s)P_{\text{AG}}$, so *the higher the slip, the smaller the fraction of air-gap power actually converted to mechanical form*, and thus the lower the motor's efficiency. A motor with high rotor resistance has a good starting torque but poor efficiency at normal operating conditions. On the other hand, a motor with low rotor resistance has a low starting torque and high starting current, but its efficiency at normal operating conditions is quite high. An induction motor designer

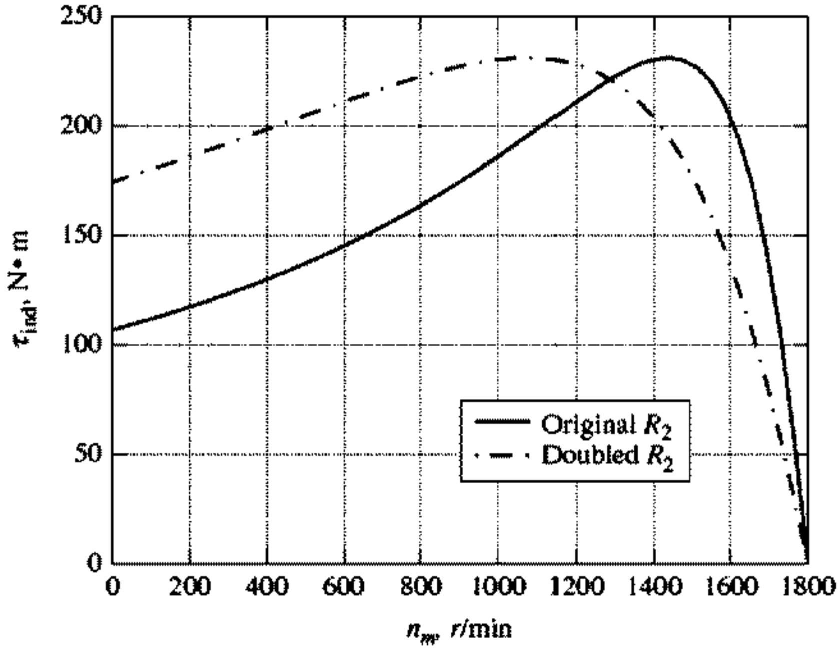


FIGURE 7–23
Torque–speed characteristics for the motor of Example 7–5.

is forced to compromise between the conflicting requirements of high starting torque and good efficiency.

One possible solution to this difficulty was suggested in passing in Section 7.5: use a wound-rotor induction motor and insert extra resistance into the rotor during starting. The extra resistance could be completely removed for better efficiency during normal operation. Unfortunately, wound-rotor motors are more expensive, need more maintenance, and require a more complex automatic control circuit than cage rotor motors. Also, it is sometimes important to completely seal a motor when it is placed in a hazardous or explosive environment, and this is easier to do with a completely self-contained rotor. It would be nice to figure out some way to add extra rotor resistance at starting and to remove it during normal running without slip rings and *without operator or control circuit intervention.*

Figure 7–24 illustrates the desired motor characteristic. This figure shows two wound-rotor motor characteristics, one with high resistance and one with low resistance. At high slips, the desired motor should behave like the high-resistance wound-rotor motor curve; at low slips, it should behave like the low-resistance wound-rotor motor curve.

Fortunately, it is possible to accomplish just this effect by properly taking advantage of *leakage reactance* in induction motor rotor design.

Control of Motor Characteristics by Cage Rotor Design

The reactance X_2 in an induction motor equivalent circuit represents the referred form of the rotor's leakage reactance. Recall that leakage reactance is the reactance due to the rotor flux lines that do not also couple with the stator windings. In

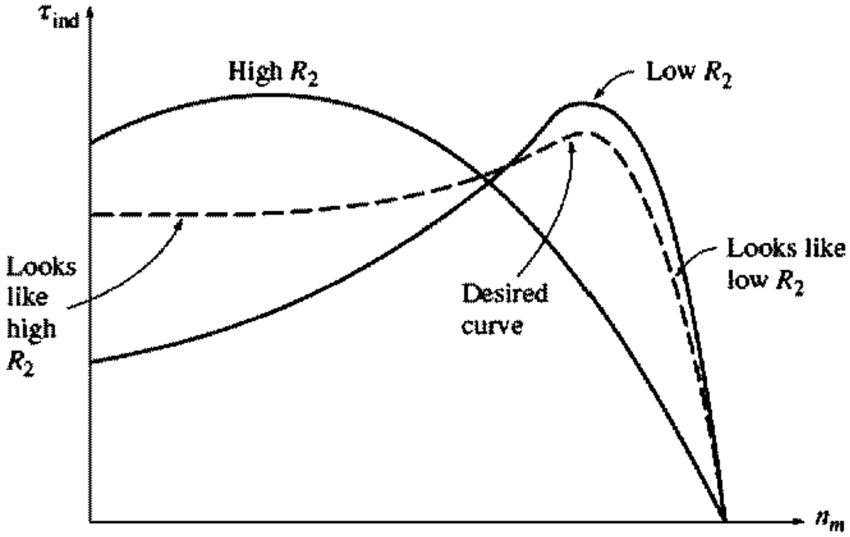


FIGURE 7–24
A torque–speed characteristic curve combining high-resistance effects at low speeds (high slip) with low-resistance effects at high speed (low slip).

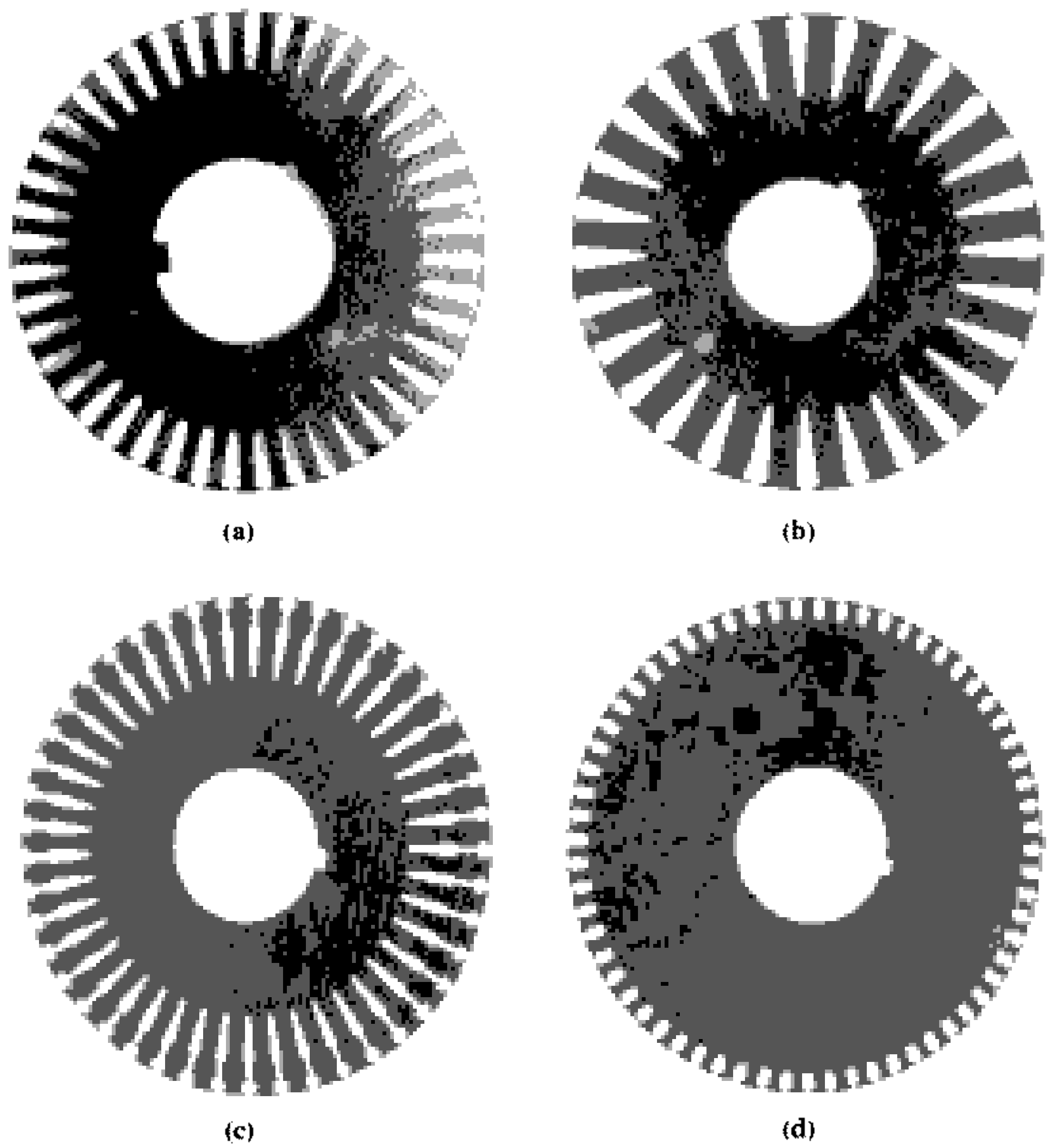
general, the farther away from the stator a rotor bar or part of a bar is, the greater its leakage reactance, since a smaller percentage of the bar's flux will reach the stator. Therefore, if the bars of a cage rotor are placed near the surface of the rotor, they will have only a small leakage flux and the reactance X_2 will be small in the equivalent circuit. On the other hand, if the rotor bars are placed deeper into the rotor surface, there will be more leakage and the rotor reactance X_2 will be larger.

For example, Figure 7–25a is a photograph of a rotor lamination showing the cross section of the bars in the rotor. The rotor bars in the figure are quite large and are placed near the surface of the rotor. Such a design will have a low resistance (due to its large cross section) and a low leakage reactance and X_2 (due to the bar's location near the stator). Because of the low rotor resistance, the pullout torque will be quite near synchronous speed [see Equation (7–53)], and the motor will be quite efficient. Remember that

$$P_{\text{conv}} = (1 - s)P_{\text{AG}} \tag{7–33}$$

so very little of the air-gap power is lost in the rotor resistance. However, since R_2 is small, the motor's starting torque will be small, and its starting current will be high. This type of design is called the National Electrical Manufacturers Association (NEMA) design class A. It is more or less a typical induction motor, and its characteristics are basically the same as those of a wound-rotor motor with no extra resistance inserted. Its torque–speed characteristic is shown in Figure 7–26.

Figure 7–25d, however, shows the cross section of an induction motor rotor with *small* bars placed near the surface of the rotor. Since the cross-sectional area of the bars is small, the rotor resistance is relatively high. Since the bars are located near the stator, the rotor leakage reactance is still small. This motor is very much like a wound-rotor induction motor with extra resistance inserted into the rotor. Because of the large rotor resistance, this motor has a pullout torque occur-



(a) (b) (c) (d)

FIGURE 7–25
Laminations from typical cage induction motor rotors, showing the cross section of the rotor bars: (a) NEMA design class A—large bars near the surface; (b) NEMA design class B—large, deep rotor bars; (c) NEMA design class C—double-cage rotor design; (d) NEMA design class D—small bars near the surface. (*Courtesy of MagneTek, Inc.*)

ring at a high slip, and its starting torque is quite high. A cage motor with this type of rotor construction is called NEMA design class D. Its torque–speed characteristic is also shown in Figure 7–26.

Deep-Bar and Double-Cage Rotor Designs

Both of the previous rotor designs are essentially similar to a wound-rotor motor with a set rotor resistance. How can a *variable* rotor resistance be produced to combine the high starting torque and low starting current of a class D design with the low normal operating slip and high efficiency of a class A design?

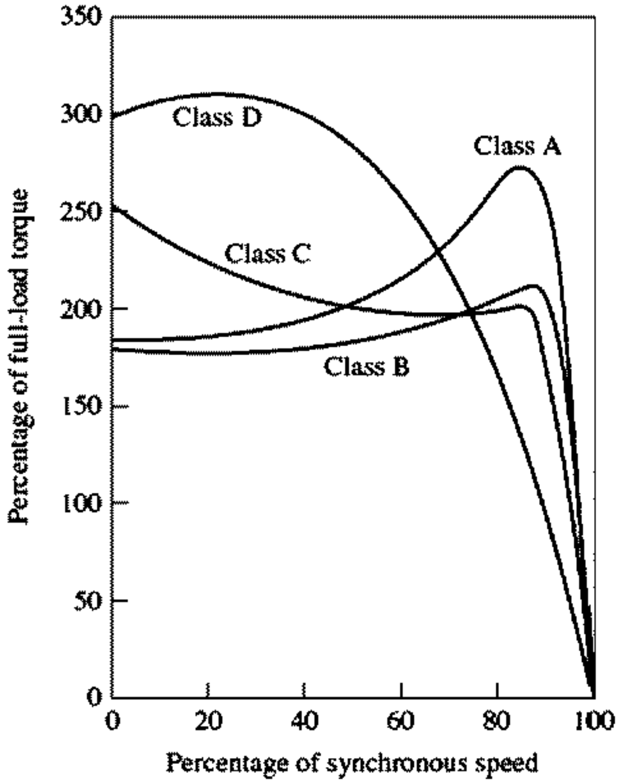


FIGURE 7–26
Typical torque–speed curves for different rotor designs.

It is possible to produce a variable rotor resistance by the use of deep rotor bars or double-cage rotors. The basic concept is illustrated with a deep-bar rotor in Figure 7–27. Figure 7–27a shows a current flowing through the upper part of a deep rotor bar. Since current flowing in that area is tightly coupled to the stator, the leakage inductance is small for this region. Figure 7–27b shows current flowing deeper in the bar. Here, the leakage inductance is higher. Since all parts of the rotor bar are in parallel electrically, the bar essentially represents a series of parallel electric circuits, the upper ones having a smaller inductance and the lower ones having a larger inductance (Figure 7–27c).

At low slip, the rotor's frequency is very small, and the reactances of all the parallel paths through the bar are small compared to their resistances. The impedances of all parts of the bar are approximately equal, so current flows through all parts of the bar equally. The resulting large cross-sectional area makes the rotor resistance quite small, resulting in good efficiency at low slips. At high slip (starting conditions), the reactances are large compared to the resistances in the rotor bars, so all the current is forced to flow in the low-reactance part of the bar near the stator. Since the *effective* cross section is lower, the rotor resistance is higher than before. With a high rotor resistance at starting conditions, the starting torque is relatively higher and the starting current is relatively lower than in a class A design. A typical torque–speed characteristic for this construction is the design class B curve in Figure 7–26.

A cross-sectional view of a double-cage rotor is shown in Figure 7–25c. It consists of a large, low-resistance set of bars buried deeply in the rotor and a small, high-resistance set of bars set at the rotor surface. It is similar to the deep-bar rotor, except that the difference between low-slip and high-slip operation is

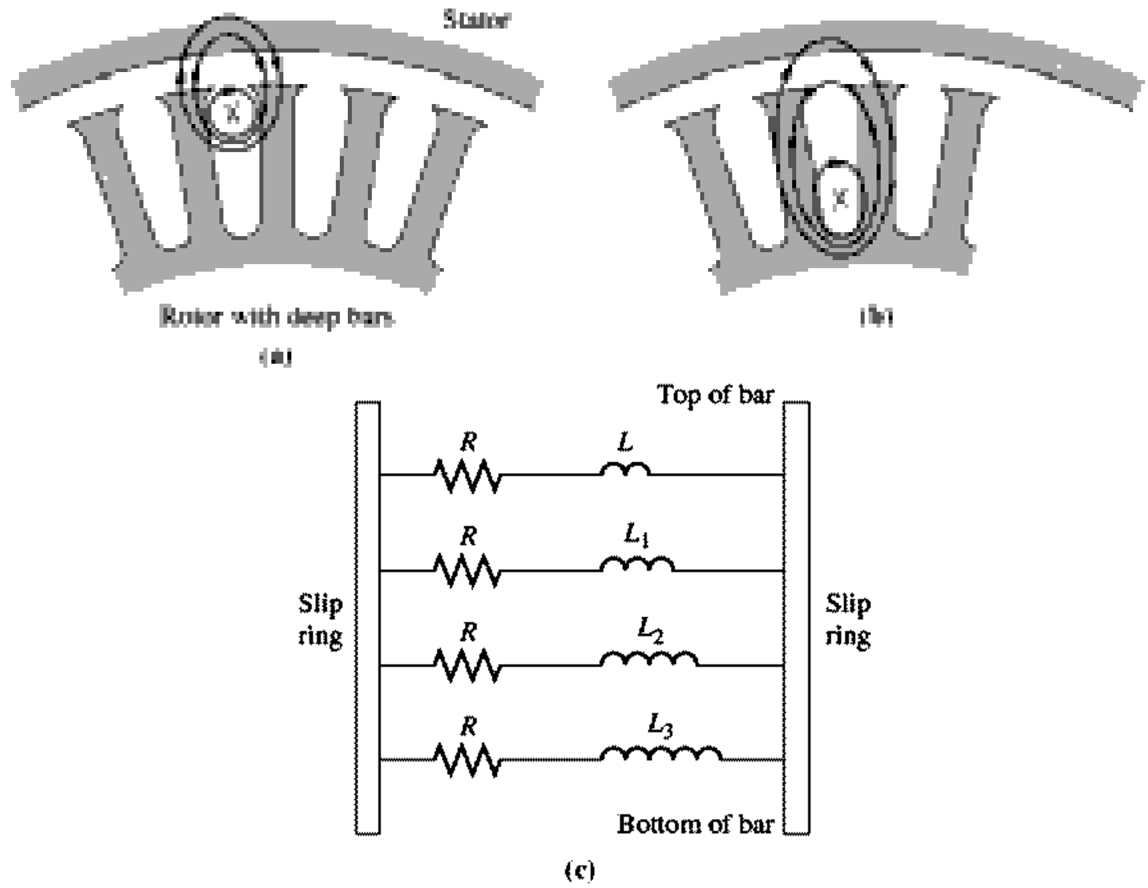


FIGURE 7–27
Flux linkage in a deep-bar rotor. (a) For a current flowing in the top of the bar, the flux is tightly linked to the stator, and leakage inductance is small; (b) for a current flowing in the bottom of the bar, the flux is loosely linked to the stator, and leakage inductance is large; (c) resulting equivalent circuit of the rotor bar as a function of depth in the rotor.

even more exaggerated. At starting conditions, only the small bar is effective, and the rotor resistance is *quite* high. This high resistance results in a large starting torque. However, at normal operating speeds, both bars are effective, and the resistance is almost as low as in a deep-bar rotor. Double-cage rotors of this sort are used to produce NEMA class B and class C characteristics. Possible torque–speed characteristics for a rotor of this design are designated design class B and design class C in Figure 7–26.

Double-cage rotors have the disadvantage that they are more expensive than the other types of cage rotors, but they are cheaper than wound-rotor designs. They allow some of the best features possible with wound-rotor motors (high starting torque with a low starting current and good efficiency at normal operating conditions) at a lower cost and without the need of maintaining slip rings and brushes.

Induction Motor Design Classes

It is possible to produce a large variety of torque–speed curves by varying the rotor characteristics of induction motors. To help industry select appropriate motors for varying applications in the integral-horsepower range, NEMA in the United

States and the International Electrotechnical Commission (IEC) in Europe have defined a series of standard designs with different torque–speed curves. These standard designs are referred to as *design classes*, and an individual motor may be referred to as a design class X motor. It is these NEMA and IEC design classes that were referred to earlier. Figure 7–26 shows typical torque–speed curves for the four standard NEMA design classes. The characteristic features of each standard design class are given below.

DESIGN CLASS A. Design class A motors are the standard motor design, with a normal starting torque, a normal starting current, and low slip. The full-load slip of design A motors must be less than 5 percent and must be less than that of a design B motor of equivalent rating. The pullout torque is 200 to 300 percent of the full-load torque and occurs at a low slip (less than 20 percent). The starting torque of this design is at least the rated torque for larger motors and is 200 percent or more of the rated torque for smaller motors. The principal problem with this design class is its extremely high inrush current on starting. Current flows at starting are typically 500 to 800 percent of the rated current. In sizes above about 7.5 hp, some form of reduced-voltage starting must be used with these motors to prevent voltage dip problems on starting in the power system they are connected to. In the past, design class A motors were the standard design for most applications below 7.5 hp and above about 200 hp, but they have largely been replaced by design class B motors in recent years. Typical applications for these motors are driving fans, blowers, pumps, lathes, and other machine tools.

DESIGN CLASS B. Design class B motors have a normal starting torque, a lower starting current, and low slip. This motor produces about the same starting torque as the class A motor with about 25 percent less current. The pullout torque is greater than or equal to 200 percent of the rated load torque, but less than that of the class A design because of the increased rotor reactance. Rotor slip is still relatively low (less than 5 percent) at full load. Applications are similar to those for design A, but design B is preferred because of its lower starting-current requirements. Design class B motors have largely replaced design class A motors in new installations.

DESIGN CLASS C. Design class C motors have a high starting torque with low starting currents and low slip (less than 5 percent) at full load. The pullout torque is slightly lower than that for class A motors, while the starting torque is up to 250 percent of the full-load torque. These motors are built from double-cage rotors, so they are more expensive than motors in the previous classes. They are used for high-starting-torque loads, such as loaded pumps, compressors, and conveyors.

DESIGN CLASS D. Design class D motors have a very high starting torque (275 percent or more of the rated torque) and a low starting current, but they also have a high slip at full load. They are essentially ordinary class A induction motors, but with the rotor bars made smaller and with a higher-resistance material. The high rotor resistance shifts the peak torque to a very low speed. It is even possible for

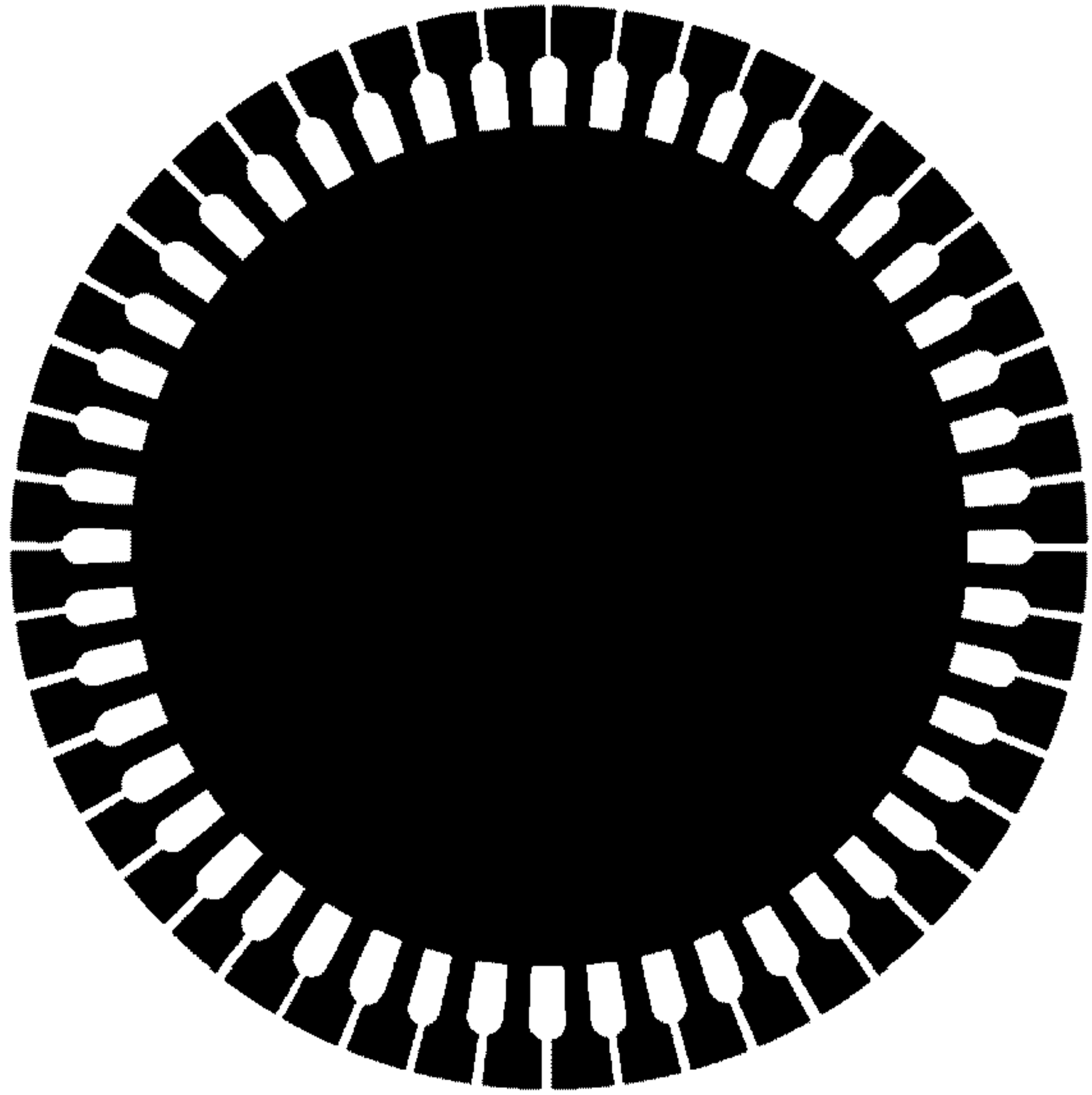


FIGURE 7–28
Rotor cross section, showing the construction of the former design class F induction motor. Since the rotor bars are deeply buried, they have a very high leakage reactance. The high leakage reactance reduces the starting torque and current of this motor, so it is called a *soft-start design*. (*Courtesy of MagneTek, Inc.*)

the highest torque to occur at zero speed (100 percent slip). Full-load slip for these motors is quite high because of the high rotor resistance. It is typically 7 to 11 percent, but may go as high as 17 percent or more. These motors are used in applications requiring the acceleration of extremely high-inertia-type loads, especially large flywheels used in punch presses or shears. In such applications, these motors gradually accelerate a large flywheel up to full speed, which then drives the punch. After a punching operation, the motor then reaccelerates the flywheel over a fairly long time for the next operation.

In addition to these four design classes, NEMA used to recognize design classes E and F, which were called *soft-start* induction motors (see Figure 7–28). These designs were distinguished by having very low starting currents and were used for low-starting-torque loads in situations where starting currents were a problem. These designs are now obsolete.

Example 7–6. A 460-V, 30-hp, 60-Hz, four-pole, Y-connected induction motor has two possible rotor designs, a single-cage rotor and a double-cage rotor. (The stator is identical for either rotor design.) The motor with the single-cage rotor may be modeled by the following impedances in ohms per phase referred to the stator circuit:

$$R_1 = 0.641\ \Omega \qquad R_2 = 0.300\ \Omega$$
$$X_1 = 0.750\ \Omega \qquad X_2 = 0.500\ \Omega \qquad X_M = 26.3\ \Omega$$

The motor with the double-cage rotor may be modeled as a tightly coupled, high-resistance outer cage in parallel with a loosely coupled, low-resistance inner cage (similar to the structure of Figure 7–25c). The stator and magnetization resistance and reactances will be identical with those in the single-cage design.

The resistance and reactance of the rotor outer cage are:

$$R_{2o} = 3.200\ \Omega \qquad X_{2o} = 0.500\ \Omega$$

Note that the resistance is high because the outer bar has a small cross section, while the reactance is the same as the reactance of the single-cage rotor, since the outer cage is very close to the stator, and the leakage reactance is small.

The resistance and reactance of the inner cage are

$$R_{2i} = 0.400\ \Omega \qquad X_{2i} = 3.300\ \Omega$$

Here the resistance is low because the bars have a large cross-sectional area, but the leakage reactance is quite high.

Calculate the torque–speed characteristics associated with the two rotor designs. How do they compare?

Solution

The torque–speed characteristic of the motor with the single-cage rotor can be calculated in exactly the same manner as Example 7–5. The torque–speed characteristic of the motor with the double-cage rotor can also be calculated in the same fashion, *except* that at each slip the rotor resistance and reactance will be the parallel combination of the impedances of the inner and outer cages. At low slips, the rotor reactance will be relatively unimportant, and the large inner cage will play a major part in the machine's operation. At high slips, the high reactance of the inner cage almost removes it from the circuit.

A MATLAB M-file to calculate and plot the two torque–speed characteristics is shown below:

```
% M-file: torque_speed_2.m
% M-file create and plot of the torque-speed curve of an
%   induction motor with a double-cage rotor design.

% First, initialize the values needed in this program.
r1 = 0.641;                  % Stator resistance
x1 = 0.750;                  % Stator reactance
r2 = 0.300;                  % Rotor resistance for single-
                             %   cage motor
r2i = 0.400;                 % Rotor resistance for inner
                             %   cage of double-cage motor
r2o = 3.200;                 % Rotor resistance for outer
                             %   cage of double-cage motor
x2 = 0.500;                  % Rotor reactance for single-
                             %   cage motor
```

```
x2i = 3.300;                  % Rotor reactance for inner
                              %   cage of double-cage motor
x2o = 0.500;                  % Rotor reactance for outer
                              %   cage of double-cage motor
xm = 26.3;                    % Magnetization branch reactance
v_phase = 460 / sqrt(3);      % Phase voltage
n_sync = 1800;                % Synchronous speed (r/min)
w_sync = 188.5;               % Synchronous speed (rad/s)

% Calculate the Thevenin voltage and impedance from Equations
% 7-41a and 7-43.
v_th = v_phase * ( xm / sqrt(r1^2 + (x1 + xm)^2) );
z_th = ((j*xm) * (r1 + j*x1)) / (r1 + j*(x1 + xm));
r_th = real(z_th);
x_th = imag(z_th);

% Now calculate the motor speed for many slips between
% 0 and 1.  Note that the first slip value is set to
% 0.001 instead of exactly 0 to avoid divide-by-zero
% problems.
s = (0:1:50) / 50;               % Slip
s(1) = 0.001;                    % Avoid division-by-zero
nm = (1 - s) * n_sync;           % Mechanical speed

% Calculate torque for the single-cage rotor.
for ii = 1:51
  t_ind1(ii) = (3 * v_th^2 * r2 / s(ii)) / ...
       (w_sync * ((r_th + r2/s(ii))^2 + (x_th + x2)^2) );
end

% Calculate resistance and reactance of the double-cage
% rotor at this slip, and then use those values to
% calculate the induced torque.
for ii = 1:51
  y_r = 1/(r2i + j*s(ii)*x2i) + 1/(r2o + j*s(ii)*x2o);
  z_r = 1/y_r;                % Effective rotor impedance
  r2eff = real(z_r);          % Effective rotor resistance
  x2eff = imag(z_r);          % Effective rotor reactance

  % Calculate induced torque for double-cage rotor.
  t_ind2(ii) = (3 * v_th^2 * r2eff / s(ii)) / ...
       (w_sync * ((r_th + r2eff/s(ii))^2 + (x_th + x2eff)^2) );
end

% Plot the torque-speed curves
plot(nm,t_ind1,'Color','k','LineWidth',2.0);
hold on;
plot(nm,t_ind2,'Color','k','LineWidth',2.0,'LineStyle','-.');
xlabel('\itn_{m}','Fontweight','Bold');
ylabel('\tau_{ind}','Fontweight','Bold');
title ('Induction motor torque-speed characteristics', ...
    'Fontweight','Bold');
legend ('Single-Cage Design','Double-Cage Design');
grid on;
hold off;
```

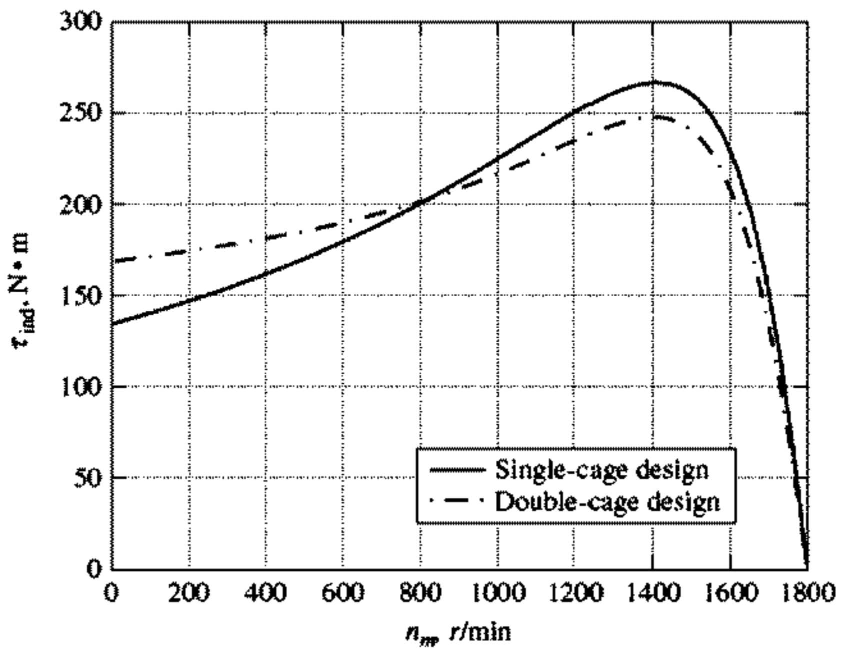


FIGURE 7–29
Comparison of torque–speed characteristics for the single- and double-cage rotors of Example 7–6.

The resulting torque–speed characteristics are shown in Figure 7–29. Note that the double-cage design has a slightly higher slip in the normal operating range, a smaller maximum torque and a higher starting torque compared to the corresponding single-cage rotor design. This behavior matches our theoretical discussions in this section.

7.7 TRENDS IN INDUCTION MOTOR DESIGN

The fundamental ideas behind the induction motor were developed during the late 1880s by Nicola Tesla, who received a patent on his ideas in 1888. At that time, he presented a paper before the American Institute of Electrical Engineers [AIEE, predecessor of today's Institute of Electrical and Electronics Engineers (IEEE)] in which he described the basic principles of the wound-rotor induction motor, along with ideas for two other important ac motors—the synchronous motor and the reluctance motor.

Although the basic idea of the induction motor was described in 1888, the motor itself did not spring forth in full-fledged form. There was an initial period of rapid development, followed by a series of slow, evolutionary improvements which have continued to this day.

The induction motor assumed recognizable modern form between 1888 and 1895. During that period, two- and three-phase power sources were developed to produce the rotating magnetic fields within the motor, distributed stator windings were developed, and the cage rotor was introduced. By 1896, fully functional and recognizable three-phase induction motors were commercially available.

Between then and the early 1970s, there was continual improvement in the quality of the steels, the casting techniques, the insulation, and the construction

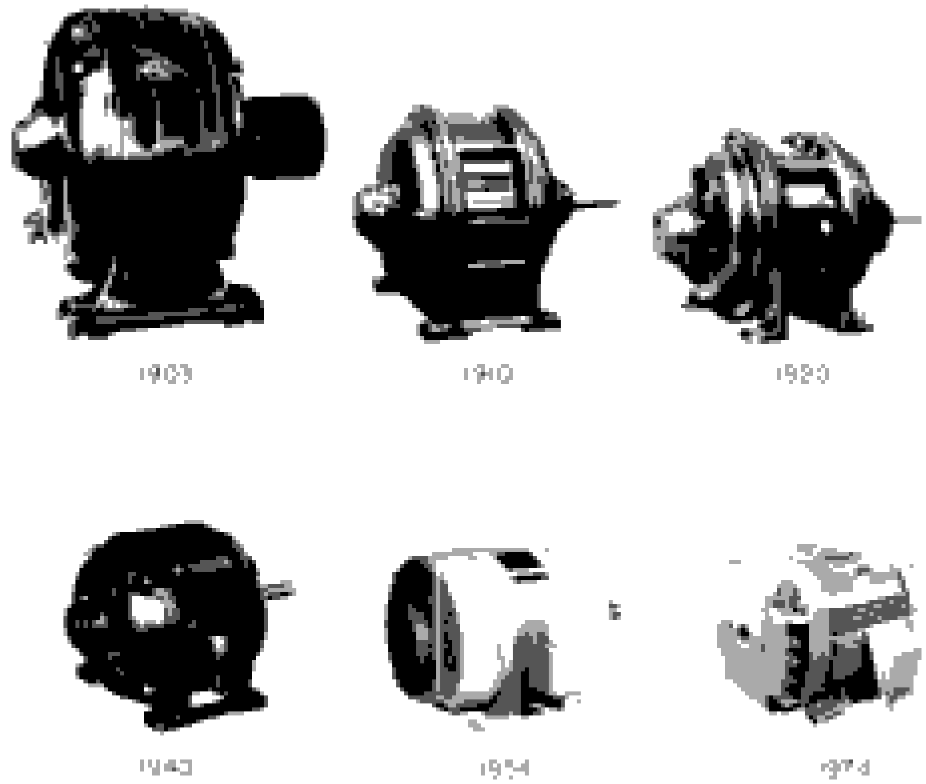


FIGURE 7–30
The evolution of the induction motor. The motors shown in this figure are all rated at 220 V and 15 hp. There has been a dramatic decrease in motor size and material requirements in induction motors since the first practical ones were produced in the 1890s. (*Courtesy of General Electric Company.*)

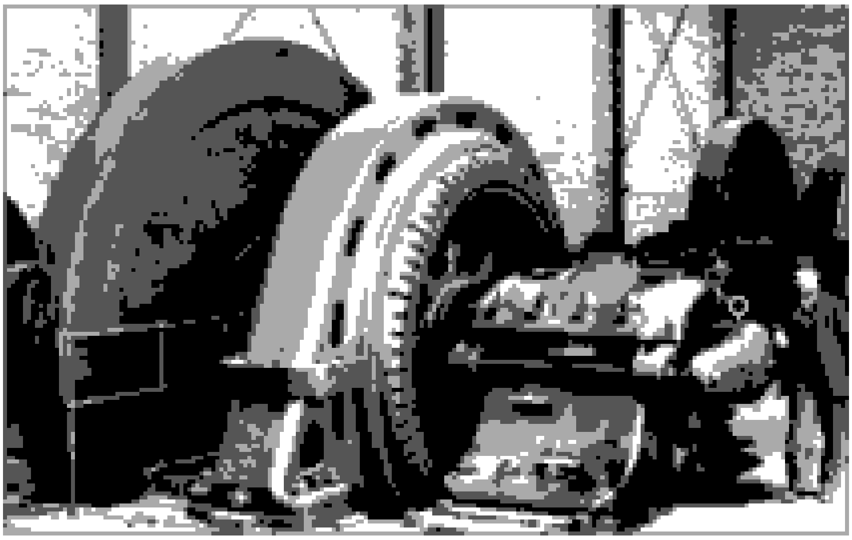


FIGURE 7–31
Typical early large induction motors. The motors shown were rated at 2000 hp. (*Courtesy of General Electric Company.*)

features used in induction motors. These trends resulted in a smaller motor for a given power output, yielding considerable savings in construction costs. In fact, a modern 100-hp motor is the same physical size as a 7.5-hp motor of 1897. This progression is vividly illustrated by the 15-hp induction motors shown in Figure 7–30. (See also Figure 7–31.)

However, these improvements in induction motor design did *not* necessarily lead to improvements in motor operating efficiency. The major design effort was directed toward reducing the initial materials cost of the machines, not toward increasing their efficiency. The design effort was oriented in that direction because electricity was so inexpensive, making the up-front cost of a motor the principal criterion used by purchasers in its selection.

Since the price of oil began its spectacular climb in 1973, the lifetime operating cost of machines has become more and more important, and the initial installation cost has become relatively less important. As a result of these trends, new emphasis has been placed on motor efficiency both by designers and by end users of the machines.

New lines of high-efficiency induction motors are now being produced by all major manufacturers, and they are forming an ever-increasing share of the induction motor market. Several techniques are used to improve the efficiency of these motors compared to the traditional standard-efficiency designs. Among these techniques are

1. More copper is used in the stator windings to reduce copper losses.
2. The rotor and stator core length is increased to reduce the magnetic flux density in the air gap of the machine. This reduces the magnetic saturation of the machine, decreasing core losses.
3. More steel is used in the stator of the machine, allowing a greater amount of heat transfer out of the motor and reducing its operating temperature. The rotor's fan is then redesigned to reduce windage losses.
4. The steel used in the stator is a special high-grade electrical steel with low hysteresis losses.
5. The steel is made of an especially thin gauge (i.e., the laminations are very close together), and the steel has a very high internal resistivity. Both effects tend to reduce the eddy current losses in the motor.
6. The rotor is carefully machined to produce a uniform air gap, reducing the stray load losses in the motor.

In addition to the general techniques described above, each manufacturer has his own unique approaches to improving motor efficiency. A typical high-efficiency induction motor is shown in Figure 7–32.

To aid in the comparison of motor efficiencies, NEMA has adopted a standard technique for measuring motor efficiency based on Method B of the IEEE Standard 112, *Test Procedure for Polyphase Induction Motors and Generators*. NEMA has also introduced a rating called *NEMA nominal efficiency*, which appears on the nameplates of design class A, B, and C motors. The nominal efficiency identifies the average efficiency of a large number of motors of a given model, and it also guarantees a certain minimum efficiency for that type of motor. The standard NEMA nominal efficiencies are shown in Figure 7–33.

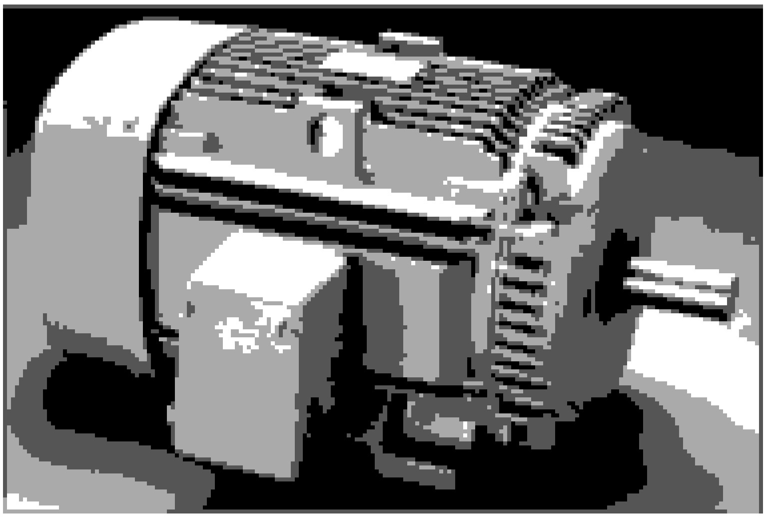


FIGURE 7–32
A General Electric Energy Saver motor, typical of modern high-efficiency induction motors. (*Courtesy of General Electric Company.*)

Nominal efficiency, %	Guaranteed minimum efficiency, %	Nominal efficiency, %	Guaranteed minimum efficiency, %
95.0	94.1	80.0	77.0
94.5	93.6	78.5	75.5
94.1	93.0	77.0	74.0
93.6	92.4	75.5	72.0
93.0	91.7	74.0	70.0
92.4	91.0	72.0	68.0
91.7	90.2	70.0	66.0
91.0	89.5	68.0	64.0
90.2	88.5	66.0	62.0
89.5	87.5	64.0	59.5
88.5	86.5	62.0	57.5
87.5	85.5	59.5	55.0
86.5	84.0	57.5	52.5
85.5	82.5	55.0	50.5
84.0	81.5	52.5	48.0
82.5	80.0	50.5	46.0
81.5	78.5		

FIGURE 7–33
Table of NEMA nominal efficiency standards. The nominal efficiency represents the mean efficiency of a large number of sample motors, and the guaranteed minimum efficiency represents the lowest permissible efficiency for any given motor of the class. (*Reproduced by permission from Motors and Generators, NEMA Publication MG-1, copyright 1987 by NEMA.*)

Other standards organizations have also established efficiency standards for induction motors, the most important of which are the British (BS-269), IEC (IEC 34-2), and Japanese (JEC-37) standards. However, the techniques prescribed for measuring induction motor efficiency are different in each standard and yield *different results for the same physical machine*. If two motors are each rated at 82.5 percent efficiency, but they are measured according to different standards, then they may not be equally efficient. When two motors are compared, it is important to compare efficiencies measured under the same standard.

7.8 STARTING INDUCTION MOTORS

Induction motors do not present the types of starting problems that synchronous motors do. In many cases, induction motors can be started by simply connecting them to the power line. However, there are sometimes good reasons for not doing this. For example, the starting current required may cause such a dip in the power system voltage that *across-the-line starting* is not acceptable.

For wound-rotor induction motors, starting can be achieved at relatively low currents by inserting extra resistance in the rotor circuit during starting. This extra resistance not only increases the starting torque but also reduces the starting current.

For cage induction motors, the starting current can vary widely depending primarily on the motor's rated power and on the effective rotor resistance at starting conditions. To estimate the rotor current at starting conditions, all cage motors now have a starting *code letter* (not to be confused with their *design class* letter) on their nameplates. The code letter sets limits on the amount of current the motor can draw at starting conditions.

These limits are expressed in terms of the starting apparent power of the motor as a function of its horsepower rating. Figure 7–34 is a table containing the starting kilovoltamperes per horsepower for each code letter.

To determine the starting current for an induction motor, read the rated voltage, horsepower, and code letter from its nameplate. Then the starting apparent power for the motor will be

$$S_{\text{start}} = (\text{rated horsepower})(\text{code letter factor}) \tag{7–55}$$

and the starting current can be found from the equation

$$I_L = \frac{S_{\text{start}}}{\sqrt{3}V_T} \tag{7–56}$$

Example 7–7. What is the starting current of a 15-hp, 208-V, code-letter-F, three-phase induction motor?

Solution

According to Figure 7–34, the maximum kilovoltamperes per horsepower is 5.6. Therefore, the maximum starting kilovoltamperes of this motor is

$$S_{\text{start}} = (15 \text{ hp})(5.6) = 84 \text{ kVA}$$

Nominal code letter	Locked rotor, kVA/hp	Nominal code letter	Locked rotor, kVA/hp
A	0–3.15	L	9.00–10.00
B	3.15–3.55	M	10.00–11.00
C	3.55–4.00	N	11.20–12.50
D	4.00–4.50	P	12.50–14.00
E	4.50–5.00	R	14.00–16.00
F	5.00–5.60	S	16.00–18.00
G	5.60–6.30	T	18.00–20.00
H	6.30–7.10	U	20.00–22.40
J	7.7–8.00	V	22.40 and up
K	8.00–9.00		

FIGURE 7–34
Table of NEMA code letters, indicating the starting kilovoltamperes per horsepower of rating for a motor. Each code letter extends up to, but does not include, the lower bound of the next higher class. (*Reproduced by permission from Motors and Generators, NEMA Publication MG-1, copyright 1987 by NEMA.*)

The starting current is thus

$$I_L = \frac{S_{\text{start}}}{\sqrt{3}V_T} \tag{7–56}$$

$$= \frac{84\text{ kVA}}{\sqrt{3}(208\text{ V})} = 233\text{ A}$$

If necessary, the starting current of an induction motor may be reduced by a starting circuit. However, if this is done, it will also reduce the starting torque of the motor.

One way to reduce the starting current is to insert extra inductors or resistors into the power line during starting. While formerly common, this approach is rare today. An alternative approach is to reduce the motor's terminal voltage during starting by using autotransformers to step it down. Figure 7–35 shows a typical reduced-voltage starting circuit using autotransformers. During starting, contacts 1 and 3 are shut, supplying a lower voltage to the motor. Once the motor is nearly up to speed, those contacts are opened and contacts 2 are shut. These contacts put full line voltage across the motor.

It is important to realize that while the starting current is reduced in direct proportion to the decrease in terminal voltage, the starting torque decreases as the *square* of the applied voltage. Therefore, only a certain amount of current reduction can be done if the motor is to start with a shaft load attached.

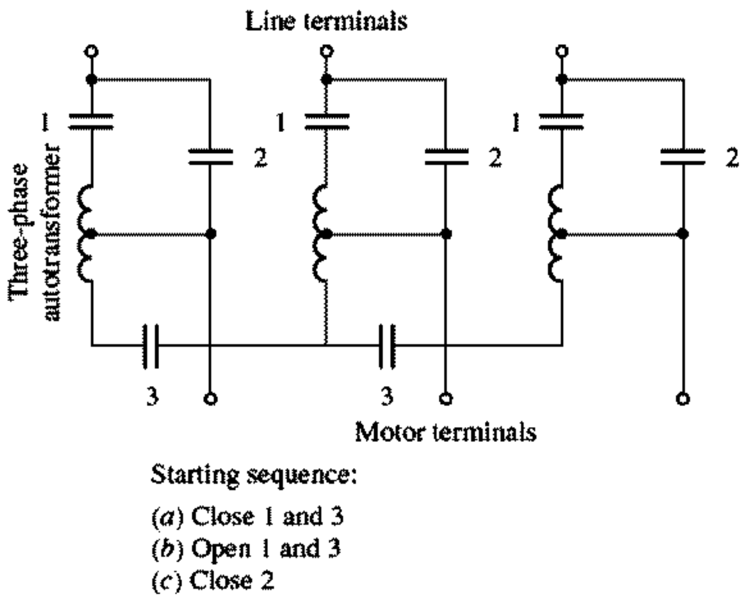


FIGURE 7–35
An autotransformer starter for an induction motor.

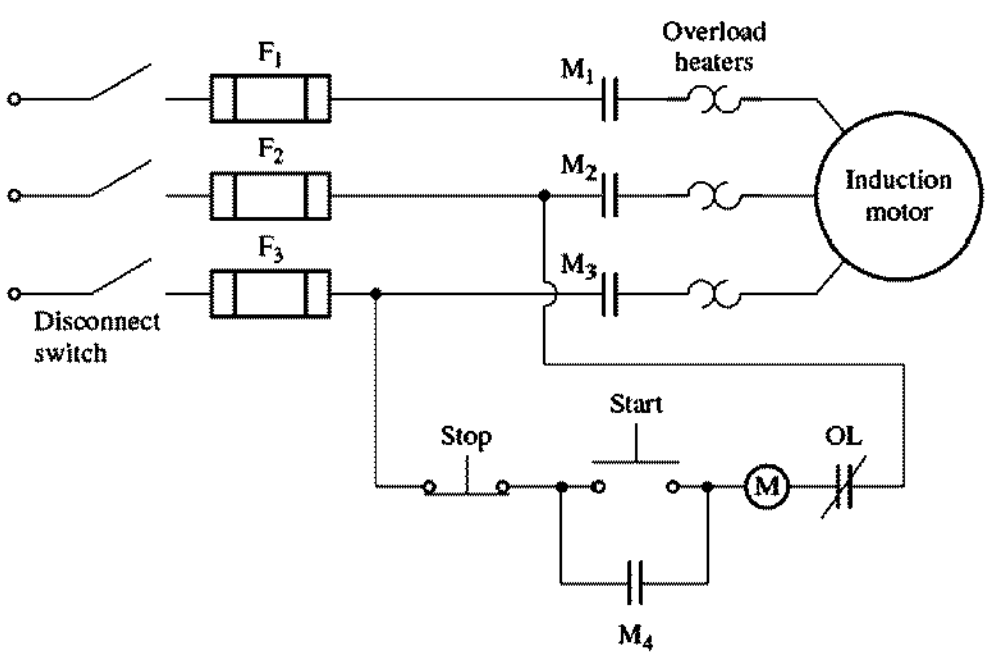


FIGURE 7–36
A typical across-the-line starter for an induction motor.

Induction Motor Starting Circuits

A typical full-voltage or across-the-line magnetic induction motor starter circuit is shown in Figure 7–36, and the meanings of the symbols used in the figure are explained in Figure 7–37. This operation of this circuit is very simple. When the start button is pressed, the relay (or *contactor*) coil M is energized, causing the normally open contacts M_1, M_2, and M_3 to shut. When these contacts shut, power is applied to the induction motor, and the motor starts. Contact M_4 also shuts,

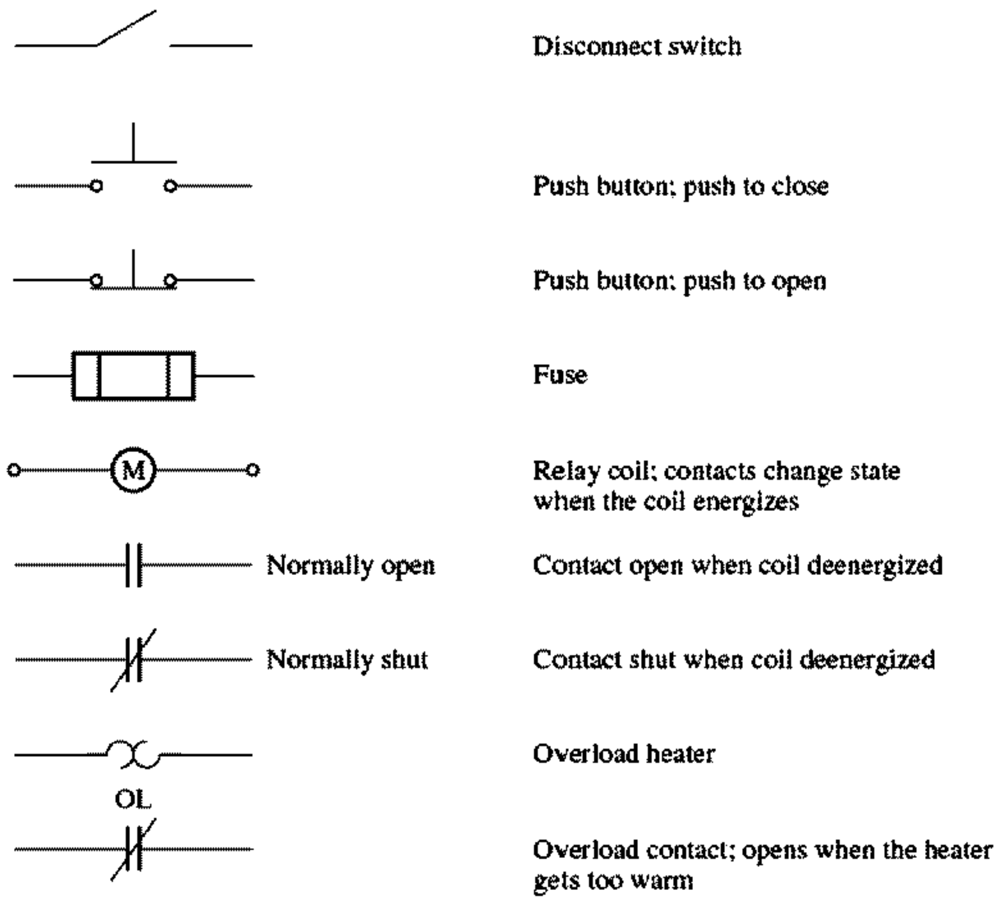


FIGURE 7–37
Typical components found in induction motor control circuits.

which shorts out the starting switch, allowing the operator to release it without removing power from the M relay. When the stop button is pressed, the M relay is deenergized, and the M contacts open, stopping the motor.

A magnetic motor starter circuit of this sort has several built-in protective features:

1. Short-circuit protection
2. Overload protection
3. Undervoltage protection

Short-circuit protection for the motor is provided by fuses F_1, F_2, and F_3. If a sudden short circuit develops within the motor and causes a current flow many times larger than the rated current, these fuses will blow, disconnecting the motor from the power supply and preventing it from burning up. However, these fuses must *not* burn up during normal motor starting, so they are designed to require currents many times greater than the full-load current before they open the circuit. This means that short circuits through a high resistance and/or excessive motor loads will not be cleared by the fuses.

Overload protection for the motor is provided by the devices labeled OL in the figure. These overload protection devices consist of two parts, an overload heater element and overload contacts. Under normal conditions, the overload contacts are shut. However, when the temperature of the heater elements rises far enough, the OL contacts open, deenergizing the M relay, which in turn opens the normally open M contacts and removes power from the motor.

When an induction motor is overloaded, it is eventually damaged by the excessive heating caused by its high currents. However, this damage takes time, and an induction motor will not normally be hurt by brief periods of high currents (such as starting currents). Only if the high current is sustained will damage occur. The overload heater elements also depend on heat for their operation, so they will not be affected by brief periods of high current during starting, and yet they will operate during long periods of high current, removing power from the motor before it can be damaged.

Undervoltage protection is provided by the controller as well. Notice from the figure that the control power for the M relay comes from directly across the lines to the motor. If the voltage applied to the motor falls too much, the voltage applied to the M relay will also fall and the relay will deenergize. The M contacts then open, removing power from the motor terminals.

An induction motor starting circuit with resistors to reduce the starting current flow is shown in Figure 7–38. This circuit is similar to the previous one, except that there are additional components present to control removal of the starting resistor. Relays 1TD, 2TD, and 3TD in Figure 7–38 are so-called time-delay relays, meaning that when they are energized there is a set time delay before their contacts shut.

When the start button is pushed in this circuit, the M relay energizes and power is applied to the motor as before. Since the 1TD, 2TD, and 3TD contacts are all open, the full starting resistor is in series with the motor, reducing the starting current.

When the M contacts close, notice that the 1TD relay is energized. However, there is a finite delay before the 1TD contacts close. During that time, the motor partially speeds up, and the starting current drops off some. After that time, the 1TD contacts close, cutting out part of the starting resistance and simultaneously energizing the 2TD relay. After another delay, the 2TD contacts shut, cutting out the second part of the resistor and energizing the 3TD relay. Finally, the 3TD contacts close, and the entire starting resistor is out of the circuit.

By a judicious choice of resistor values and time delays, this starting circuit can be used to prevent the motor starting current from becoming dangerously large, while still allowing enough current flow to ensure prompt acceleration to normal operating speeds.

7.9 SPEED CONTROL OF INDUCTION MOTORS

Until the advent of modern solid-state drives, induction motors in general were not good machines for applications requiring considerable speed control. The normal operating range of a typical induction motor (design classes A, B, and C)

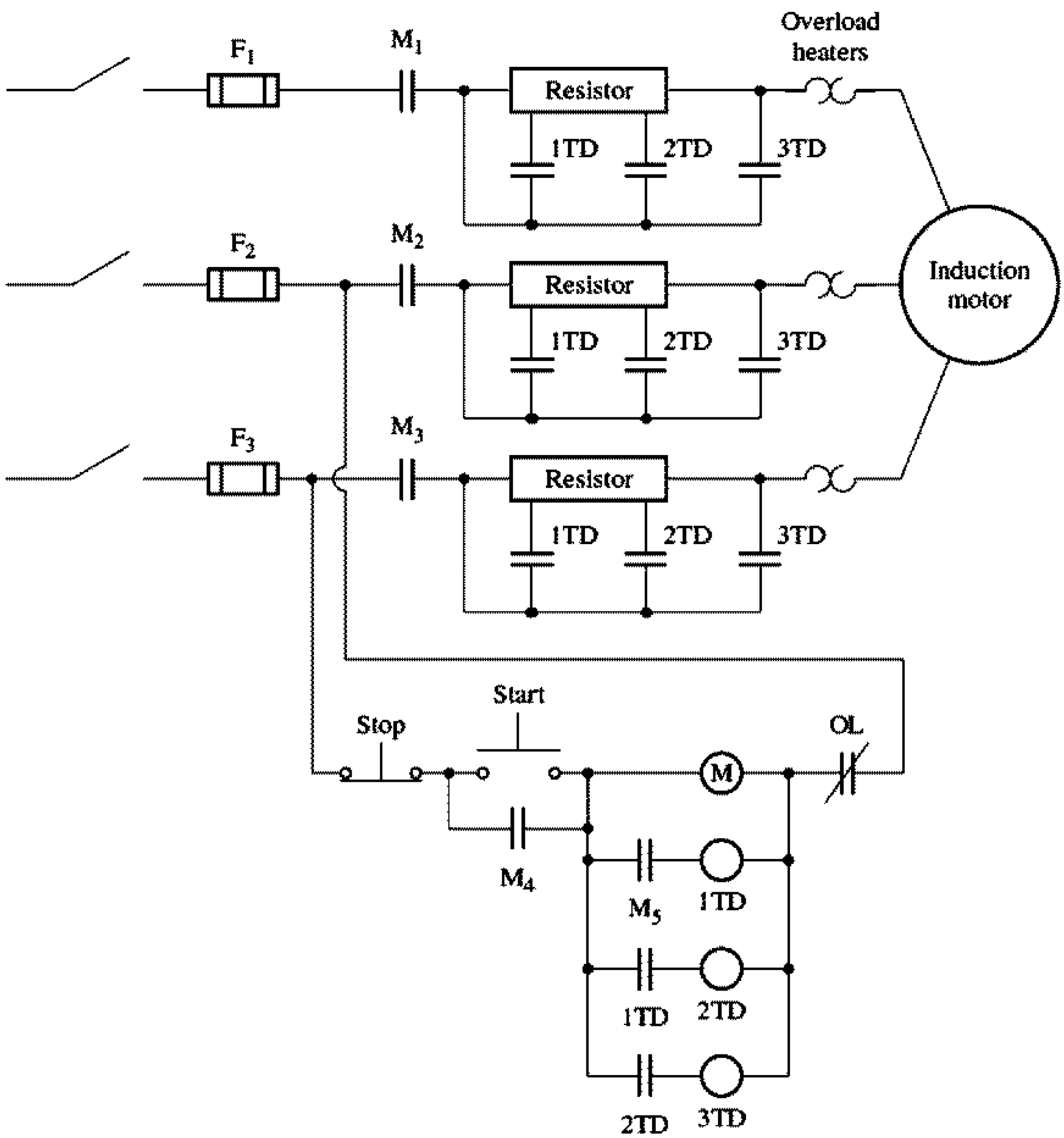


FIGURE 7–38
A three-step resistive starter for an induction motor.

is confined to less than 5 percent slip, and the speed variation over that range is more or less directly proportional to the load on the shaft of the motor. Even if the slip could be made larger, the efficiency of the motor would become very poor, since the rotor copper losses are directly proportional to the slip on the motor (remember that $P_{RCL} = sP_{AG}$).

There are really only two techniques by which the speed of an induction motor can be controlled. One is to vary the synchronous speed, which is the speed of the stator and rotor magnetic fields, since the rotor speed always remains near n_{sync}. The other technique is to vary the slip of the motor for a given load. Each of these approaches will be taken up in more detail.

The synchronous speed of an induction motor is given by

$$n_{\text{sync}} = \frac{120 f_e}{P} \tag{7-1}$$

so the only ways in which the synchronous speed of the machine can be varied are (1) by changing the electrical frequency and (2) by changing the number of poles on the machine. Slip control may be accomplished by varying either the rotor resistance or the terminal voltage of the motor.

Induction Motor Speed Control by Pole Changing

There are two major approaches to changing the number of poles in an induction motor:

1. The method of consequent poles
2. Multiple stator windings

The *method of consequent poles* is quite an old method for speed control, having been originally developed in 1897. It relies on the fact that the number of poles in the stator windings of an induction motor can easily be changed by a factor of 2:1 with only simple changes in coil connections. Figure 7–39 shows a

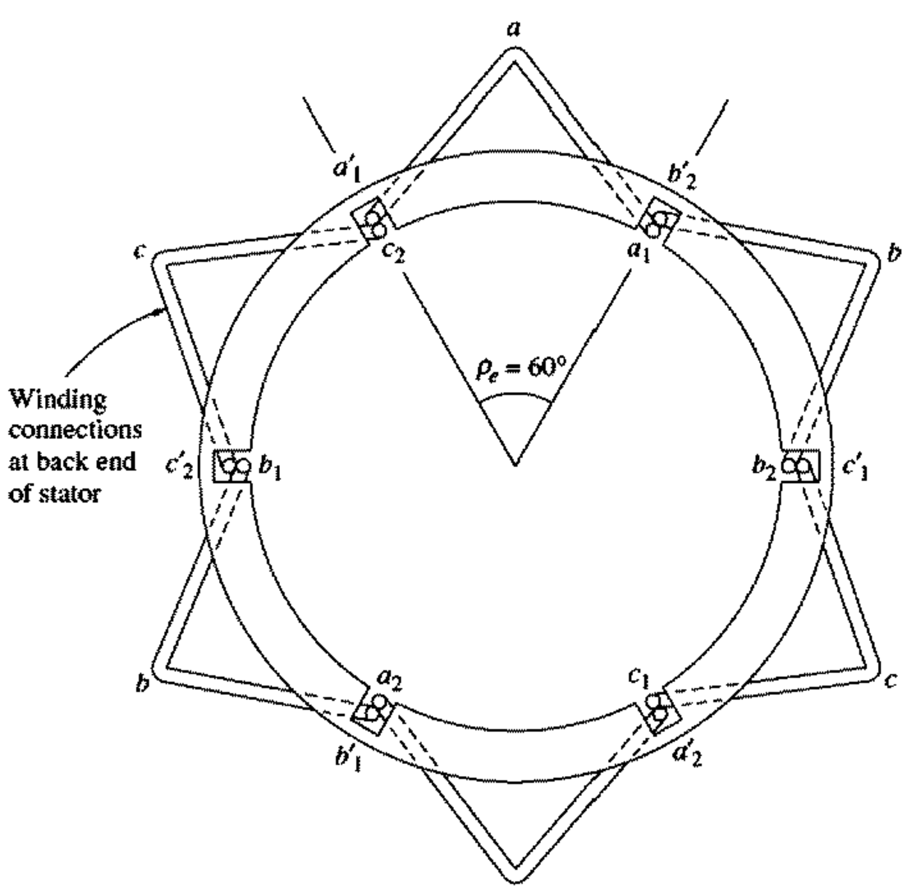


FIGURE 7–39
A two-pole stator winding for pole changing. Notice the very small rotor pitch of these windings.

simple two-pole induction motor stator suitable for pole changing. Notice that the individual coils are of very short pitch (60 to 90°). Figure 7–40 shows phase *a* of these windings separately for more clarity of detail.

Figure 7–40a shows the current flow in phase *a* of the stator windings at an instant of time during normal operation. Note that the magnetic field leaves the stator in the upper phase group (a north pole) and enters the stator in the lower phase group (a south pole). This winding is thus producing two stator magnetic poles.

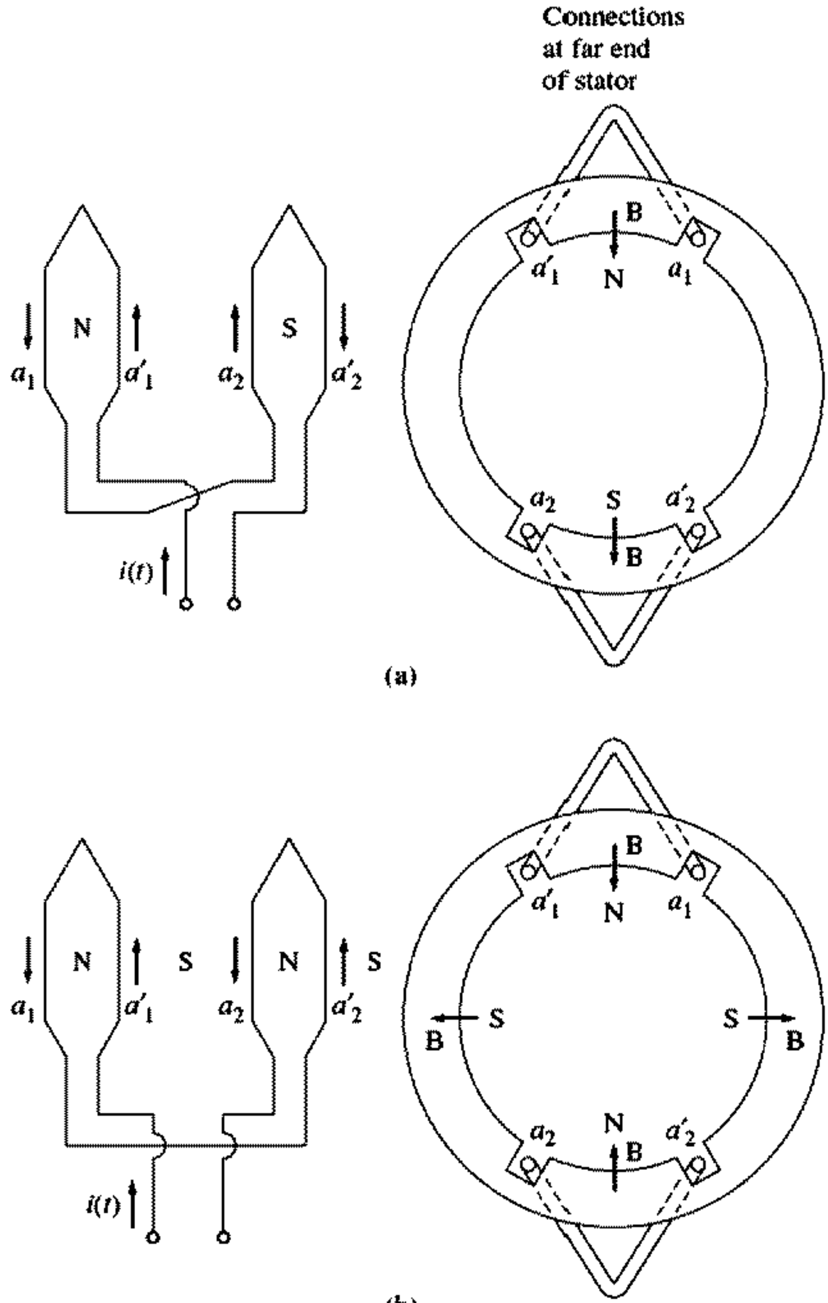


FIGURE 7–40

A close-up view of one phase of a pole-changing winding. (a) In the two-pole configuration, one coil is a north pole and the other one is a south pole. (b) When the connection on one of the two coils is reversed, they are both north poles, and the magnetic flux returns to the stator at points halfway between the two coils. The south poles are called *consequent poles*, and the winding is now a four-pole winding.

Now suppose that the direction of current flow in the *lower* phase group on the stator is reversed (Figure 7–40b). Then the magnetic field will leave the stator in *both* the upper phase group *and* the lower phase group—each one will be a north magnetic pole. The magnetic flux in this machine must return to the stator *between* the two phase groups, producing a pair of *consequent* south magnetic poles. Notice that now the stator has four magnetic poles—twice as many as before.

The rotor in such a motor is of the cage design, since a cage rotor always has as many poles induced in it as there are in the stator and can thus adapt when the number of stator poles changes.

When the motor is reconnected from two-pole to four-pole operation, the resulting maximum torque of the induction motor can be the same as before (constant-torque connection), half of its previous value (square-law-torque connection, used for fans, etc.), or twice its previous value (constant-output-power connection), depending on how the stator windings are rearranged. Figure 7–41 shows the possible stator connections and their effect on the torque–speed curve.

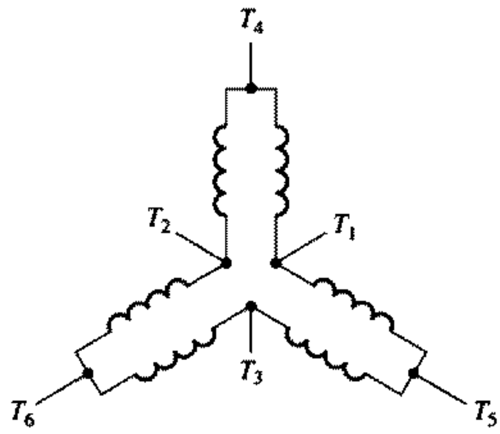
The major disadvantage of the consequent-pole method of changing speed is that the speeds *must* be in a ratio of 2:1. The traditional approach to overcoming this limitation was to employ *multiple stator windings* with different numbers of poles and to energize only one set at a time. For example, a motor might be wound with a four-pole and a six-pole set of stator windings, and its synchronous speed on a 60-Hz system could be switched from 1800 to 1200 r/min simply by supplying power to the other set of windings. Unfortunately, multiple stator windings increase the expense of the motor and are therefore used only when absolutely necessary.

By combining the method of consequent poles with multiple stator windings, it is possible to build a four-speed induction motor. For example, with separate four- and six-pole windings, it is possible to produce a 60-Hz motor capable of running at 600, 900, 1200, and 1800 r/min.

Speed Control by Changing the Line Frequency

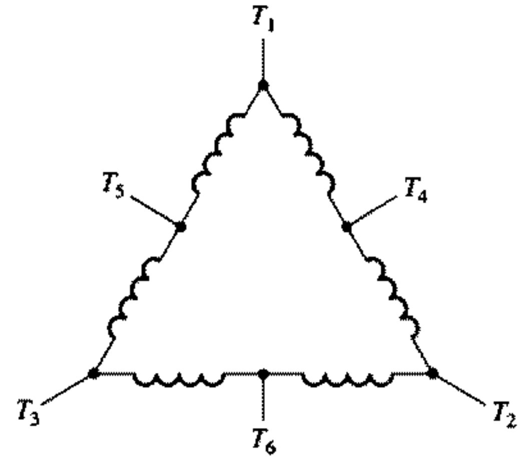
If the electrical frequency applied to the stator of an induction motor is changed, the rate of rotation of its magnetic fields n_{sync} will change in direct proportion to the change in electrical frequency, and the no-load point on the torque–speed characteristic curve will change with it (see Figure 7–42). The synchronous speed of the motor at rated conditions is known as the *base speed*. By using variable frequency control, it is possible to adjust the speed of the motor either above or below base speed. A properly designed variable-frequency induction motor drive can be *very* flexible. It can control the speed of an induction motor over a range from as little as 5 percent of base speed up to about twice base speed. However, it is important to maintain certain voltage and torque limits on the motor as the frequency is varied, to ensure safe operation.

When running at speeds below the base speed of the motor, it is necessary to reduce the terminal voltage applied to the stator for proper operation. The terminal voltage applied to the stator should be decreased linearly with decreasing



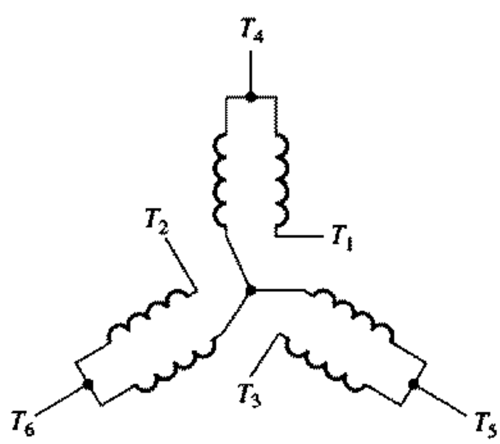
Speed	Lines			
	L_1	L_2	L_3	
Low	T_1	T_2	T_3	T_4, T_5, T_6 open
High	T_4	T_5	T_6	$T_1 - T_2 - T_3$ together

(a)



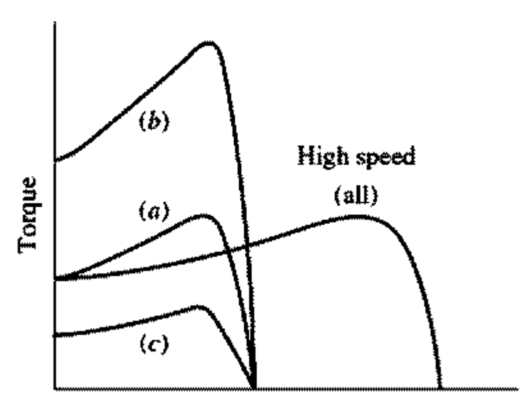
Speed	Lines			
	L_1	L_2	L_3	
Low	T_4	T_5	T_6	$T_1 - T_2 - T_3$ together
High	T_1	T_2	T_3	T_4, T_5, T_6 open

(b)



Speed	Lines			
	L_1	L_2	L_3	
Low	T_1	T_2	T_3	T_4, T_5, T_6 open
High	T_4	T_5	T_6	$T_1 - T_2 - T_3$ together

(c)



(d)

FIGURE 7–41
Possible connections of the stator coils in a pole-changing motor, together with the resulting torque–speed characteristics: (a) *Constant-torque connection*—the torque capabilities of the motor remain approximately constant in both high-speed and low-speed connections. (b) *Constant-horsepower connection*—the power capabilities of the motor remain approximately constant in both high-speed and low-speed connections. (c) *Fan torque connection*—the torque capabilities of the motor change with speed in the same manner as fan-type loads.

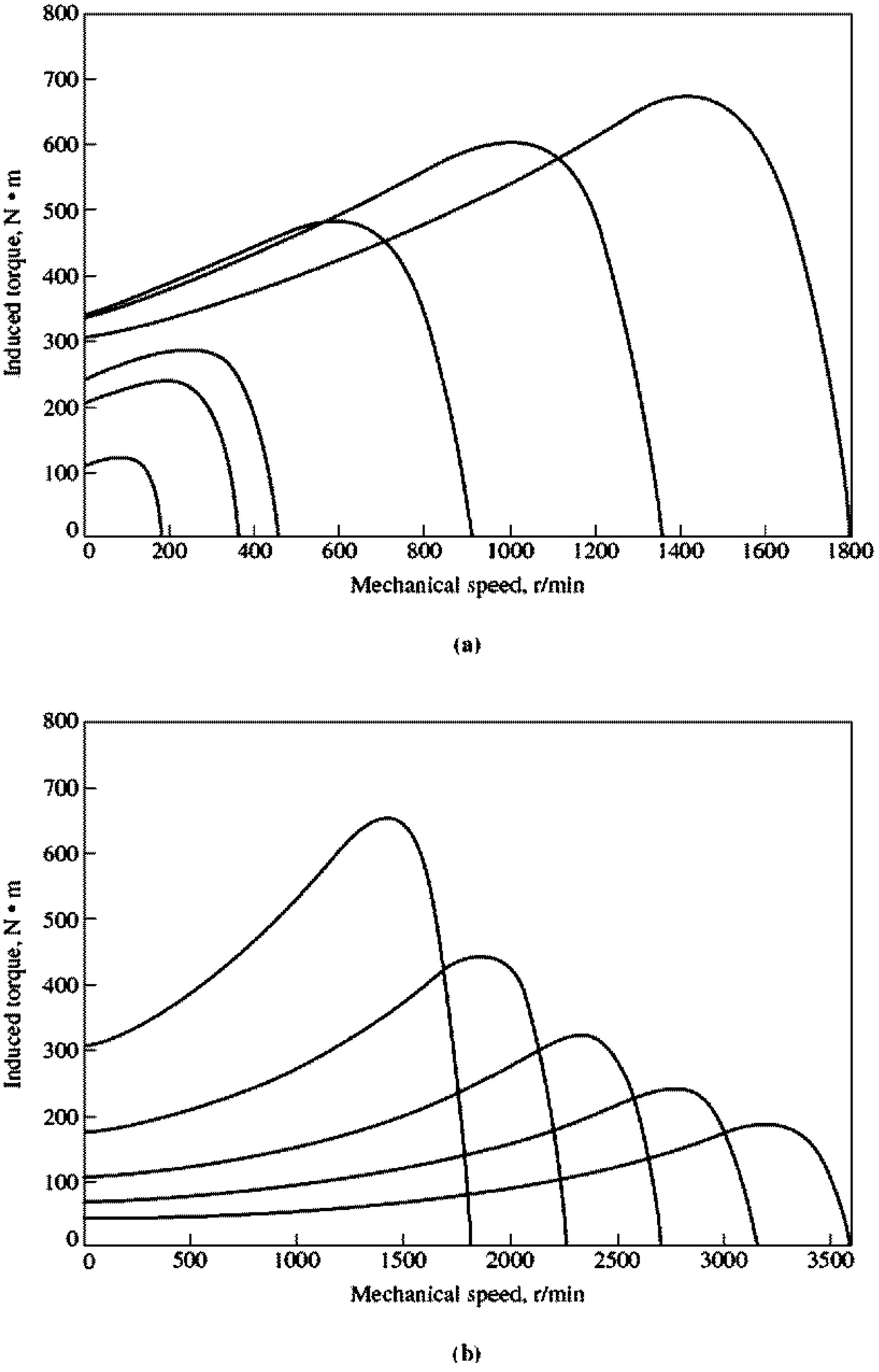
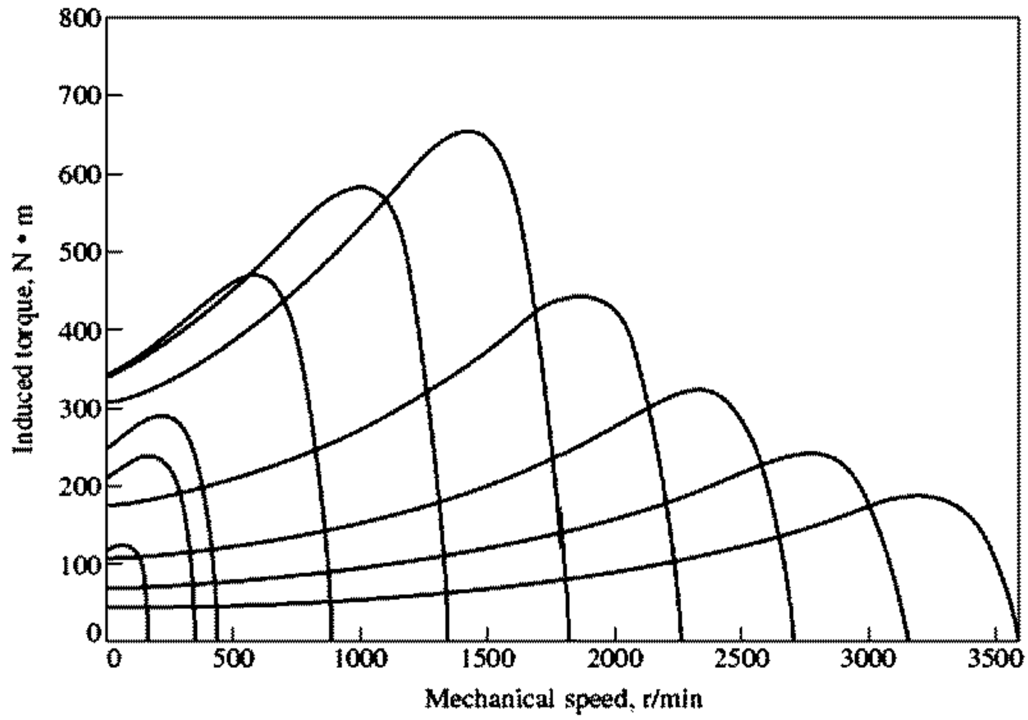


FIGURE 7–42
Variable-frequency speed control in an induction motor: (a) The family of torque–speed characteristic curves for speeds below base speed, assuming that the line voltage is derated linearly with frequency. (b) The family of torque–speed characteristic curves for speeds above base speed, assuming that the line voltage is held constant.



(c)

FIGURE 7–42 (concluded)
(c) The torque–speed characteristic curves for all frequencies.

stator frequency. This process is called *derating*. If it is not done, the steel in the core of the induction motor will saturate and excessive magnetization currents will flow in the machine.

To understand the necessity for derating, recall that an induction motor is basically a rotating transformer. As with any transformer, the flux in the core of an induction motor can be found from Faraday's law:

$$v(t) = -N\frac{d\phi}{dt} \tag{1–36}$$

If a voltage $v(t) = V_M \sin \omega t$ is applied to the core, the resulting flux ϕ is

$$\phi(t) = \frac{1}{N_P}\int v(t)\,dt$$

$$= \frac{I}{N_P}\int V_M \sin \omega t\,dt$$

$$\boxed{\phi(t) = -\frac{V_M}{\omega N_P}\cos \omega t} \tag{7–57}$$

Note that the electrical frequency appears in the *denominator* of this expression. Therefore, if the electrical frequency applied to the stator *decreases* by 10 percent while the magnitude of the voltage applied to the stator remains constant, the flux in the core of the motor will *increase* by about 10 percent and the magnetization current of the motor will increase. In the unsaturated region of the motor's magnetization curve, the increase in magnetization current will also be about 10 percent. However, in the saturated region of the motor's magnetization curve, a 10 percent increase in flux requires a much larger increase in magnetization current. Induction motors are normally designed to operate near the saturation point on their magnetization curves, so the increase in flux due to a decrease in frequency will cause excessive magnetization currents to flow in the motor. (This same problem was observed in transformers; see Section 2.12.)

To avoid excessive magnetization currents, it is customary to decrease the applied stator voltage in direct proportion to the decrease in frequency whenever the frequency falls below the rated frequency of the motor. Since the applied voltage v appears in the numerator of Equation (7–57) and the frequency ω appears in the denominator of Equation (7–57), the two effects counteract each other, and the magnetization current is unaffected.

When the voltage applied to an induction motor is varied linearly with frequency below the base speed, the flux in the motor will remain approximately constant. Therefore, the maximum torque which the motor can supply remains fairly high. However, the maximum power rating of the motor must be decreased linearly with decreases in frequency to protect the stator circuit from overheating. The power supplied to a three-phase induction motor is given by

$$P = \sqrt{3}V_L I_L \cos \theta$$

If the voltage V_L is decreased, then the maximum power P must also be decreased, or else the current flowing in the motor will become excessive, and the motor will overheat.

Figure 7–42a shows a family of induction motor torque–speed characteristic curves for speeds below base speed, assuming that the magnitude of the stator voltage varies linearly with frequency.

When the electrical frequency applied to the motor exceeds the rated frequency of the motor, the stator voltage is held constant at the rated value. Although saturation considerations would permit the voltage to be raised above the rated value under these circumstances, it is limited to the rated voltage to protect the winding insulation of the motor. The higher the electrical frequency above base speed, the larger the denominator of Equation (7–57) becomes. Since the numerator term is held constant above rated frequency, the resulting flux in the machine decreases and the maximum torque decreases with it. Figure 7–42b shows a family of induction motor torque–speed characteristic curves for speeds above base speed, assuming that the stator voltage is held constant.

If the stator voltage is varied linearly with frequency below base speed and is held constant at rated value above base speed, then the resulting family of torque–speed characteristics is as shown in Figure 7–42c. The rated speed for the motor shown in Figure 7–42 is 1800 r/min.

In the past, the principal disadvantage of electrical frequency control as a method of speed changing was that a dedicated generator or mechanical frequency changer was required to make it operate. This problem has disappeared with the development of modern solid-state variable-frequency motor drives. In

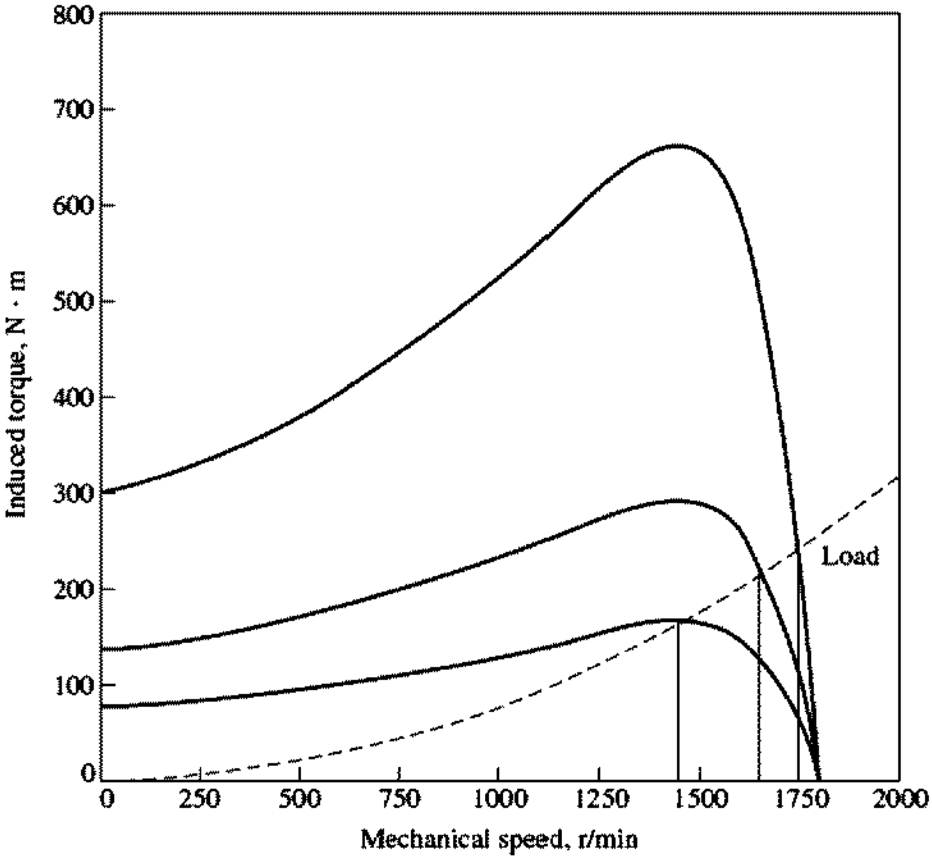


FIGURE 7–43
Variable-line-voltage speed control in an induction motor.

fact, changing the line frequency with solid-state motor drives has become the method of choice for induction motor speed control. Note that this method can be used with *any* induction motor, unlike the pole-changing technique, which requires a motor with special stator windings.

A typical solid-state variable-frequency induction motor drive will be described in Section 7.10.

Speed Control by Changing the Line Voltage

The torque developed by an induction motor is proportional to the square of the applied voltage. If a load has a torque–speed characteristic such as the one shown in Figure 7–43, then the speed of the motor may be controlled over a limited range by varying the line voltage. This method of speed control is sometimes used on small motors driving fans.

Speed Control by Changing the Rotor Resistance

In wound-rotor induction motors, it is possible to change the shape of the torque–speed curve by inserting extra resistances into the rotor circuit of the machine. The resulting torque–speed characteristic curves are shown in Figure 7–44.

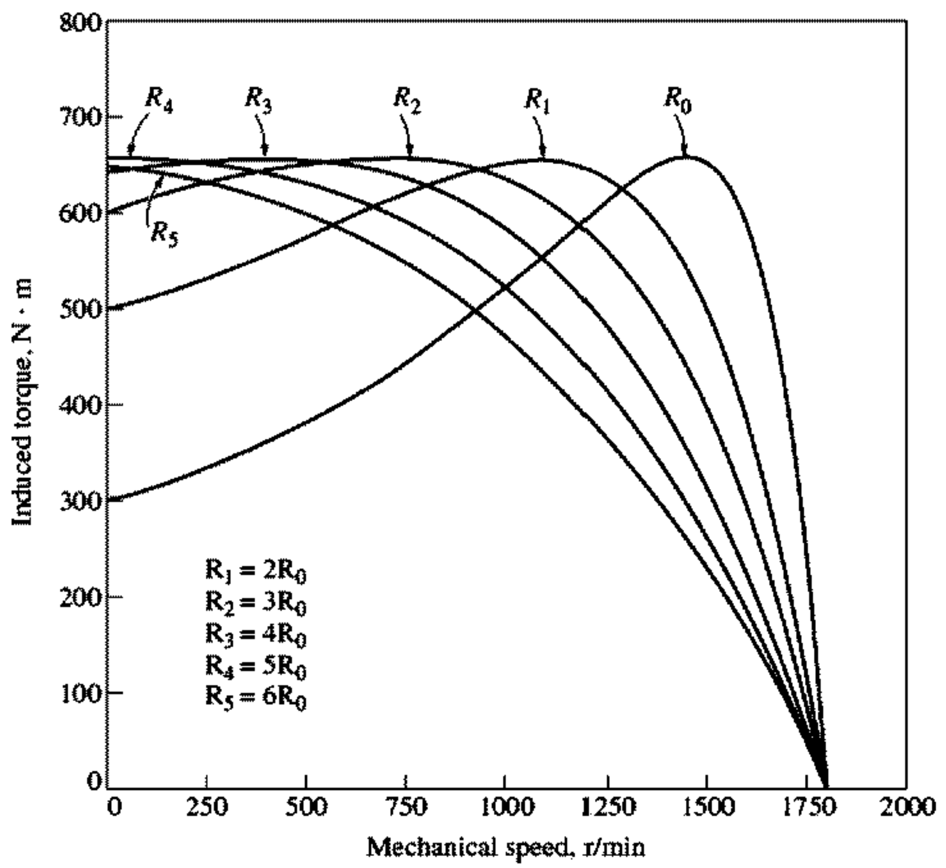


FIGURE 7–44
Speed control by varying the rotor resistance of a wound-rotor induction motor.

If the torque–speed curve of the load is as shown in the figure, then changing the rotor resistance will change the operating speed of the motor. However, inserting extra resistances into the rotor circuit of an induction motor seriously reduces the efficiency of the machine. Such a method of speed control is normally used only for short periods because of this efficiency problem.

7.10 SOLID-STATE INDUCTION MOTOR DRIVES

As mentioned in the previous section, the method of choice today for induction motor speed control is the solid-state variable-frequency induction motor drive. A typical drive of this sort is shown in Figure 7–45. The drive is very flexible: its input power can be either single-phase or three-phase, either 50 or 60 Hz, and anywhere from 208 to 230 V. The output from this drive is a three-phase set of voltages whose frequency can be varied from 0 up to 120 Hz and whose voltage can be varied from 0 V up to the rated voltage of the motor.

The output voltage and frequency control is achieved by using the pulse-width modulation (PWM) techniques described in Chapter 3. Both output frequency and output voltage can be controlled independently by pulse-width modulation. Figure 7–46 illustrates the manner in which the PWM drive can control the output frequency while maintaining a constant rms voltage level, while Figure 7–47 illustrates

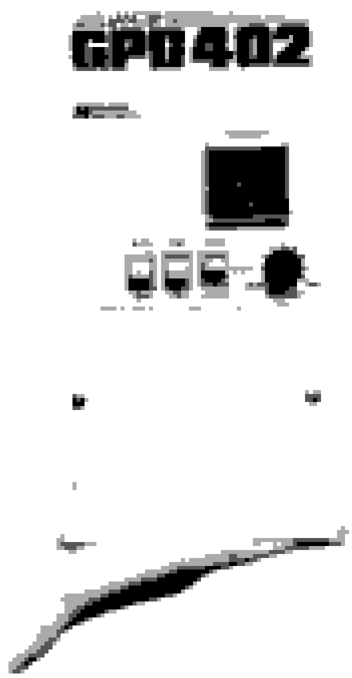


FIGURE 7–45
A typical solid-state variable-frequency induction motor drive. (*Courtesy of MagneTek, Inc.*)

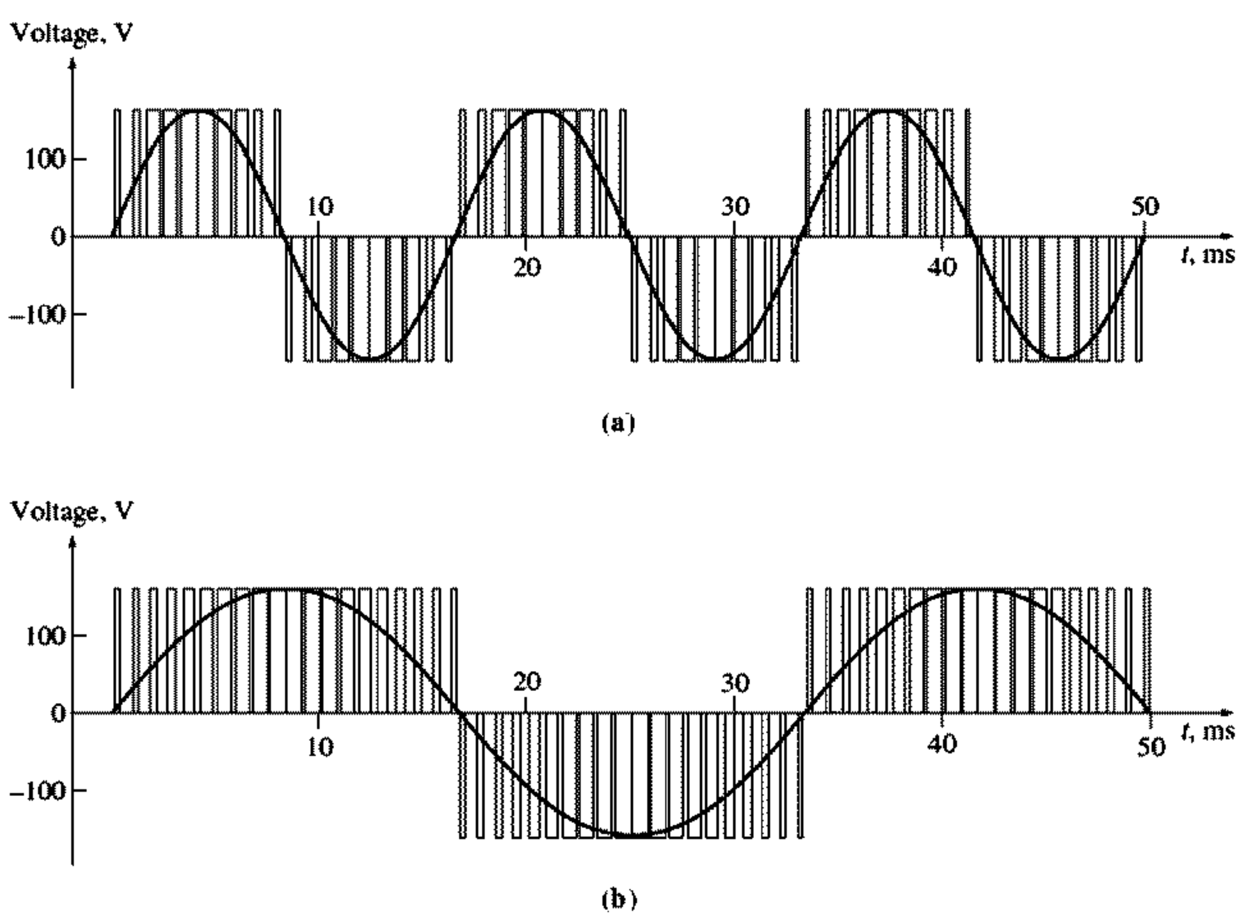


FIGURE 7–46
Variable-frequency control with a PWM waveform: (a) 60-Hz, 120-V PWM waveform; (b) 30-Hz, 120-V PWM waveform.

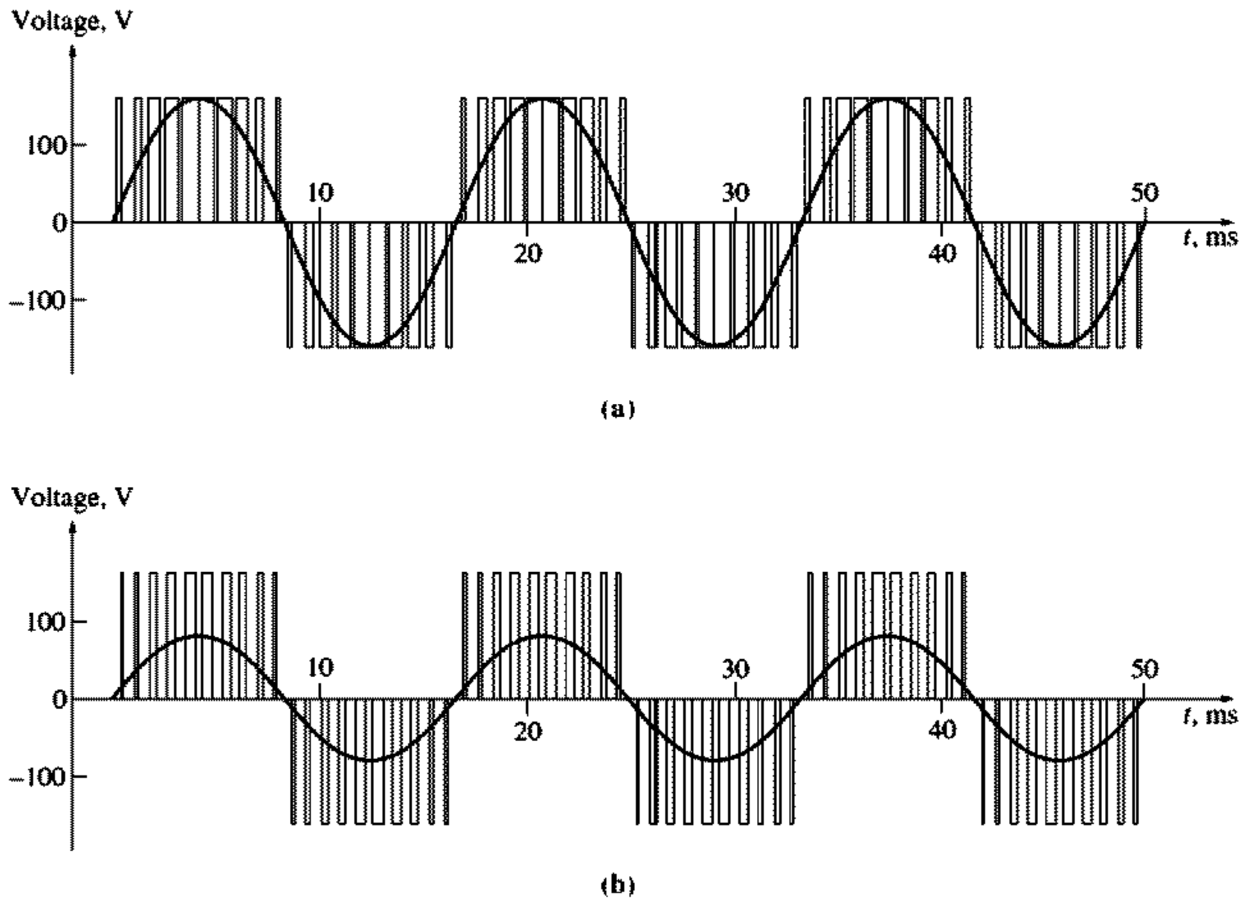


FIGURE 7–47
Variable voltage control with a PWM waveform: (a) 60-Hz, 120-V PWM waveform; (b) 60-Hz, 60-V PWM waveform.

the manner in which the PWM drive can control the rms voltage level while maintaining a constant frequency.

As we described in Section 7.9, it is often desirable to vary the output frequency and output rms voltage together in a linear fashion. Figure 7–48 shows typical output voltage waveforms from one phase of the drive for the situation in which frequency and voltage are varied simultaneously in a linear fashion.* Figure 7–48a shows the output voltage adjusted for a frequency of 60 Hz and an rms voltage of 120 V. Figure 7–48b shows the output adjusted for a frequency of 30 Hz and an rms voltage of 60 V, and Figure 7–48c shows the output adjusted for a frequency of 20 Hz and an rms voltage of 40 V. Notice that the peak voltage out of the drive remains the same in all three cases; the rms voltage level is controlled by the fraction of time the voltage is switched on, and the frequency is controlled by the rate at which the polarity of the pulses switches from positive to negative and back again.

The typical induction motor drive shown in Figure 7–45 has many built-in features which contribute to its adjustability and ease of use. Here is a summary of some of these features.

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*The output waveforms in Figure 7–47 are actually simplified waveforms. The real induction motor drive has a much higher carrier frequency than that shown in the figure.

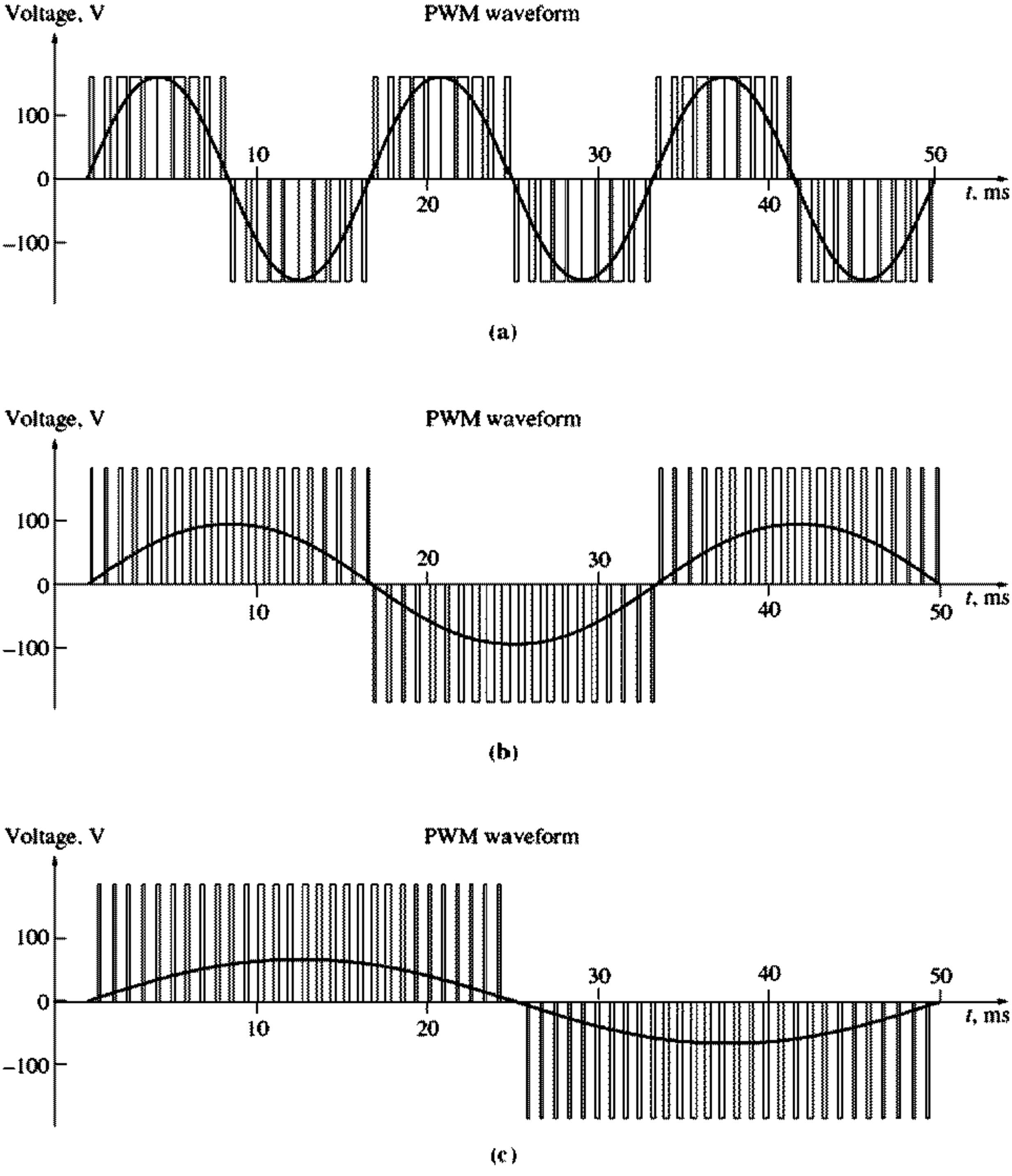


FIGURE 7–48
Simultaneous voltage and frequency control with a PWM waveform: (a) 60-Hz, 120-V PWM waveform; (b) 30-Hz, 60-V PWM waveform; (c) 20-Hz, 40-V PWM waveform.

Frequency (Speed) Adjustment

The output frequency of the drive can be controlled manually from a control mounted on the drive cabinet, or it can be controlled remotely by an external voltage or current signal. The ability to adjust the frequency of the drive in response to some external signal is very important, since it permits an external computer or process controller to control the speed of the motor in accordance with the overall needs of the plant in which it is installed.

A Choice of Voltage and Frequency Patterns

The types of mechanical loads which might be attached to an induction motor vary greatly. Some loads such as fans require very little torque when starting (or running at low speeds) and have torques which increase as the square of the speed. Other loads might be harder to start, requiring more than the rated full-load torque of the motor just to get the load moving. This drive provides a variety of voltage-versus-frequency patterns which can be selected to match the torque from the induction motor to the torque required by its load. Three of these patterns are shown in Figures 7–49 through 7–51.

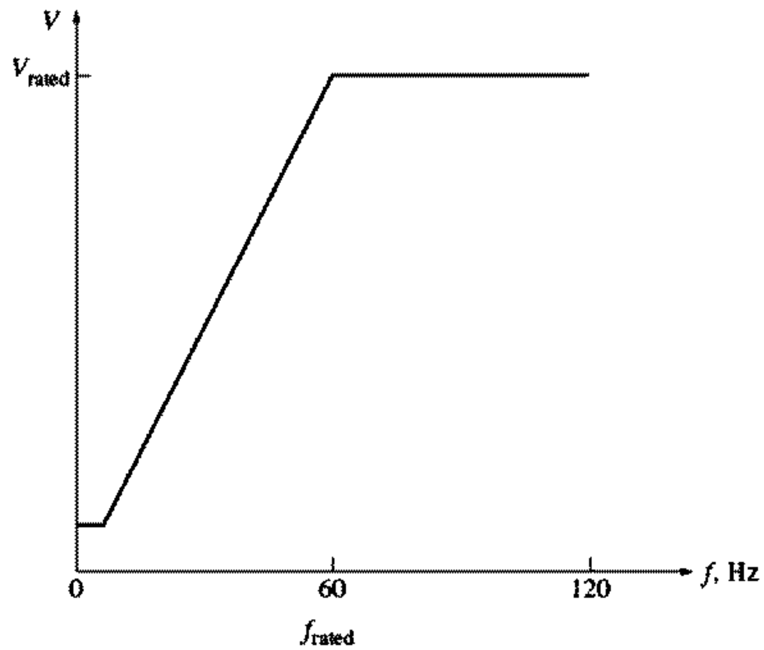
Figure 7–49a shows the standard or general-purpose voltage-versus-frequency pattern, described in the previous section. This pattern changes the output voltage linearly with changes in output frequency for speeds below base speed and holds the output voltage constant for speeds above base speed. (The small constant-voltage region at very low frequencies is necessary to ensure that there will be some starting torque at the very lowest speeds.) Figure 7–49b shows the resulting induction motor torque–speed characteristics for several operating frequencies below base speed.

Figure 7–50a shows the voltage-versus-frequency pattern used for loads with high starting torques. This pattern also changes the output voltage linearly with changes in output frequency for speeds below base speed, but it has a shallower slope at frequencies below 30 Hz. For any given frequency below 30 Hz, the output voltage will be *higher* than it was with the previous pattern. This higher voltage will produce a higher torque, but at the cost of increased magnetic saturation and higher magnetization currents. The increased saturation and higher currents are often acceptable for the short periods required to start heavy loads. Figure 7–50b shows the induction motor torque–speed characteristics for several operating frequencies below base speed. Notice the increased torque available at low frequencies compared to Figure 7–49b.

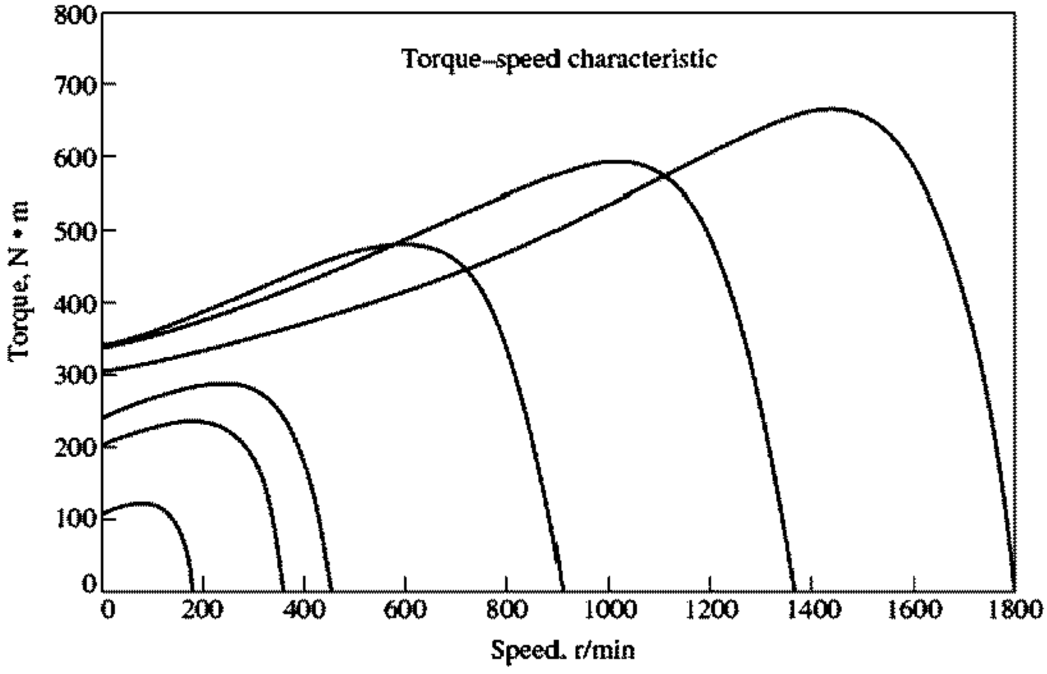
Figure 7–51a shows the voltage-versus-frequency pattern used for loads with low starting torques (called *soft-start loads*). This pattern changes the output voltage parabolically with changes in output frequency for speeds below base speed. For any given frequency below 60 Hz, the output voltage will be lower than it was with the standard pattern. This lower voltage will produce a lower torque, providing a slow, smooth start for low-torque loads. Figure 7–51b shows the induction motor torque–speed characteristics for several operating frequencies below base speed. Notice the decreased torque available at low frequencies compared to Figure 7–49.

Independently Adjustable Acceleration and Deceleration Ramps

When the desired operating speed of the motor is changed, the drive controlling it will change frequency to bring the motor to the new operating speed. If the speed change is sudden (e.g., an instantaneous jump from 900 to 1200 r/min), the drive



(a)



(b)

FIGURE 7–49
(a) Possible voltage-versus-frequency patterns for the solid-state variable-frequency induction motor drive: *general-purpose pattern*. This pattern consists of a linear voltage–frequency curve below rated frequency and a constant voltage above rated frequency. (b) The resulting torque–speed characteristic curves for speeds below rated frequency (speeds above rated frequency look like Figure 7–41b).

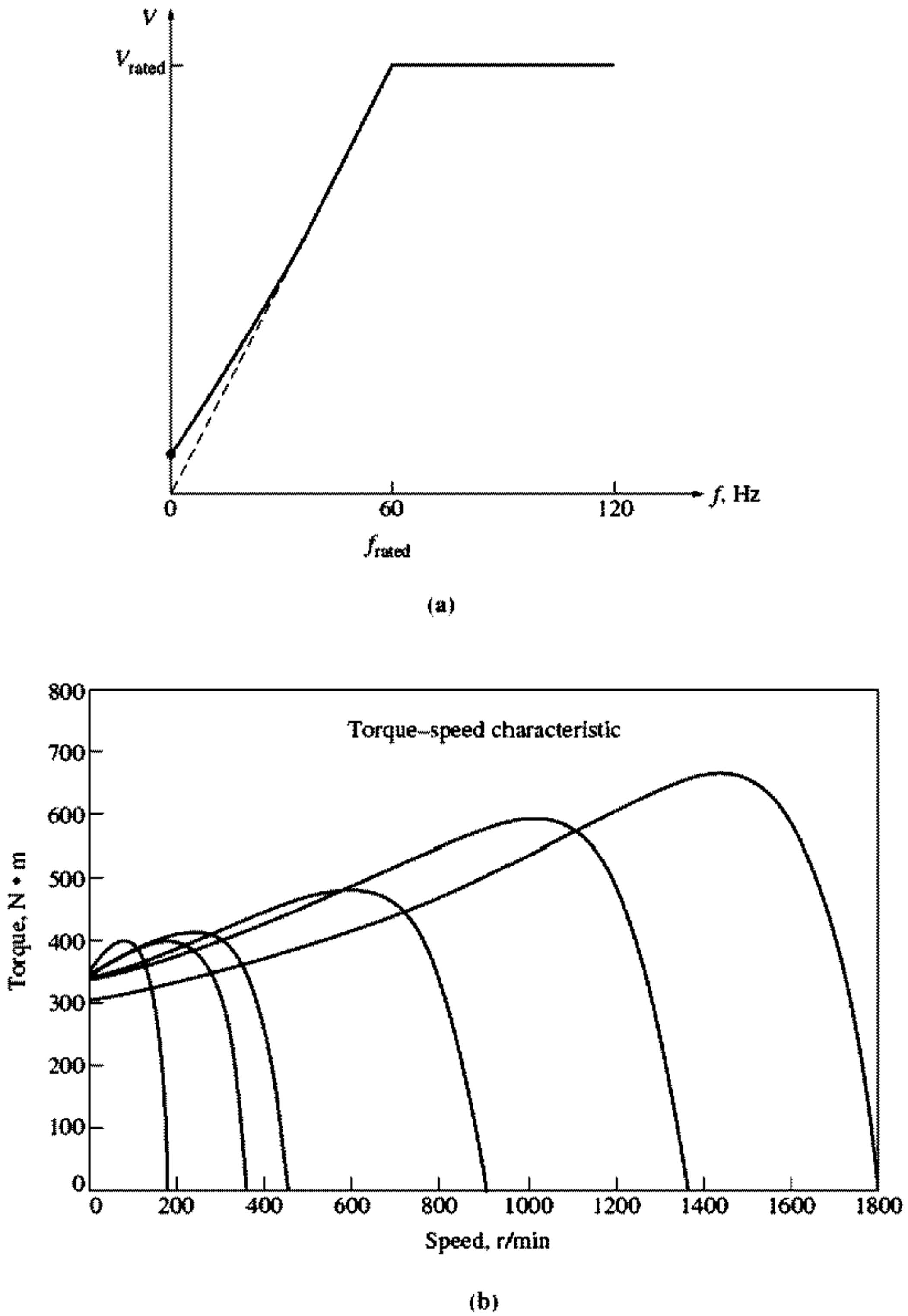
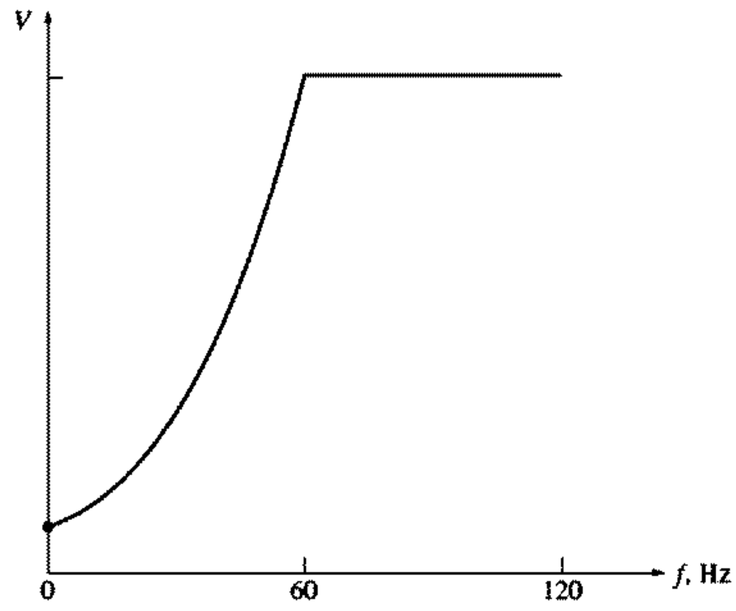
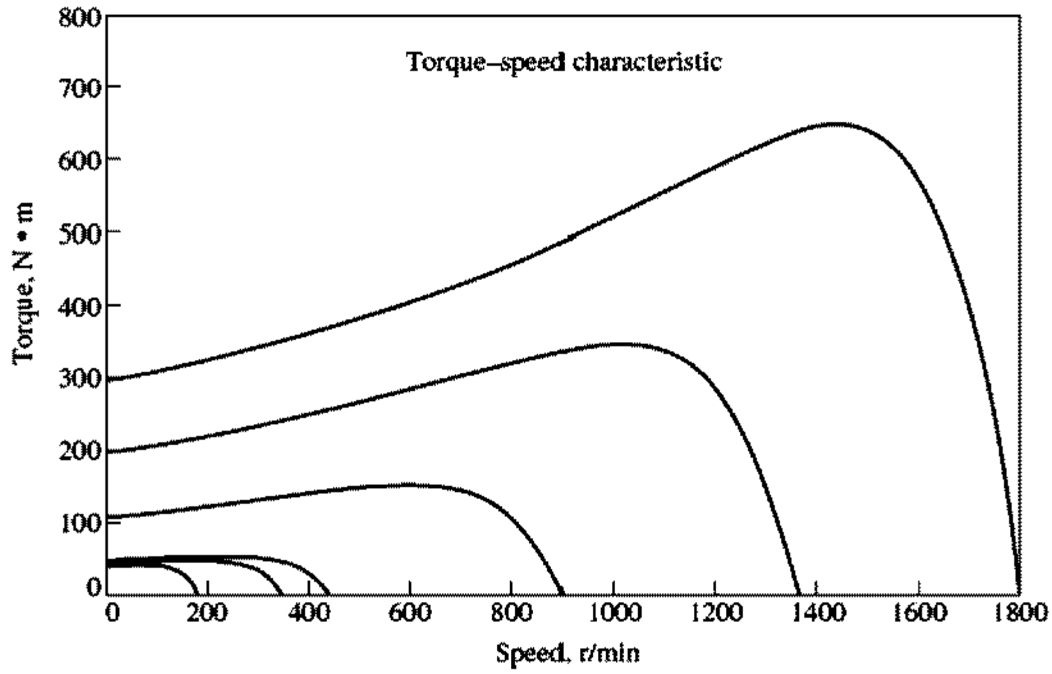


FIGURE 7–50
(a) Possible voltage-versus-frequency patterns for the solid-state variable-frequency induction motor drive: *high-starting-torque pattern.* This is a modified voltage-frequency pattern suitable for loads requiring high starting torques. It is the same as the linear voltage–frequency pattern except at low speeds. The voltage is disproportionately high at very low speeds, which produces extra torque at the cost of a higher magnetization current. (b) The resulting torque–speed characteristic curves for speeds below rated frequency (speeds above rated frequency look like Figure 7–41b).



(a)



(b)

FIGURE 7–51

(a) Possible voltage-versus-frequency patterns for the solid-state variable-frequency induction motor drive: *fan torque pattern*. This is a voltage–frequency pattern suitable for use with motors driving fans and centrifugal pumps, which have a very low starting torque. (b) The resulting torque–speed characteristic curves for speeds below rated frequency (speeds above rated frequency look like Figure 7–41b).

does not try to make the motor instantaneously jump from the old desired speed to the new desired speed. Instead, the rate of motor acceleration or deceleration is limited to a safe level by special circuits built into the electronics of the drive. These rates can be adjusted independently for accelerations and decelerations.

Motor Protection

The induction motor drive has built into it a variety of features designed to protect the motor attached to the drive. The drive can detect excessive steady-state currents (an overload condition), excessive instantaneous currents, overvoltage conditions, or undervoltage conditions. In any of the above cases, it will shut down the motor.

Induction motor drives like the one described above are now so flexible and reliable that induction motors with these drives are displacing dc motors in many applications which require a wide range of speed variation.

7.11 DETERMINING CIRCUIT MODEL PARAMETERS

The equivalent circuit of an induction motor is a very useful tool for determining the motor's response to changes in load. However, if a model is to be used for a real machine, it is necessary to determine what the element values are that go into the model. How can R_1, R_2, X_1, X_2, and X_M be determined for a real motor?

These pieces of information may be found by performing a series of tests on the induction motor that are analogous to the short-circuit and open-circuit tests in a transformer. The tests must be performed under precisely controlled conditions, since the resistances vary with temperature and the rotor resistance also varies with rotor frequency. The exact details of how each induction motor test must be performed in order to achieve accurate results are described in IEEE Standard 112. Although the details of the tests are very complicated, the concepts behind them are relatively straightforward and will be explained here.

The No-Load Test

The no-load test of an induction motor measures the rotational losses of the motor and provides information about its magnetization current. The test circuit for this test is shown in Figure 7–52a. Wattmeters, a voltmeter, and three ammeters are connected to an induction motor, which is allowed to spin freely. The only load on the motor is the friction and windage losses, so all P_{conv} in this motor is consumed by mechanical losses, and the slip of the motor is very small (possibly as small as 0.001 or less). The equivalent circuit of this motor is shown in Figure 7–52b. With its very small slip, the resistance corresponding to its power converted, $R_2(1 - s)/s$, is much much larger than the resistance corresponding to the rotor copper losses R_2 and much larger than the rotor reactance X_2. In this case, the equivalent circuit reduces approximately to the last circuit in Figure 7–52b. There,

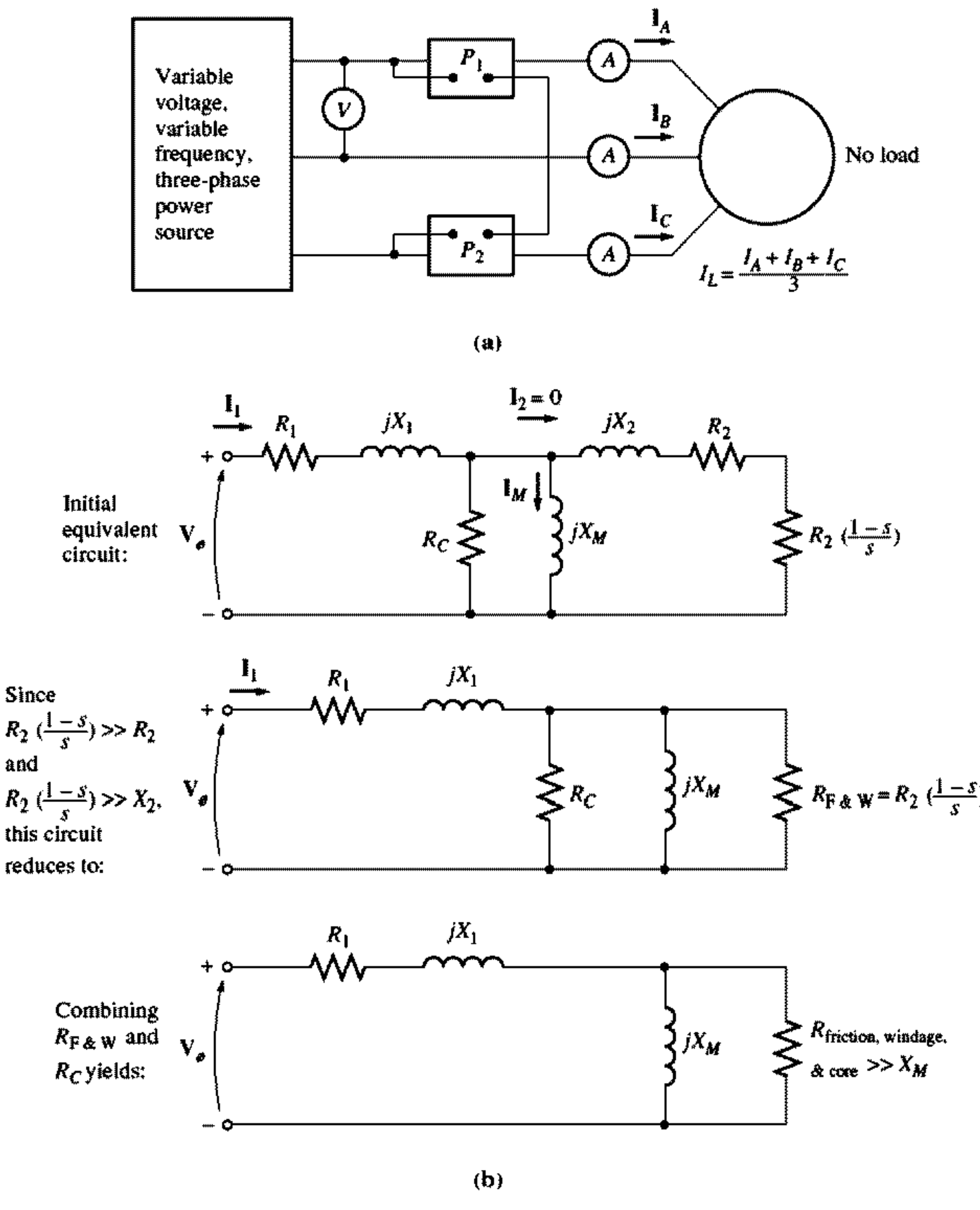


FIGURE 7–52
The no-load test of an induction motor: (a) test circuit; (b) the resulting motor equivalent circuit. Note that at no load the motor's impedance is essentially the series combination of R_1, jX_1, and jX_M.

the output resistor is in parallel with the magnetization reactance X_M and the core losses R_C.

In this motor at no-load conditions, the input power measured by the meters must equal the losses in the motor. The rotor copper losses are negligible because the current I_2 is *extremely* small [because of the large load resistance $R_2(1 - s)/s$], so they may be neglected. The stator copper losses are given by

$$\boxed{P_{\mathrm{SCL}} = 3I_1^2 R_1} \tag{7-25}$$

so the input power must equal

$$\begin{aligned} P_{\mathrm{in}} &= P_{\mathrm{SCL}} + P_{\mathrm{core}} + P_{\mathrm{F\&W}} + P_{\mathrm{misc}} \\ &= 3I_1^2 R_1 + P_{\mathrm{rot}} \end{aligned} \tag{7-58}$$

where P_{rot} is the rotational losses of the motor:

$$P_{\mathrm{rot}} = P_{\mathrm{core}} + P_{\mathrm{F\&W}} + P_{\mathrm{misc}} \tag{7-59}$$

Thus, given the input power to the motor, the rotational losses of the machine may be determined.

The equivalent circuit that describes the motor operating in this condition contains resistors R_C and $R_2(1-s)/s$ in parallel with the magnetizing reactance X_M. The current needed to establish a magnetic field is quite large in an induction motor, because of the high reluctance of its air gap, so the reactance X_M will be much smaller than the resistances in parallel with it and the overall input power factor will be very small. With the large lagging current, most of the voltage drop will be across the inductive components in the circuit. The equivalent input impedance is thus approximately

$$\boxed{|Z_{\mathrm{eq}}| = \frac{V_\phi}{I_{1,\mathrm{nl}}} \approx X_1 + X_M} \tag{7-60}$$

and if X_1 can be found in some other fashion, the magnetizing impedance X_M will be known for the motor.

The DC Test for Stator Resistance

The rotor resistance R_2 plays an extremely critical role in the operation of an induction motor. Among other things, R_2 determines the shape of the torque–speed curve, determining the speed at which the pullout torque occurs. A standard motor test called the *locked-rotor test* can be used to determine the total motor circuit resistance (this test is taken up in the next section). However, this test finds only the *total* resistance. To find the rotor resistance R_2 accurately, it is necessary to know R_1 so that it can be subtracted from the total.

There is a test for R_1 independent of R_2, X_1 and X_2. This test is called the *dc test*. Basically, a dc voltage is applied to the stator windings of an induction motor. Because the current is dc, there is no induced voltage in the rotor circuit and no resulting rotor current flow. Also, the reactance of the motor is zero at direct current. Therefore, the only quantity limiting current flow in the motor is the stator resistance, and that resistance can be determined.

The basic circuit for the dc test is shown in Figure 7–53. This figure shows a dc power supply connected to two of the three terminals of a Y-connected induction motor. To perform the test, the current in the stator windings is adjusted to the rated value, and the voltage between the terminals is measured. The current in

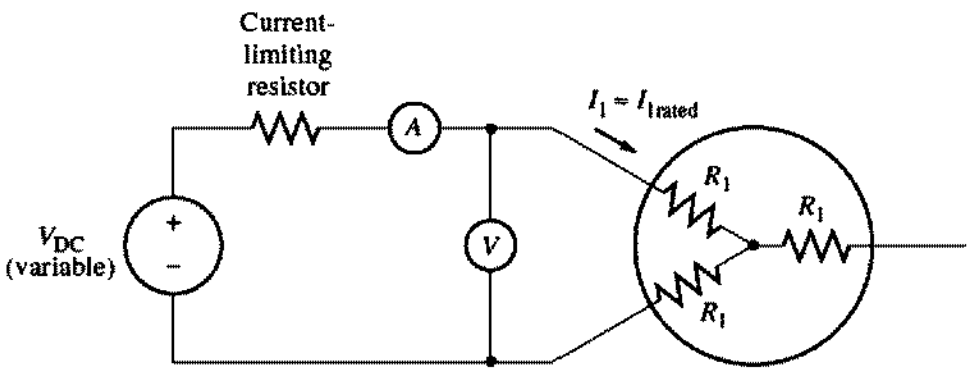


FIGURE 7–53
Test circuit for a dc resistance test.

the stator windings is adjusted to the rated value in an attempt to heat the windings to the same temperature they would have during normal operation (remember, winding resistance is a function of temperature).

The current in Figure 7–53 flows through two of the windings, so the total resistance in the current path is $2R_1$. Therefore,

$$2R_1 = \frac{V_{DC}}{I_{DC}}$$

or

$$\boxed{R_1 = \frac{V_{DC}}{2I_{DC}}} \tag{7–61}$$

With this value of R_1 the stator copper losses at no load may be determined, and the rotational losses may be found as the difference between the input power at no load and the stator copper losses.

The value of R_1 calculated in this fashion is not completely accurate, since it neglects the skin effect that occurs when an ac voltage is applied to the windings. More details concerning corrections for temperature and skin effect can be found in IEEE Standard 112.

The Locked-Rotor Test

The third test that can be performed on an induction motor to determine its circuit parameters is called the *locked-rotor test*, or sometimes the *blocked-rotor test*. This test corresponds to the short-circuit test on a transformer. In this test, the rotor is locked or blocked so that it *cannot* move, a voltage is applied to the motor, and the resulting voltage, current, and power are measured.

Figure 7–54a shows the connections for the locked-rotor test. To perform the locked-rotor test, an ac voltage is applied to the stator, and the current flow is adjusted to be approximately full-load value. When the current is full-load value, the voltage, current, and power flowing into the motor are measured. The equivalent circuit for this test is shown in Figure 7–54b. Notice that since the rotor is not moving, the slip $s = 1$, and so the rotor resistance R_2/s is just equal to R_2 (quite a

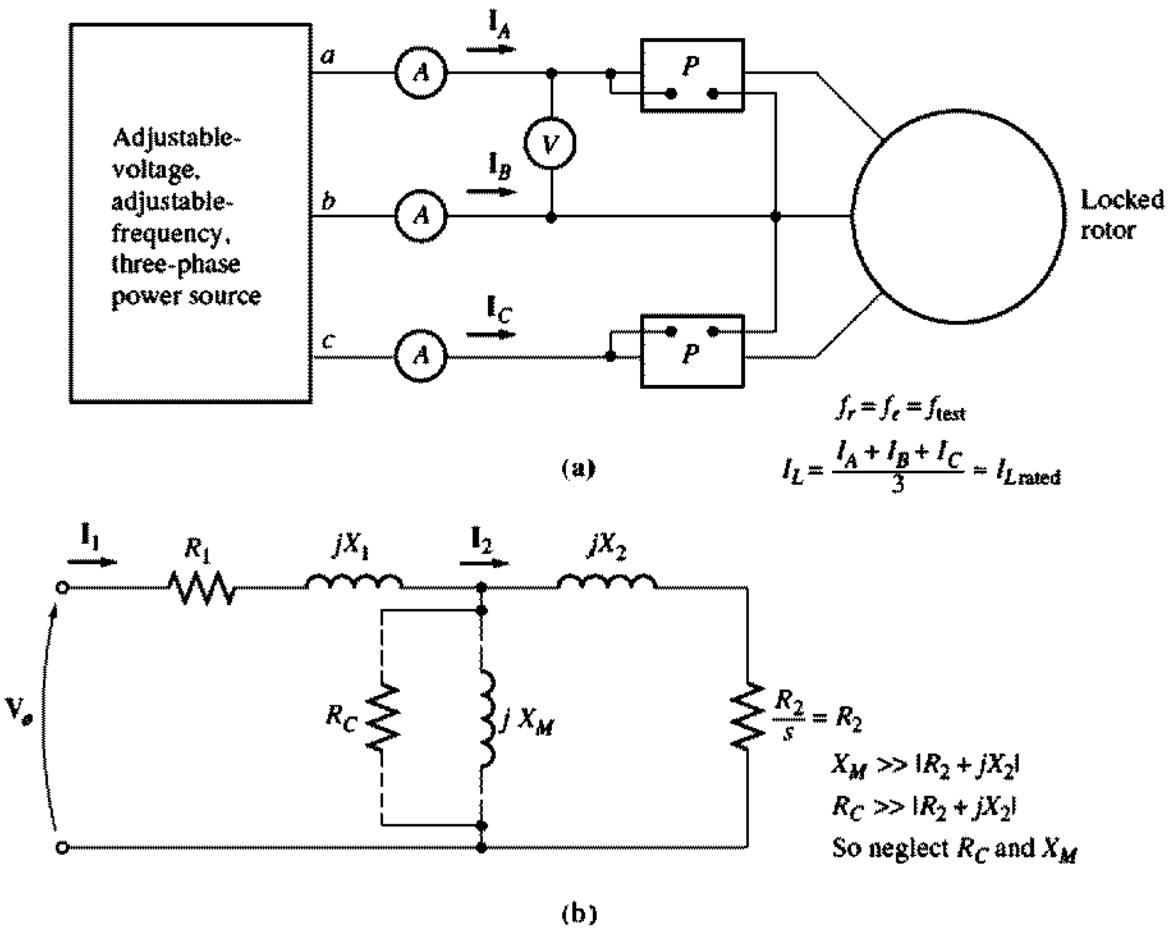


FIGURE 7–54
The locked-rotor test for an induction motor: (a) test circuit; (b) motor equivalent circuit.

small value). Since R_2 and X_2 are so small, almost all the input current will flow through them, instead of through the much larger magnetizing reactance X_M. Therefore, the circuit under these conditions looks like a series combination of X_1, R_1, X_2, and R_2.

There is one problem with this test, however. In normal operation, the stator frequency is the line frequency of the power system (50 or 60 Hz). At starting conditions, the rotor is also at line frequency. However, at normal operating conditions, the slip of most motors is only 2 to 4 percent, and the resulting rotor frequency is in the range of 1 to 3 Hz. This creates a problem in that *the line frequency does not represent the normal operating conditions of the rotor.* Since effective rotor resistance is a strong function of frequency for design class B and C motors, the incorrect rotor frequency can lead to misleading results in this test. A typical compromise is to use a frequency 25 percent or less of the rated frequency. While this approach is acceptable for essentially constant resistance rotors (design classes A and D), it leaves a lot to be desired when one is trying to find the normal rotor resistance of a variable-resistance rotor. Because of these and similar problems, a great deal of care must be exercised in taking measurements for these tests.

After a test voltage and frequency have been set up, the current flow in the motor is quickly adjusted to about the rated value, and the input power, voltage, and current are measured before the rotor can heat up too much. The input power to the motor is given by

$$P = \sqrt{3} V_T I_L \cos\theta$$

so the locked-rotor power factor can be found as

$$\boxed{\mathrm{PF} = \cos\theta = \frac{P_{\text{in}}}{\sqrt{3} V_T I_L}} \tag{7–62}$$

and the impedance angle θ is just equal to $\cos^{-1}$ PF.

The magnitude of the total impedance in the motor circuit at this time is

$$\boxed{|Z_{\text{LR}}| = \frac{V_\phi}{I_1} = \frac{V_T}{\sqrt{3} I_L}} \tag{7–63}$$

and the angle of the total impedance is θ. Therefore,

$$\begin{aligned} Z_{\text{LR}} &= R_{\text{LR}} + jX'_{\text{LR}} \\ &= |Z_{\text{LR}}|\cos\theta + j|Z_{\text{LR}}|\sin\theta \end{aligned} \tag{7–64}$$

The locked-rotor resistance R_{LR} is equal to

$$\boxed{R_{\text{LR}} = R_1 + R_2} \tag{7–65}$$

while the locked-rotor reactance X'_{LR} is equal to

$$X'_{\text{LR}} = X'_1 + X'_2 \tag{7–66}$$

where X'_1 and X'_2 are the stator and rotor reactances *at the test frequency*, respectively.

The rotor resistance R_2 can now be found as

$$R_2 = R_{\text{LR}} - R_1 \tag{7–67}$$

where R_1 was determined in the dc test. The total rotor reactance referred to the stator can also be found. Since the reactance is directly proportional to the frequency, the total equivalent reactance at the normal operating frequency can be found as

$$\boxed{X_{\text{LR}} = \frac{f_{\text{rated}}}{f_{\text{test}}} X'_{\text{LR}} = X_1 + X_2} \tag{7–68}$$

Unfortunately, there is no simple way to separate the contributions of the stator and rotor reactances from each other. Over the years, experience has shown that motors of certain design types have certain proportions between the rotor and stator reactances. Figure 7–55 summarizes this experience. In normal practice, it really does not matter just how X_{LR} is broken down, since the reactance appears as the sum $X_1 + X_2$ in all the torque equations.

	X_1 and X_2 as functions of X_{LR}	
Rotor Design	X_1	X_2
Wound rotor	$0.5\ X_{LR}$	$0.5\ X_{LR}$
Design A	$0.5\ X_{LR}$	$0.5\ X_{LR}$
Design B	$0.4\ X_{LR}$	$0.6\ X_{LR}$
Design C	$0.3\ X_{LR}$	$0.7\ X_{LR}$
Design D	$0.5\ X_{LR}$	$0.5\ X_{LR}$

FIGURE 7–55
Rules of thumb for dividing rotor and stator circuit reactance.

Example 7–8. The following test data were taken on a 7.5-hp, four-pole, 208-V, 60-Hz, design A, Y-connected induction motor having a rated current of 28 A.

DC test:

$$V_{DC} = 13.6 \text{ V} \qquad I_{DC} = 28.0 \text{ A}$$

No-load test:

$$V_T = 208 \text{ V} \qquad f = 60 \text{ Hz}$$
$$I_A = 8.12 \text{ A} \qquad P_{in} = 420 \text{ W}$$
$$I_B = 8.20 \text{ A}$$
$$I_C = 8.18 \text{ A}$$

Locked-rotor test:

$$V_T = 25 \text{ V} \qquad f = 15 \text{ Hz}$$
$$I_A = 28.1 \text{ A} \qquad P_{in} = 920 \text{ W}$$
$$I_B = 28.0 \text{ A}$$
$$I_C = 27.6 \text{ A}$$

(*a*) Sketch the per-phase equivalent circuit for this motor.

(*b*) Find the slip at the pullout torque, and find the value of the pullout torque itself.

Solution

(*a*) From the dc test,

$$R_1 = \frac{V_{DC}}{2I_{DC}} = \frac{13.6 \text{ V}}{2(28.0 \text{ A})} = 0.243\ \Omega$$

From the no-load test,

$$I_{L,av} = \frac{8.12 \text{ A} + 8.20 \text{ A} + 8.18 \text{ A}}{3} = 8.17 \text{ A}$$

$$V_{\phi,nl} = \frac{208 \text{ V}}{\sqrt{3}} = 120 \text{ V}$$

Therefore,

$$|Z_{nl}| = \frac{120\ \text{V}}{8.17\ \text{A}} = 14.7\ \Omega = X_1 + X_M$$

When X_1 is known, X_M can be found. The stator copper losses are

$$P_{SCL} = 3I_1^2 R_1 = 3(8.17\ \text{A})^2(0.243\ \Omega) = 48.7\ \text{W}$$

Therefore, the no-load rotational losses are

$$\begin{aligned} P_{rot} &= P_{in,nl} - P_{SCL,nl} \\ &= 420\ \text{W} - 48.7\ \text{W} = 371.3\ \text{W} \end{aligned}$$

From the locked-rotor test,

$$I_{L,av} = \frac{28.1\ \text{A} + 28.0\ \text{A} + 27.6\ \text{A}}{3} = 27.9\ \text{A}$$

The locked-rotor impedance is

$$|Z_{LR}| = \frac{V_\phi}{I_A} = \frac{V_T}{\sqrt{3}I_A} = \frac{25\ \text{V}}{\sqrt{3}(27.9\ \text{A})} = 0.517\ \Omega$$

and the impedance angle θ is

$$\begin{aligned} \theta &= \cos^{-1} \frac{P_{in}}{\sqrt{3}V_T I_L} \\ &= \cos^{-1} \frac{920\ \text{W}}{\sqrt{3}(25\ \text{V})(27.9\ \text{A})} \\ &= \cos^{-1} 0.762 = 40.4° \end{aligned}$$

Therefore, $R_{LR} = 0.517 \cos 40.4° = 0.394\ \Omega = R_1 + R_2$. Since $R_1 = 0.243\ \Omega$, R_2 must be $0.151\ \Omega$. The reactance at 15 Hz is

$$X'_{LR} = 0.517 \sin 40.4° = 0.335\ \Omega$$

The equivalent reactance at 60 Hz is

$$X_{LR} = \frac{f_{rated}}{f_{test}} X'_{LR} = \left(\frac{60\ \text{Hz}}{15\ \text{Hz}}\right) 0.335\ \Omega = 1.34\ \Omega$$

For design class A induction motors, this reactance is assumed to be divided equally between the rotor and stator, so

$$X_1 = X_2 = 0.67\ \Omega$$

$$X_M = |Z_{nl}| - X_1 = 14.7\ \Omega - 0.67\ \Omega = 14.03\ \Omega$$

The final per-phase equivalent circuit is shown in Figure 7–56.

(b) For this equivalent circuit, the Thevenin equivalents are found from Equations (7–41b), (7–44), and (7–45) to be

$$V_{TH} = 114.6\ \text{V} \qquad R_{TH} = 0.221\ \Omega \qquad X_{TH} = 0.67\ \Omega$$

Therefore, the slip at the pullout torque is given by

$$s_{max} = \frac{R_2}{\sqrt{R_{TH}^2 + (X_{TH} + X_2)^2}} \tag{7–53}$$

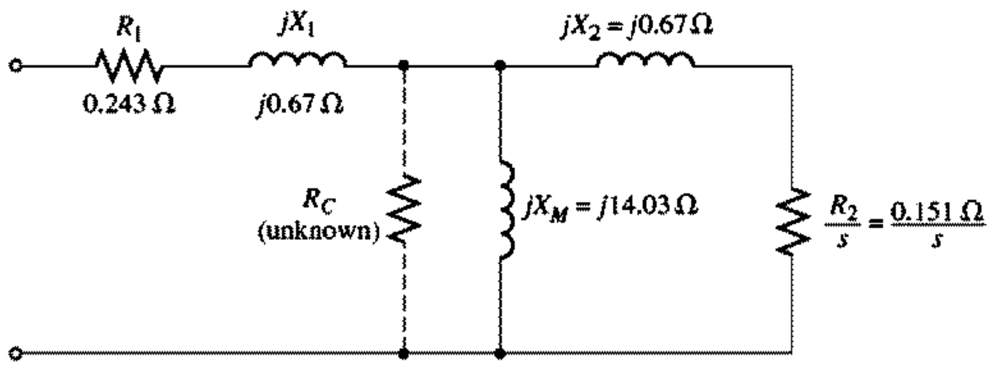


FIGURE 7–56
Motor per-phase equivalent circuit for Example 7–8.

$$= \frac{0.151\,\Omega}{\sqrt{(0.243\,\Omega)^2 + (0.67\,\Omega + 0.67\,\Omega)^2}} = 0.111 = 11.1\%$$

The maximum torque of this motor is given by

$$\tau_{\max} = \frac{3V_{TH}^2}{2\omega_{sync}[R_{TH} + \sqrt{R_{TH}^2 + (X_{TH} + X^2)}]} \tag{7–54}$$

$$= \frac{3(114.6\text{ V})^2}{2(188.5\text{ rad/s})[0.221\,\Omega + \sqrt{(0.221\,\Omega)^2 + (0.67\,\Omega + 0.67\,\Omega)^2}]}$$

$$= 66.2\text{ N}\cdot\text{m}$$

7.12 THE INDUCTION GENERATOR

The torque–speed characteristic curve in Figure 7–20 shows that if an induction motor is driven at a speed *greater* than n_{sync} by an external prime mover, the direction of its inducted torque will reverse and it will act as a generator. As the torque applied to its shaft by the prime mover increases, the amount of power produced by the induction generator increases. As Figure 7–57 shows, there is a maximum possible induced torque in the generator mode of operation. This torque is known as the *pushover torque* of the generator. If a prime mover applies a torque greater than the pushover torque to the shaft of an induction generator, the generator will overspeed.

As a generator, an induction machine has severe limitations. Because it lacks a separate field circuit, an induction generator *cannot* produce reactive power. In fact, it consumes reactive power, and an external source of reactive power must be connected to it at all times to maintain its stator magnetic field. This external source of reactive power must also control the terminal voltage of the generator—with no field current, an induction generator cannot control its own output voltage. Normally, the generator's voltage is maintained by the external power system to which it is connected.

The one great advantage of an induction generator is its simplicity. An induction generator does not need a separate field circuit and does not have to be driven continuously at a fixed speed. As long as the machine's speed is some

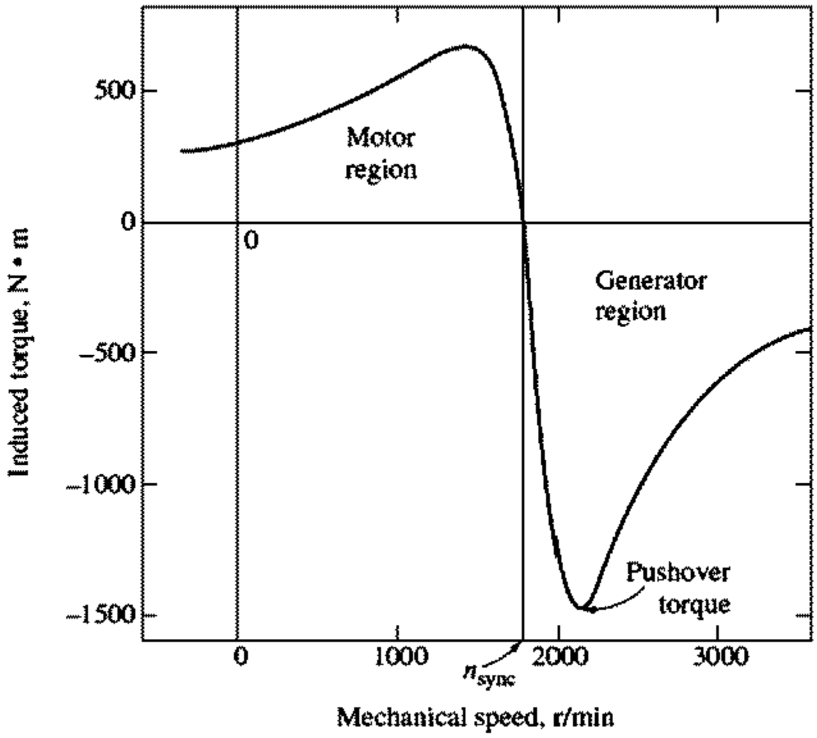


FIGURE 7–57
The torque–speed characteristic of an induction machine, showing the generator region of operation. Note the pushover torque.

value greater than n_{sync} for the power system to which it is connected, it will function as a generator. The greater the torque applied to its shaft (up to a certain point), the greater its resulting output power. The fact that no fancy regulation is required makes this generator a good choice for windmills, heat recovery systems, and similar supplementary power sources attached to an existing power system. In such applications, power-factor correction can be provided by capacitors, and the generator's terminal voltage can be controlled by the external power system.

The Induction Generator Operating Alone

It is also possible for an induction machine to function as an isolated generator, independent of any power system, as long as capacitors are available to supply the reactive power required by the generator and by any attached loads. Such an isolated induction generator is shown in Figure 7–58.

The magnetizing current I_M required by an induction machine as a function of terminal voltage can be found by running the machine as a motor at no load and measuring its armature current as a function of terminal voltage. Such a magnetization curve is shown in Figure 7–59a. To achieve a given voltage level in an induction generator, external capacitors must supply the magnetization current corresponding to that level.

Since the reactive current that a capacitor can produce is *directly proportional* to the voltage applied to it, the locus of all possible combinations of voltage and current through a capacitor is a straight line. Such a plot of voltage versus current

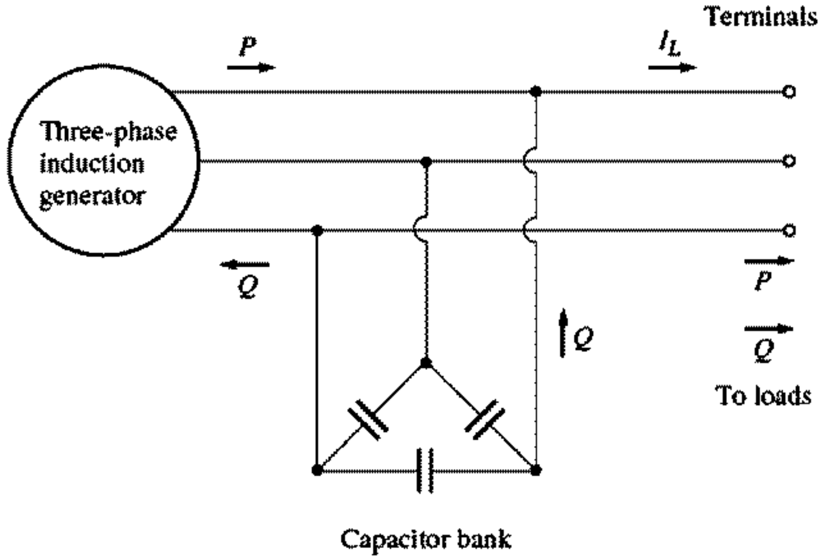


FIGURE 7–58
An induction generator operating alone with a capacitor bank to supply reactive power.

for a given frequency is shown in Figure 7–59b. *If a three-phase set of capacitors is connected across the terminals of an induction generator, the no-load voltage of the induction generator will be the intersection of the generator's magnetization curve and the capacitor's load line.* The no-load terminal voltage of an induction generator for three different sets of capacitance is shown in Figure 7–59c.

How does the voltage build up in an induction generator when it is first started? When an induction generator first starts to turn, the residual magnetism in its field circuit produces a small voltage. That small voltage produces a capacitive current flow, which increases the voltage, further increasing the capacitive current, and so forth until the voltage is fully built up. If no residual flux is present in the induction generator's rotor, then its voltage will not build up, and it must be magnetized by momentarily running it as a motor.

The most serious problem with an induction generator is that its voltage varies wildly with changes in load, especially reactive load. Typical terminal characteristics of an induction generator operating alone with a constant parallel capacitance are shown in Figure 7–60. Notice that, in the case of inductive loading, the voltage collapses *very* rapidly. This happens because the fixed capacitors must supply all the reactive power needed by both the load and the generator, and any reactive power diverted to the load moves the generator back along its magnetization curve, causing a major drop in generator voltage. It is therefore very difficult to start an induction motor on a power system supplied by an induction generator—special techniques must be employed to increase the effective capacitance during starting and then decrease it during normal operation.

Because of the nature of the induction machine's torque–speed characteristic, an induction generator's frequency varies with changing loads: but since the torque–speed characteristic is very steep in the normal operating range, the total frequency variation is usually limited to less than 5 percent. This amount of variation may be quite acceptable in many isolated or emergency generator applications.

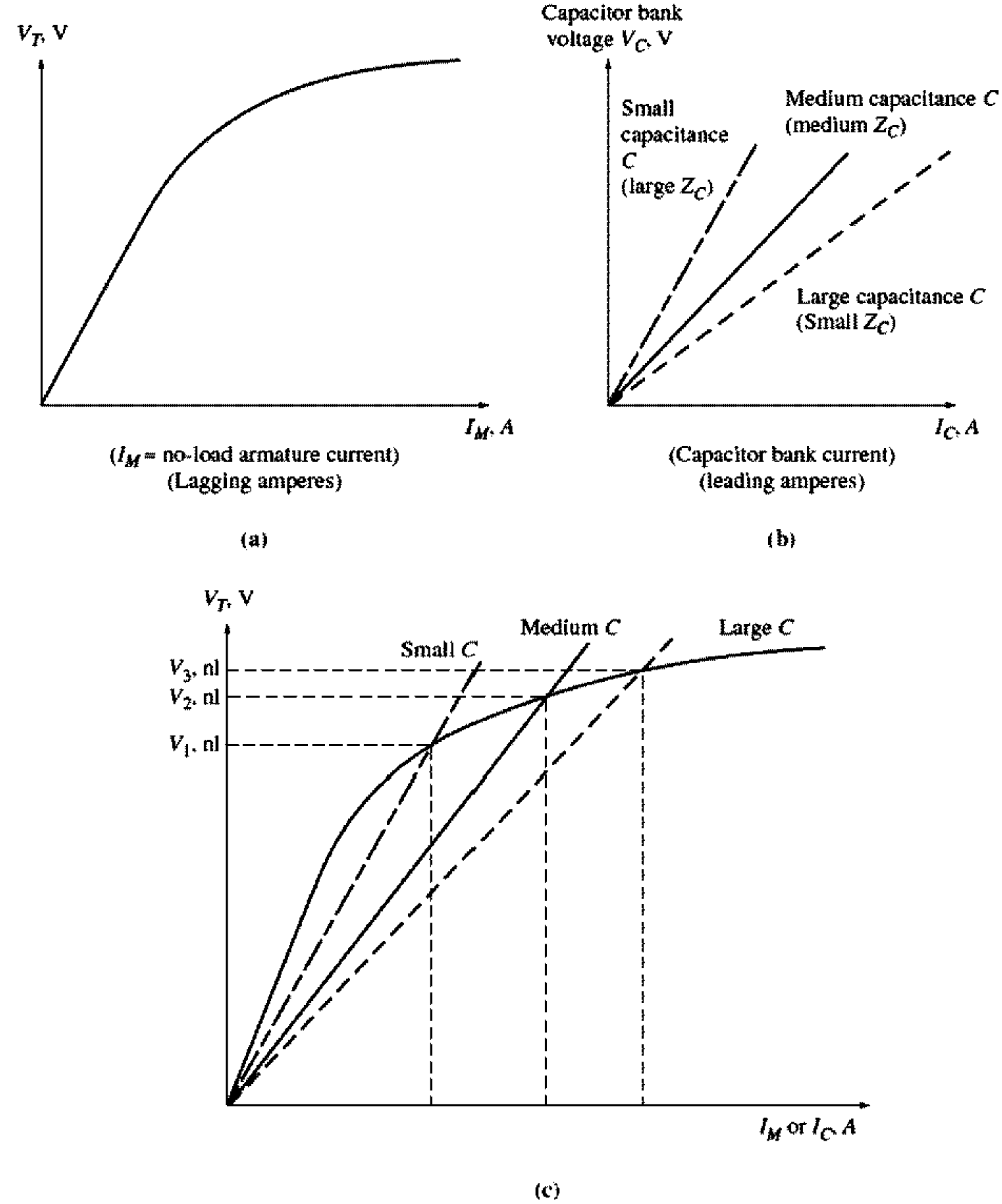


FIGURE 7–59
(a) The magnetization curve of an induction machine. It is a plot of the terminal voltage of the machine as a function of its magnetization current (which *lags* the phase voltage by approximately 90°). (b) Plot of the voltage–current characteristic of a capacitor bank. Note that the larger the capacitance, the greater its current for a given voltage. This current *leads* the phase voltage by approximately 90°. (c) The no-load terminal voltage for an isolated induction generator can be found by plotting the generator terminal characteristic and the capacitor voltage–current characteristic on a single set of axes. The intersection of the two curves is the point at which the reactive power demanded by the generator is exactly supplied by the capacitors, and this point gives the *no-load terminal voltage* of the generator.

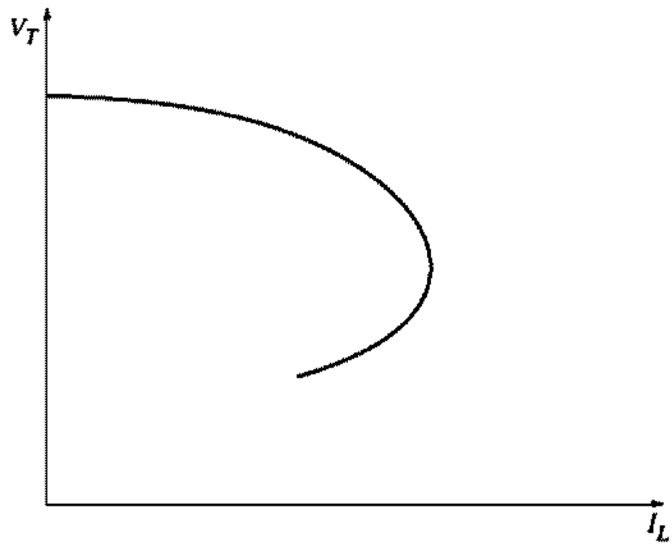


FIGURE 7–60
The terminal voltage–current characteristic of an induction generator for a load with a constant lagging power factor.

Induction Generator Applications

Induction generators have been used since early in the twentieth century, but by the 1960s and 1970s they had largely disappeared from use. However, the induction generator has made a comeback since the oil price shocks of 1973. With energy costs so high, energy recovery became an important part of the economics of most industrial processes. The induction generator is ideal for such applications because it requires very little in the way of control systems or maintenance.

Because of their simplicity and small size per kilowatt of output power, induction generators are also favored very strongly for small windmills. Many commercial windmills are designed to operate in parallel with large power systems, supplying a fraction of the customer's total power needs. In such operation, the power system can be relied on for voltage and frequency control, and static capacitors can be used for power-factor correction.

7.13 INDUCTION MOTOR RATINGS

A nameplate for a typical high-efficiency integral-horsepower induction motor is shown in Figure 7–61. The most important ratings present on the nameplate are

1. Output power
2. Voltage
3. Current
4. Power factor
5. Speed
6. Nominal efficiency

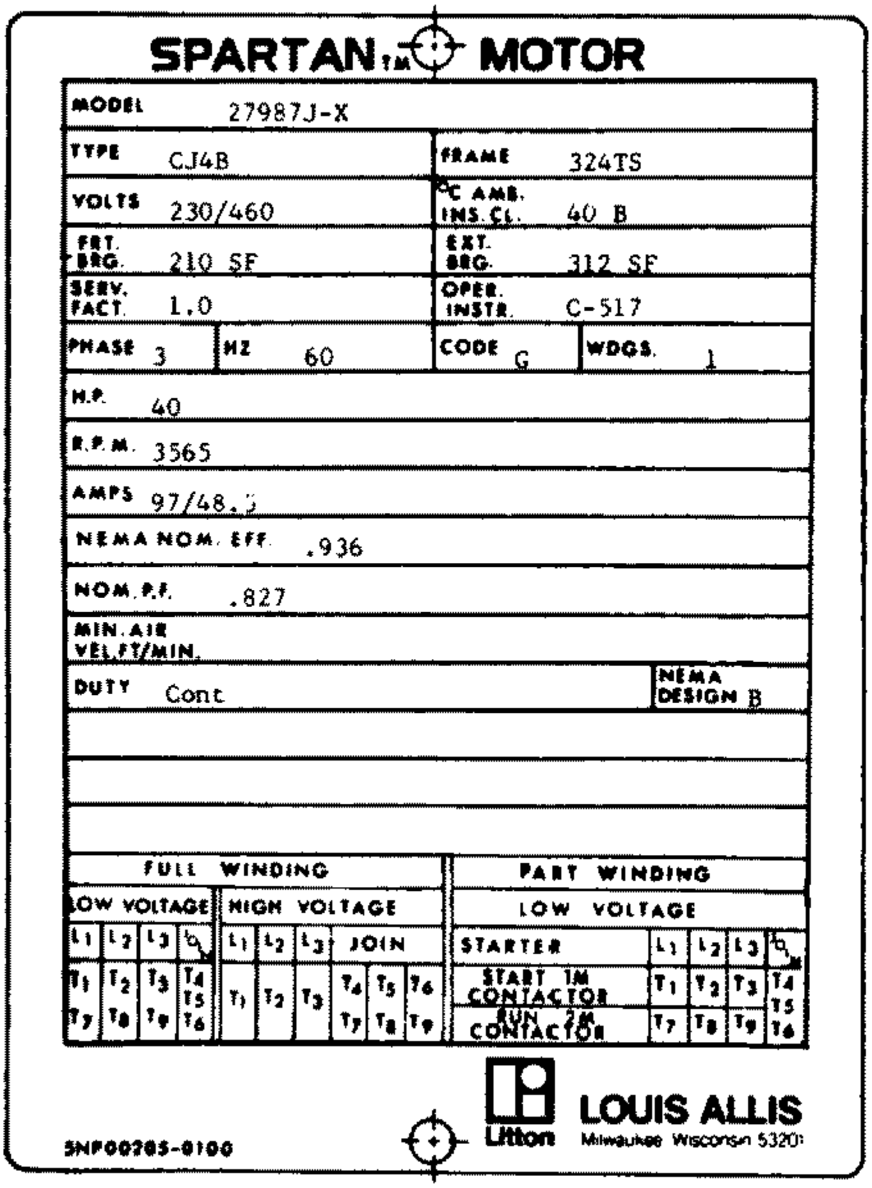


FIGURE 7–61
The nameplate of a typical high-efficiency induction motor. (*Courtesy of MagneTek, Inc.*)

7. NEMA design class
8. Starting code

A nameplate for a typical standard-efficiency induction motor would be similar, except that it might not show a nominal efficiency.

The voltage limit on the motor is based on the maximum acceptable magnetization current flow, since the higher the voltage gets, the more saturated the motor's iron becomes and the higher its magnetization current becomes. Just as in the case of transformers and synchronous machines, a 60-Hz induction motor may be used on a 50-Hz power system, but only if the voltage rating is decreased by an amount proportional to the decrease in frequency. This derating is necessary because the flux in the core of the motor is proportional to the integral of the applied voltage. To keep the maximum flux in the core constant while the period of integration is increasing, the average voltage level must decrease.

The current limit on an induction motor is based on the maximum acceptable heating in the motor's windings, and the power limit is set by the combination of the voltage and current ratings with the machine's power factor and efficiency.

NEMA design classes, starting code letters, and nominal efficiencies were discussed in previous sections of this chapter.

7.14 SUMMARY

The induction motor is the most popular type of ac motor because of its simplicity and ease of operation. An induction motor does not have a separate field circuit; instead, it depends on transformer action to induce voltages and currents in its field circuit. In fact, an induction motor is basically a rotating transformer. Its equivalent circuit is similar to that of a transformer, except for the effects of varying speed.

An induction motor normally operates at a speed near synchronous speed, but it can never operate at exactly n_{sync}. There must always be some relative motion in order to induce a voltage in the induction motor's field circuit. The rotor voltage induced by the relative motion between the rotor and the stator magnetic field produces a rotor current, and that rotor current interacts with the stator magnetic field to produce the induced torque in the motor.

In an induction motor, the slip or speed at which the maximum torque occurs can be controlled by varying the rotor resistance. The *value* of that maximum torque is independent of the rotor resistance. A high rotor resistance lowers the speed at which maximum torque occurs and thus increases the starting torque of the motor. However, it pays for this starting torque by having very poor speed regulation in its normal operating range. A low rotor resistance, on the other hand, reduces the motor's starting torque while improving its speed regulation. Any normal induction motor design must be a compromise between these two conflicting requirements.

One way to achieve such a compromise is to employ deep-bar or double-cage rotors. These rotors have a high effective resistance at starting and a low effective resistance under normal running conditions, thus yielding both a high starting torque and good speed regulation in the same motor. The same effect can be achieved with a wound-rotor induction motor if the rotor field resistance is varied.

Speed control of induction motors can be accomplished by changing the number of poles on the machine, by changing the applied electrical frequency, by changing the applied terminal voltage, or by changing the rotor resistance in the case of a wound-rotor induction motor.

The induction machine can also be used as a generator as long as there is some source of reactive power (capacitors or a synchronous machine) available in the power system. An induction generator operating alone has serious voltage regulation problems, but when it operates in parallel with a large power system, the power system can control the machine's voltage. Induction generators are usually rather small machines and are used principally with alternative energy sources, such as windmills, or with energy recovery systems. Almost all the really large generators in use are synchronous generators.

QUESTIONS

7–1. What are slip and slip speed in an induction motor?

7–2. How does an induction motor develop torque?

7–3. Why is it impossible for an induction motor to operate at synchronous speed?

7–4. Sketch and explain the shape of a typical induction motor torque–speed characteristic curve.

7–5. What equivalent circuit element has the most direct control over the speed at which the pullout torque occurs?

7–6. What is a deep-bar cage rotor? Why is it used? What NEMA design class(es) can be built with it?

7–7. What is a double-cage cage rotor? Why is it used? What NEMA design class(es) can be built with it?

7–8. Describe the characteristics and uses of wound-rotor induction motors and of each NEMA design class of cage motors.

7–9. Why is the efficiency of an induction motor (wound-rotor or cage) so poor at high slips?

7–10. Name and describe four means of controlling the speed of induction motors.

7–11. Why is it necessary to reduce the voltage applied to an induction motor as electrical frequency is reduced?

7–12. Why is terminal voltage speed control limited in operating range?

7–13. What are starting code factors? What do they say about the starting current of an induction motor?

7–14. How does a resistive starter circuit for an induction motor work?

7–15. What information is learned in a locked-rotor test?

7–16. What information is learned in a no-load test?

7–17. What actions are taken to improve the efficiency of modern high-efficiency induction motors?

7–18. What controls the terminal voltage of an induction generator operating alone?

7–19. For what applications are induction generators typically used?

7–20. How can a wound-rotor induction motor be used as a frequency changer?

7–21. How do different voltage-frequency patterns affect the torque–speed characteristics of an induction motor?

7–22. Describe the major features of the solid-state induction motor drive featured in Section 7.10.

7–23. Two 480-V, 100-hp induction motors are manufactured. One is designed for 50-Hz operation, and one is designed for 60-Hz operation, but they are otherwise similar. Which of these machines is larger?

7–24. An induction motor is running at the rated conditions. If the shaft load is now increased, how do the following quantities change?

(a) Mechanical speed

(b) Slip

(c) Rotor induced voltage
(d) Rotor current
(e) Rotor frequency
(f) P_{RCL}
(g) Synchronous speed

PROBLEMS

7–1. A dc test is performed on a 460-V, Δ-connected, 100-hp induction motor. If V_{DC} = 24 V and I_{DC} = 80 A, what is the stator resistance R_1? *Why is this so?*

7–2. A 220-V, three-phase, two-pole, 50-Hz induction motor is running at a slip of 5 percent. Find:
(a) The speed of the magnetic fields in revolutions per minute
(b) The speed of the rotor in revolutions per minute
(c) The slip speed of the rotor
(d) The rotor frequency in hertz

7–3. Answer the questions in Problem 7–2 for a 480-V, three-phase, four-pole, 60-Hz induction motor running at a slip of 0.035.

7–4. A three-phase, 60-Hz induction motor runs at 890 r/min at no load and at 840 r/min at full load.
(a) How many poles does this motor have?
(b) What is the slip at rated load?
(c) What is the speed at one-quarter of the rated load?
(d) What is the rotor's electrical frequency at one-quarter of the rated load?

7–5. A 50-kW, 440-V, 50-Hz, six-pole induction motor has a slip of 6 percent when operating at full-load conditions. At full-load conditions, the friction and windage losses are 300 W, and the core losses are 600 W. Find the following values for full-load conditions:
(a) The shaft speed n_m
(b) The output power in watts
(c) The load torque τ_{load} in newton-meters
(d) The induced torque τ_{ind} in newton-meters
(e) The rotor frequency in hertz

7–6. A three-phase, 60-Hz, four-pole induction motor runs at a no-load speed of 1790 r/min and a full-load speed of 1720 r/min. Calculate the slip and the electrical frequency of the rotor at no-load and full-load conditions. What is the speed regulation of this motor [Equation (4–68)]?

7–7. A 208-V, two-pole, 60-Hz, Y-connected wound-rotor induction motor is rated at 15 hp. Its equivalent circuit components are

$$R_1 = 0.200\ \Omega \qquad R_2 = 0.120\ \Omega \qquad X_M = 15.0\ \Omega$$
$$X_1 = 0.410\ \Omega \qquad X_2 = 0.410\ \Omega$$
$$P_{mech} = 250\text{ W} \qquad P_{misc} \approx 0 \qquad P_{core} = 180\text{ W}$$

For a slip of 0.05, find
(a) The line current I_L
(b) The stator copper losses P_{SCL}
(c) The air-gap power P_{AG}

(*d*) The power converted from electrical to mechanical form P_{conv}
(*e*) The induced torque τ_{ind}
(*f*) The load torque τ_{load}
(*g*) The overall machine efficiency
(*h*) The motor speed in revolutions per minute and radians per second

7–8. For the motor in Problem 7–7, what is the slip at the pullout torque? What is the pullout torque of this motor?

7–9. (*a*) Calculate and plot the torque–speed characteristic of the motor in Problem 7–7.
(*b*) Calculate and plot the output power versus speed curve of the motor in Problem 7–7.

7–10. For the motor of Problem 7–7, how much additional resistance (referred to the stator circuit) would it be necessary to add to the rotor circuit to make the maximum torque occur at starting conditions (when the shaft is not moving)? Plot the torque–speed characteristic of this motor with the additional resistance inserted.

7–11. If the motor in Problem 7–7 is to be operated on a 50-Hz power system, what must be done to its supply voltage? Why? What will the equivalent circuit component values be at 50 Hz? Answer the questions in Problem 7–7 for operation at 50 Hz with a slip of 0.05 and the proper voltage for this machine.

7–12. Figure 7–18a shows a simple circuit consisting of a voltage source, a resistor, and two reactances. Find the Thevenin equivalent voltage and impedance of this circuit at the terminals. Then derive the expressions for the magnitude of $\mathbf{V}_{TH}$ and for R_{TH} given in Equations (7–41b) and (7–44).

7–13. Figure P7–1 shows a simple circuit consisting of a voltage source, two resistors, and two reactances in series with each other. If the resistor R_L is allowed to vary but all the other components are constant, at what value of R_L will the maximum possible power be supplied to it? *Prove* your answer. (*Hint:* Derive an expression for load power in terms of V, R_S, X_S, R_L, and X_L and take the partial derivative of that expression with respect to R_L.) Use this result to derive the expression for the pullout torque [Equation (7–54)].

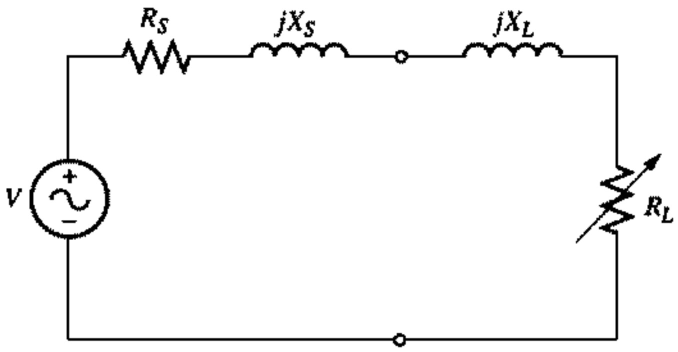


FIGURE P7–1
Circuit for Problem 7–13.

7–14. A 440-V, 50-Hz, two-pole, Y-connected induction motor is rated at 75 kW. The equivalent circuit parameters are

$R_1 = 0.075\ \Omega$	$R_2 = 0.065\ \Omega$	$X_M = 7.2\ \Omega$
$X_1 = 0.17\ \Omega$	$X_2 = 0.17\ \Omega$	
$P_{F\&W} = 1.0\ \text{kW}$	$P_{misc} = 150\ \text{W}$	$P_{core} = 1.1\ \text{kW}$

For a slip of 0.04, find

(*a*) The line current I_L
(*b*) The stator power factor
(*c*) The rotor power factor
(*d*) The stator copper losses P_{SCL}
(*e*) The air-gap power P_{AG}
(*f*) The power converted from electrical to mechanical form P_{conv}
(*g*) The induced torque τ_{ind}
(*h*) The load torque τ_{load}
(*i*) The overall machine efficiency η
(*j*) The motor speed in revolutions per minute and radians per second

7–15. For the motor in Problem 7–14, what is the pullout torque? What is the slip at the pullout torque? What is the rotor speed at the pullout torque?

7–16. If the motor in Problem 7–14 is to be driven from a 440-V, 60-Hz power supply, what will the pullout torque be? What will the slip be at pullout?

7–17. Plot the following quantities for the motor in Problem 7–14 as slip varies from 0 to 10 percent: (*a*) τ_{ind}; (*b*) P_{conv}; (*c*) P_{out}; (*d*) efficiency η. At what slip does P_{out} equal the rated power of the machine?

7–18. A 208-V, 60 Hz six-pole, Y-connected, 25-hp design class B induction motor is tested in the laboratory, with the following results:

No load:	208 V, 22.0 A, 1200 W, 60 Hz
Locked rotor:	24.6 V, 64.5 A, 2200 W, 15 Hz
DC test:	13.5 V, 64 A

Find the equivalent circuit of this motor, and plot its torque–speed characteristic curve.

7–19. A 460-V, four-pole, 50-hp, 60-Hz, Y-connected, three-phase induction motor develops its full-load induced torque at 3.8 percent slip when operating at 60 Hz and 460 V. The per-phase circuit model impedances of the motor are

$$R_1 = 0.33\ \Omega \qquad X_M = 30\ \Omega$$
$$X_1 = 0.42\ \Omega \qquad X_2 = 0.42\ \Omega$$

Mechanical, core, and stray losses may be neglected in this problem.
(*a*) Find the value of the rotor resistance R_2.
(*b*) Find τ_{max}, s_{max}, and the rotor speed at maximum torque for this motor.
(*c*) Find the starting torque of this motor.
(*d*) What code letter factor should be assigned to this motor?

7–20. Answer the following questions about the motor in Problem 7–19.
(*a*) If this motor is started from a 460-V infinite bus, how much current will flow in the motor at starting?
(*b*) If transmission line with an impedance of $0.35 + j0.25\ \Omega$ per phase is used to connect the induction motor to the infinite bus, what will the starting current of the motor be? What will the motor's terminal voltage be on starting?
(*c*) If an ideal 1.4:1 step-down autotransformer is connected between the transmission line and the motor, what will the current be in the transmission line during starting? What will the voltage be at the motor end of the transmission line during starting?

7–21. In this chapter, we learned that a step-down autotransformer could be used to reduce the starting current drawn by an induction motor. While this technique works, an autotransformer is relatively expensive. A much less expensive way to reduce the starting current is to use a device called *Y–Δ starter*. If an induction motor is normally Δ-connected, it is possible to reduce its phase voltage V_ϕ (and hence its starting current) by simply reconnecting the stator windings in Y during starting, and then restoring the connections to Δ when the motor comes up to speed. Answer the following questions about this type of starter.

(*a*) How would the phase voltage at starting compare with the phase voltage under normal running conditions?

(*b*) How would the starting current of the Y-connected motor compare to the starting current if the motor remained in a Δ-connection during starting?

7–22. A 460-V, 100-hp, four-pole, Δ-connected, 60-Hz, three-phase induction motor has a full-load slip of 5 percent, an efficiency of 92 percent, and a power factor of 0.87 lagging. At start-up, the motor develops 1.9 times the full-load torque but draws 7.5 times the rated current at the rated voltage. This motor is to be started with an autotransformer reduced-voltage starter.

(*a*) What should the output voltage of the starter circuit be to reduce the starting torque until it equals the rated torque of the motor?

(*b*) What will the motor starting current and the current drawn from the supply be at this voltage?

7–23. A wound-rotor induction motor is operating at rated voltage and frequency with its slip rings shorted and with a load of about 25 percent of the rated value for the machine. If the rotor resistance of this machine is doubled by inserting external resistors into the rotor circuit, explain what happens to the following:

(*a*) Slip s

(*b*) Motor speed n_m

(*c*) The induced voltage in the rotor

(*d*) The rotor current

(*e*) τ_{ind}

(*f*) P_{out}

(*g*) P_{RCL}

(*h*) Overall efficiency η

7–24. Answer the following questions about a 460-V, Δ-connected, two-pole, 75-hp, 60-Hz, starting-code-letter-E induction motor:

(*a*) What is the maximum current starting current that this machine's controller must be designed to handle?

(*b*) If the controller is designed to switch the stator windings from a Δ connection to a Y connection during starting, what is the maximum starting current that the controller must be designed to handle?

(*c*) If a 1.25:1 step-down autotransformer starter is used during starting, what is the maximum starting current that will be drawn from the line?

7–25. When it is necessary to stop an induction motor very rapidly, many induction motor controllers reverse the direction of rotation of the magnetic fields by switching any two stator leads. When the direction of rotation of the magnetic fields is reversed, the motor develops an induced torque opposite to the current direction of rotation, so it quickly stops and tries to start turning in the opposite direction. If power is removed from the stator circuit at the moment when the rotor speed goes through zero, then the motor has been stopped very rapidly. This technique for rapidly stopping an induction motor is called *plugging*. The motor of Problem 7–19 is running at rated conditions and is to be stopped by plugging.

(a) What is the slip s before plugging?
(b) What is the frequency of the rotor before plugging?
(c) What is the induced torque τ_{ind} before plugging?
(d) What is the slip s immediately after switching the stator leads?
(e) What is the frequency of the rotor immediately after switching the stator leads?
(f) What is the induced torque τ_{ind} immediately after switching the stator leads?

REFERENCES

1. Alger, Phillip. *Induction Machines*, 2nd ed. New York: Gordon and Breach, 1970.
2. Del Toro, V. *Electric Machines and Power Systems*. Englewood Cliffs, N.J.: Prentice-Hall, 1985.
3. Fitzgerald, A. E. and C. Kingsley, Jr. *Electric Machinery*. New York: McGraw-Hill, 1952.
4. Fitzgerald, A. E., C. Kingsley, Jr., and S. D. Umans. *Electric Machinery*, 5th ed. New York: McGraw-Hill, 1990.
5. Institute of Electrical and Electronics Engineers. *Standard Test Procedure for Polyphase Induction Motors and Generators*, IEEE Standard 112-1996. New York: IEEE, 1996.
6. Kosow, Irving L. *Control of Electric Motors*. Englewood Cliffs, N.J.: Prentice-Hall, 1972.
7. McPherson, George. *An Introduction to Electrical Machines and Transformers*. New York: Wiley, 1981.
8. National Electrical Manufacturers Association. *Motors and Generators*, Publication No. MG1-1993. Washington, D.C.: NEMA, 1993.
9. Slemon, G. R., and A. Straughen. *Electric Machines*. Reading, Mass.: Addison-Wesley, 1980.
10. Vithayathil, Joseph. *Power Electronics: Principles and Applications*. New York: McGraw-Hill, 1995.
11. Werninck, E. H. (ed.). *Electric Motor Handbook*. London: McGraw-Hill, 1978.