CHAPTER

8

DC MACHINERY FUNDAMENTALS

DC machines are generators that convert mechanical energy to dc electric energy and motors that convert dc electric energy to mechanical energy. Most dc machines are like ac machines in that they have ac voltages and currents within them—dc machines have a dc output only because a mechanism exists that converts the internal ac voltages to dc voltages at their terminals. Since this mechanism is called a commutator, dc machinery is also known as *commutating machinery.*

The fundamental principles involved in the operation of dc machines are very simple. Unfortunately, they are usually somewhat obscured by the complicated construction of real machines. This chapter will first explain the principles of dc machine operation by using simple examples and then consider some of the complications that occur in real dc machines.

8.1 A SIMPLE ROTATING LOOP BETWEEN CURVED POLE FACES

The linear machine studied in Section 1.8 served as an introduction to basic machine behavior. Its response to loading and to changing magnetic fields closely resembles the behavior of the real dc generators and motors that we will study in Chapter 9. However, real generators and motors do not move in a straight line—they *rotate*. The next step toward understanding real dc machines is to study the simplest possible example of a rotating machine.

The simplest possible rotating dc machine is shown in Figure 8–1. It consists of a single loop of wire rotating about a fixed axis. The rotating part of this machine is called the *rotor*, and the stationary part is called the *stator.* The magnetic

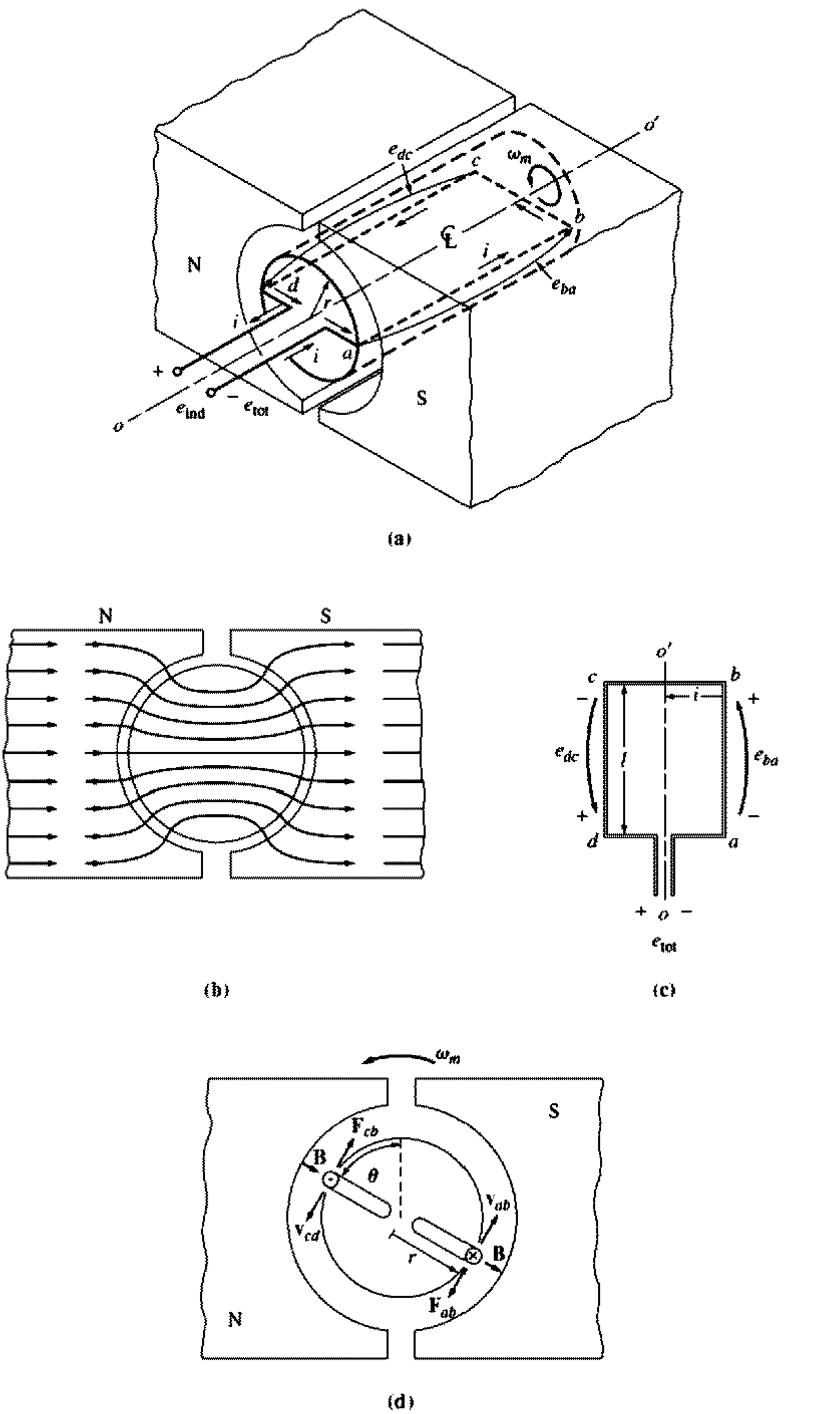


FIGURE 8–1
A simple rotating loop between curved pole faces. (a) Perspective view; (b) view of field lines; (c) top view; (d) front view.

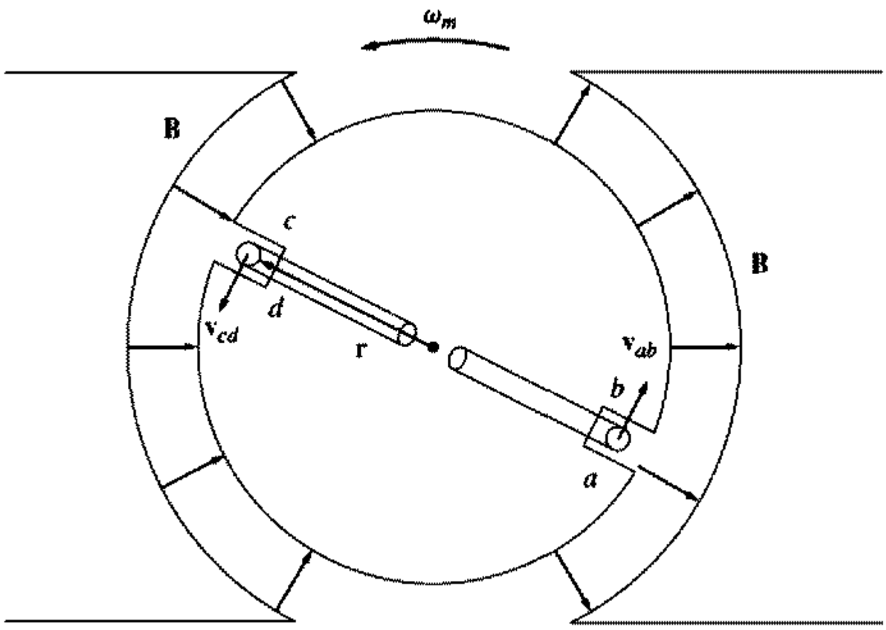


FIGURE 8–2
Derivation of an equation for the voltages induced in the loop.

field for the machine is supplied by the magnetic north and south poles shown on the stator in Figure 8–1.

Notice that the loop of rotor wire lies in a slot carved in a ferromagnetic core. The iron rotor, together with the curved shape of the pole faces, provides a constant-width air gap between the rotor and stator. Remember from Chapter 1 that the reluctance of air is much much higher than the reluctance of the iron in the machine. To minimize the reluctance of the flux path through the machine, the magnetic flux must take the shortest possible path through the air between the pole face and the rotor surface.

Since the magnetic flux must take the shortest path through the air, it is *perpendicular* to the rotor surface everywhere under the pole faces. Also, since the air gap is of uniform width, the reluctance is the same everywhere under the pole faces. The uniform reluctance means that the magnetic flux density is constant everywhere under the pole faces.

The Voltage Induced in a Rotating Loop

If the rotor of this machine is rotated, a voltage will be induced in the wire loop. To determine the magnitude and shape of the voltage, examine Figure 8–2. The loop of wire shown is rectangular, with sides *ab* and *cd* perpendicular to the plane of the page and with sides *bc* and *da* parallel to the plane of the page. The magnetic field is constant and perpendicular to the surface of the rotor everywhere under the pole faces and rapidly falls to zero beyond the edges of the poles.

To determine the total voltage e_{tot} on the loop, examine each segment of the loop separately and sum all the resulting voltages. The voltage on each segment is given by Equation (1–45):

$$e_{ind} = (\mathbf{v} \times \mathbf{B}) \cdot \mathbf{l} \tag{1–45}$$

1. *Segment ab.* In this segment, the velocity of the wire is tangential to the path of rotation. The magnetic field **B** points out perpendicular to the rotor surface everywhere under the pole face and is zero beyond the edges of the pole face. Under the pole face, velocity **v** is perpendicular to **B**, and the quantity **v** × **B** points into the page. Therefore, the induced voltage on the segment is

$$e_{ba} = (\mathbf{v} \times \mathbf{B}) \cdot \mathbf{l} = \begin{cases} vBl & \text{positive into page} \quad \text{under the pole face} \\ 0 & \text{beyond the pole edges} \end{cases} \tag{8–1}$$

2. *Segment bc.* In this segment, the quantity **v** × **B** is either into or out of the page, while length **l** is in the plane of the page, so **v** × **B** is perpendicular to **l**. Therefore the voltage in segment *bc* will be zero:

$$e_{cb} = 0 \tag{8–2}$$

3. *Segment cd.* In this segment, the velocity of the wire is tangential to the path of rotation. The magnetic field **B** points *in* perpendicular to the rotor surface everywhere under the pole face and is zero beyond the edges of the pole face. Under the pole face, velocity **v** is perpendicular to **B**, and the quantity **v** × **B** points out of the page. Therefore, the induced voltage on the segment is

$$e_{dc} = (\mathbf{v} \times \mathbf{B}) \cdot \mathbf{l} = \begin{cases} vBl & \text{positive out of page} \quad \text{under the pole face} \\ 0 & \text{beyond the pole edges} \end{cases} \tag{8–3}$$

4. *Segment da.* Just as in segment *bc*, **v** × **B** is perpendicular to **l**. Therefore the voltage in this segment will be zero too:

$$e_{ad} = 0 \tag{8–4}$$

The total induced voltage on the loop e_{ind} is given by

$$e_{\text{ind}} = e_{ba} + e_{cb} + e_{dc} + e_{ad}$$

$$\boxed{e_{\text{ind}} = \begin{cases} 2vBl & \text{under the pole faces} \\ 0 & \text{beyond the pole edges} \end{cases}} \tag{8–5}$$

When the loop rotates through 180°, segment *ab* is under the north pole face instead of the south pole face. At that time, the direction of the voltage on the segment reverses, but its magnitude remains constant. The resulting voltage e_{tot} is shown as a function of time in Figure 8–3.

There is an alternative way to express Equation (8–5), which clearly relates the behavior of the single loop to the behavior of larger, real dc machines. To derive this alternative expression, examine Figure 8–4. Notice that the tangential velocity v of the edges of the loop can be expressed as

$$v = r\omega$$

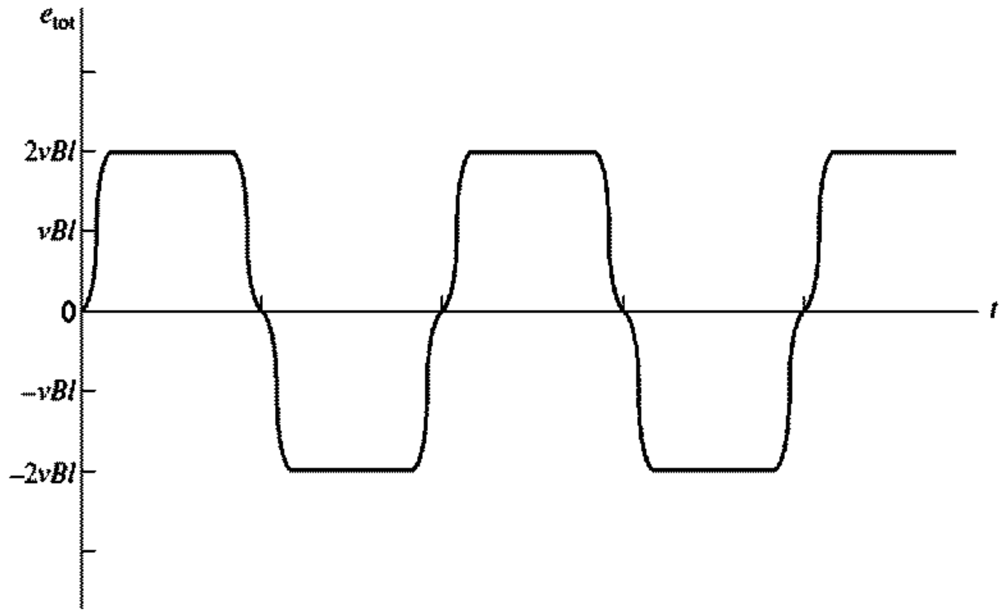


FIGURE 8–3
The output voltage of the loop.

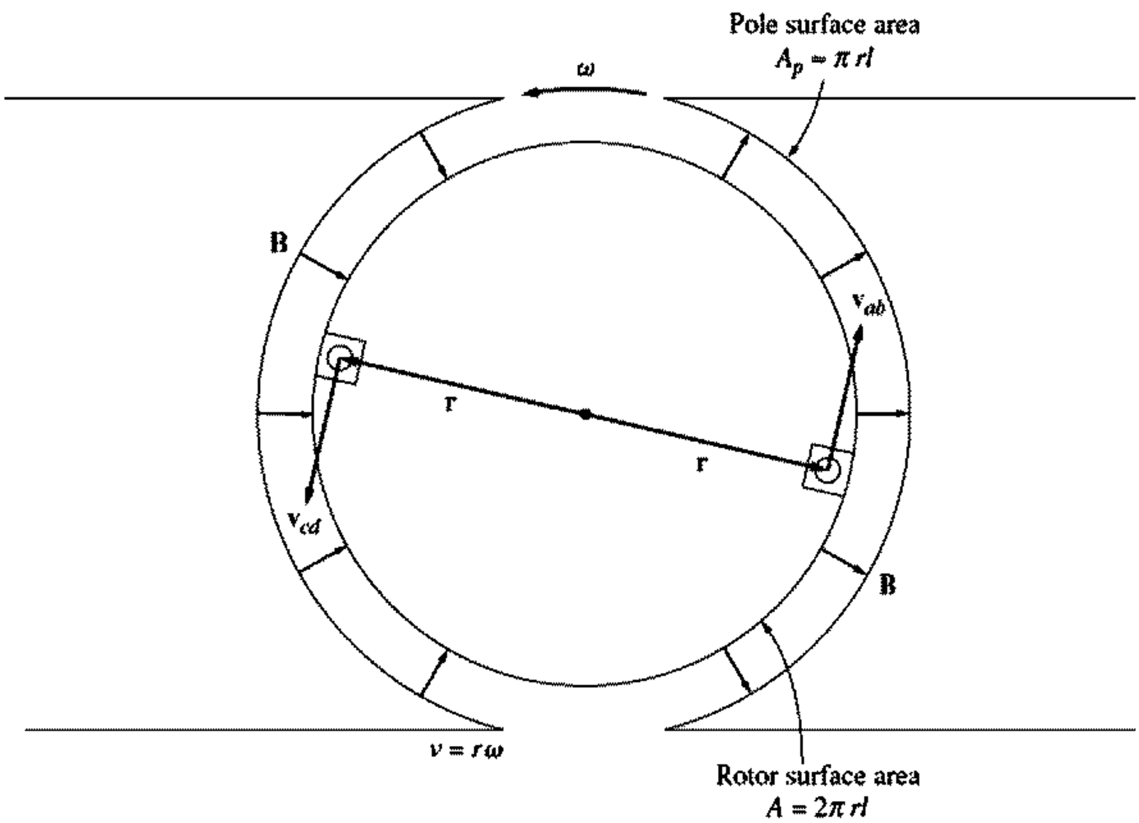


FIGURE 8–4
Derivation of an alternative form of the induced voltage equation.

where r is the radius from axis of rotation out to the edge of the loop and ω is the angular velocity of the loop. Substituting this expression into Equation (8–5) gives

$$e_{\text{ind}} = \begin{cases} 2r\omega Bl & \text{under the pole faces} \\ 0 & \text{beyond the pole edges} \end{cases}$$

$$e_{\text{ind}} = \begin{cases} 2rlB\omega & \text{under the pole faces} \\ 0 & \text{beyond the pole edges} \end{cases}$$

Notice also from Figure 8–4 that the rotor surface is a cylinder, so the area of the rotor surface A is just equal to $2\pi rl$. Since there are two poles, the area of the rotor *under each pole* (ignoring the small gaps between poles) is $A_P = \pi rl$. Therefore,

$$e_{\text{ind}} = \begin{cases} \dfrac{2}{\pi} A_P B\omega & \text{under the pole faces} \\ 0 & \text{beyond the pole edges} \end{cases}$$

Since the flux density B is constant everywhere in the air gap under the pole faces, the total flux under each pole is just the area of the pole times its flux density:

$$\phi = A_P B$$

Therefore, the final form of the voltage equation is

$$\boxed{e_{\text{ind}} = \begin{cases} \dfrac{2}{\pi} \phi\omega & \text{under the pole faces} \\ 0 & \text{beyond the pole edges} \end{cases}} \tag{8–6}$$

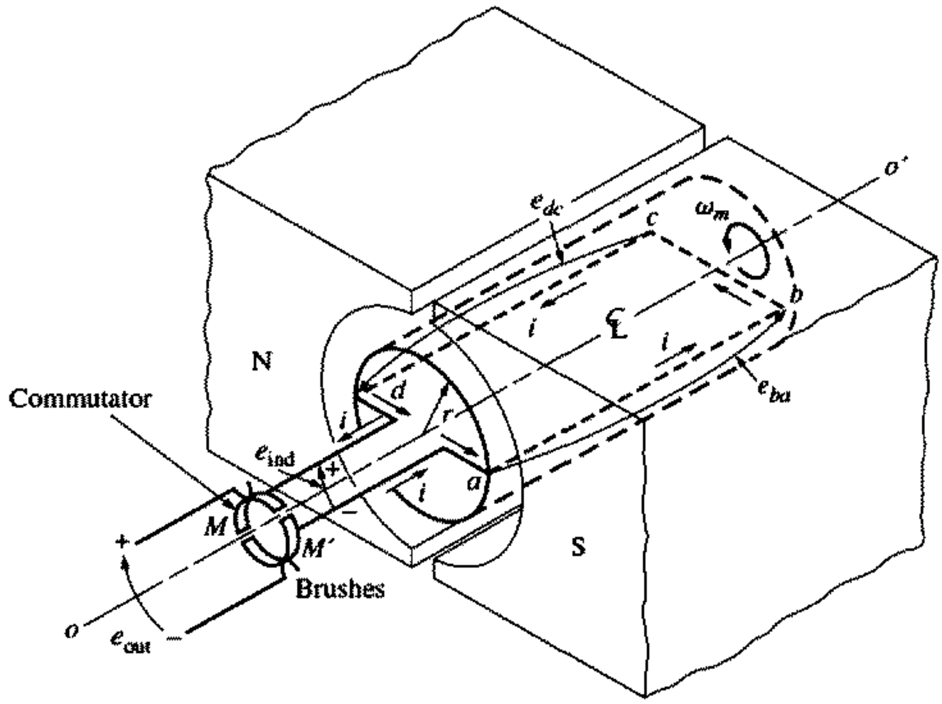
Thus, *the voltage generated in the machine is equal to the product of the flux inside the machine and the speed of rotation of the machine*, multiplied by a constant representing the mechanical construction of the machine. In general, the voltage in any real machine will depend on the same three factors:

1. The flux in the machine
2. The speed of rotation
3. A constant representing the construction of the machine

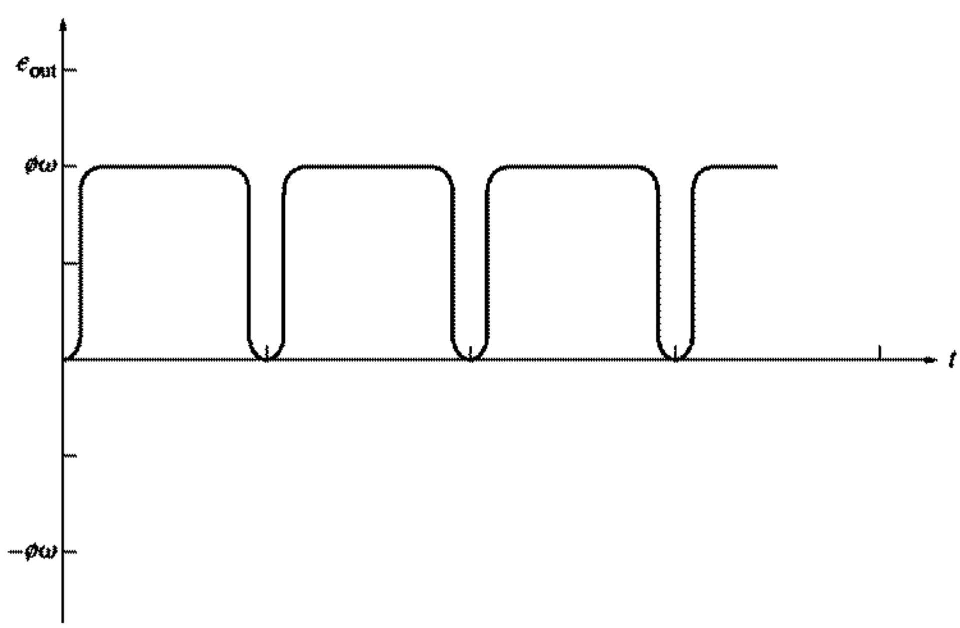
Getting DC Voltage out of the Rotating Loop

Figure 8–3 is a plot of the voltage e_{tot} generated by the rotating loop. As shown, the voltage out of the loop is alternately a constant positive value and a constant negative value. How can this machine be made to produce a dc voltage instead of the ac voltage it now has?

One way to do this is shown in Figure 8–5a. Here two semicircular conducting segments are added to the end of the loop, and two fixed contacts are set up at an angle such that at the instant when the voltage in the loop is zero, the contacts



(a)



(b)

FIGURE 8–5
Producing a dc output from the machine with a commutator and brushes. (a) Perspective view; (b) the resulting output voltage.

short-circuit the two segments. In this fashion, *every time the voltage of the loop switches direction, the contacts also switch connections, and the output of the contacts is always built up in the same way* (Figure 8–5b). This connection-switching process is known as *commutation*. The rotating semicircular segments are called *commutator segments*, and the fixed contacts are called *brushes*.

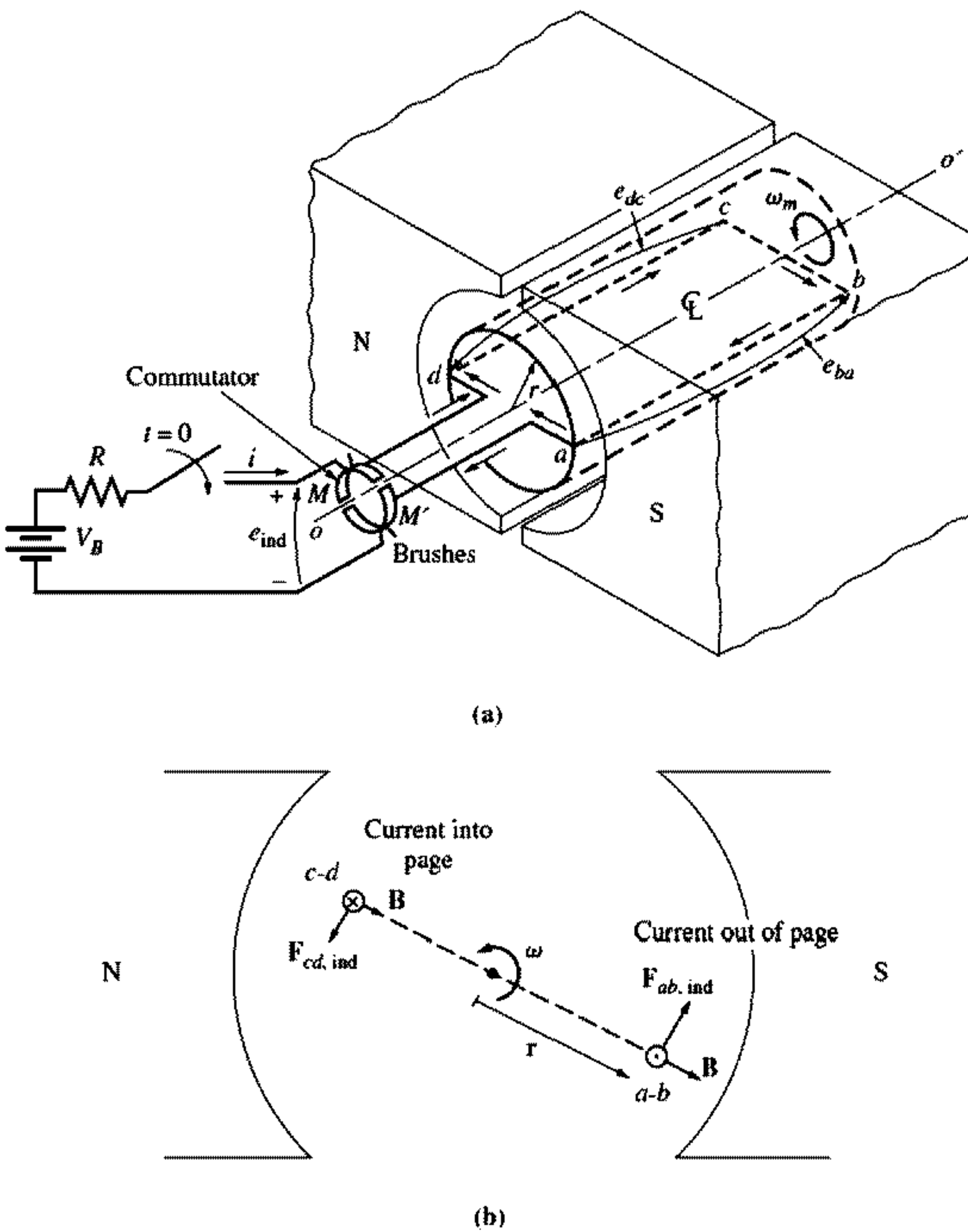


FIGURE 8–6
Derivation of an equation for the induced torque in the loop. Note that the iron core is not shown in part b for clarity.

The Induced Torque in the Rotating Loop

Suppose a battery is now connected to the machine in Figure 8–5. The resulting configuration is shown in Figure 8–6. How much torque will be produced in the loop when the switch is closed and a current is allowed to flow into it? To determine the torque, look at the close-up of the loop shown in Figure 8–6b.

The approach to take in determining the torque on the loop is to look at one segment of the loop at a time and then sum the effects of all the individual segments. The force on a segment of the loop is given by Equation (1–43):

$$\mathbf{F} = i(\mathbf{l} \times \mathbf{B}) \tag{1–43}$$

and the torque on the segment is given by

$$\tau = rF \sin \theta \tag{1–6}$$

where θ is the angle between $\mathbf{r}$ and $\mathbf{F}$. The torque is essentially zero whenever the loop is beyond the pole edges.

While the loop is under the pole faces, the torque is

1. *Segment ab.* In segment *ab*, the current from the battery is directed out of the page. The magnetic field under the pole face is pointing radially out of the rotor, so the force on the wire is given by

$$\begin{aligned} \mathbf{F}_{ab} &= i(\mathbf{l} \times \mathbf{B}) \\ &= ilB \qquad \text{tangent to direction of motion} \end{aligned} \tag{8–7}$$

The torque on the rotor caused by this force is

$$\begin{aligned} \tau_{ab} &= rF \sin \theta \\ &= r(ilB) \sin 90^\circ \\ &= rilB \qquad \text{CCW} \end{aligned} \tag{8–8}$$

2. *Segment bc.* In segment *bc*, the current from the battery is flowing from the upper left to the lower right in the picture. The force induced on the wire is given by

$$\begin{aligned} \mathbf{F}_{bc} &= i(\mathbf{l} \times \mathbf{B}) \\ &= 0 \qquad \text{since } \mathbf{l} \text{ is parallel to } \mathbf{B} \end{aligned} \tag{8–9}$$

Therefore,

$$\tau_{bc} = 0 \tag{8–10}$$

3. *Segment cd.* In segment *cd*, the current from the battery is directed into the page. The magnetic field under the pole face is pointing radially into the rotor, so the force on the wire is given by

$$\begin{aligned} \mathbf{F}_{cd} &= i(\mathbf{l} \times \mathbf{B}) \\ &= ilB \qquad \text{tangent to direction of motion} \end{aligned} \tag{8–11}$$

The torque on the rotor caused by this force is

$$\begin{aligned} \tau_{cd} &= rF \sin \theta \\ &= r(ilB) \sin 90^\circ \\ &= rilB \qquad \text{CCW} \end{aligned} \tag{8–12}$$

4. *Segment da.* In segment *da*, the current from the battery is flowing from the upper left to the lower right in the picture. The force induced on the wire is given by

$$\mathbf{F}_{da} = i(\mathbf{l} \times \mathbf{B})$$
$$= 0 \qquad \text{since } \mathbf{l} \text{ is parallel to } \mathbf{B} \tag{8–13}$$

Therefore,

$$\tau_{da} = 0 \tag{8–14}$$

The resulting total induced torque on the loop is given by

$$\tau_{ind} = \tau_{ab} + \tau_{bc} + \tau_{cd} + \tau_{da}$$

$$\boxed{\tau_{ind} = \begin{cases} 2rilB & \text{under the pole faces} \\ 0 & \text{beyond the pole edges} \end{cases}} \tag{8–15}$$

By using the facts that $A_P \approx \pi rl$ and $\phi = A_P B$, the torque expression can be reduced to

$$\boxed{\tau_{ind} = \begin{cases} \dfrac{2}{\pi}\phi i & \text{under the pole faces} \\ 0 & \text{beyond the pole edges} \end{cases}} \tag{8–16}$$

Thus, *the torque produced in the machine is the product of the flux in the machine and the current in the machine*, times some quantity representing the mechanical construction of the machine (the percentage of the rotor covered by pole faces). In general, the torque in *any* real machine will depend on the same three factors:

1. The flux in the machine
2. The current in the machine
3. A constant representing the construction of the machine

Example 8–1. Figure 8–6 shows a simple rotating loop between curved pole faces connected to a battery and a resistor through a switch. The resistor shown models the total resistance of the battery and the wire in the machine. The physical dimensions and characteristics of this machine are

$$r = 0.5 \text{ m} \qquad l = 1.0 \text{ m}$$
$$R = 0.3\ \Omega \qquad B = 0.25 \text{ T}$$
$$V_B = 120 \text{ V}$$

(a) What happens when the switch is closed?
(b) What is the machine's maximum starting current? What is its steady-state angular velocity at no load?
(c) Suppose a load is attached to the loop, and the resulting load torque is 10 N • m. What would the new steady-state speed be? How much power is supplied to the shaft of the machine? How much power is being supplied by the battery? Is this machine a motor or a generator?

(*d*) Suppose the machine is again unloaded, and a torque of 7.5 N • m is applied to the shaft in the direction of rotation. What is the new steady-state speed? Is this machine now a motor or a generator?

(*e*) Suppose the machine is running unloaded. What would the final steady-state speed of the rotor be if the flux density were reduced to 0.20 T?

Solution

(*a*) When the switch in Figure 8–6 is closed, a current will flow in the loop. Since the loop is initially stationary, $e_{ind} = 0$. Therefore, the current will be given by

$$i = \frac{V_B - e_{\text{ind}}}{R} = \frac{V_B}{R}$$

This current flows through the rotor loop, producing a torque

$$\tau_{\text{ind}} = \frac{2}{\pi}\phi i \qquad \text{CCW}$$

This induced torque produces an angular acceleration in a counterclockwise direction, so the rotor of the machine begins to turn. But as the rotor begins to turn, an induced voltage is produced in the motor, given by

$$e_{\text{ind}} = \frac{2}{\pi}\phi\omega$$

so the current i falls. As the current falls, $\tau_{\text{ind}} = (2/\pi)\phi\, i\downarrow$ decreases, and the machine winds up in steady state with $\tau_{\text{ind}} = 0$, and the battery voltage $V_B = e_{\text{ind}}$.

This is the same sort of starting behavior seen earlier in the linear dc machine.

(*b*) At *starting conditions*, the machine's current is

$$i = \frac{V_B}{R} = \frac{120\ \text{V}}{0.3\ \Omega} = 400\ \text{A}$$

At *no-load steady-state conditions*, the induced torque τ_{ind} must be zero. But $\tau_{\text{ind}} = 0$ implies that current i must equal zero, since $\tau_{\text{ind}} = (2/\pi)\phi\, i$, and the flux is nonzero. The fact that $i = 0$ A means that the battery voltage $V_B = e_{\text{ind}}$. Therefore, the speed of the rotor is

$$V_B = e_{\text{ind}} = \frac{2}{\pi}\phi\omega$$

$$\omega = \frac{V_B}{(2/\pi)\phi} = \frac{V_B}{2rlB}$$

$$= \frac{120\ \text{V}}{2(0.5\ \text{m})(1.0\ \text{m})(0.25\ \text{T})} = 480\ \text{rad/s}$$

(*c*) If a load torque of 10 N • m is applied to the shaft of the machine, it will begin to slow down. But as ω decreases, $e_{\text{ind}} = (2/\pi)\phi\,\omega\downarrow$ decreases and the rotor current increases $[i = (V_B - e_{\text{ind}}\downarrow)/R]$. As the rotor current increases, $|\tau_{\text{ind}}|$ increases too, until $|\tau_{\text{ind}}| = |\tau_{\text{load}}|$ at a lower speed ω.

At steady state, $|\tau_{\text{load}}| = |\tau_{\text{ind}}| = (2/\pi)\phi\, i$. Therefore,

$$i = \frac{\tau_{\text{ind}}}{(2/\pi)\phi} = \frac{\tau_{\text{ind}}}{2rlB}$$

$$= \frac{10\ \text{N}\cdot\text{m}}{(2)(0.5\ \text{m})(1.0\ \text{m})(0.25\ \text{T})} = 40\ \text{A}$$

By Kirchhoff's voltage law, $e_{ind} = V_B - iR$, so

$$e_{ind} = 120\text{ V} - (40\text{ A})(0.3\ \Omega) = 108\text{ V}$$

Finally, the speed of the shaft is

$$\omega = \frac{e_{ind}}{(2/\pi)\phi} = \frac{e_{ind}}{2rlB}$$

$$= \frac{108\text{ V}}{(2)(0.5\text{ m})(1.0\text{ m})(0.25\text{ T})} = 432\text{ rad/s}$$

The power supplied to the shaft is

$$P = \tau\omega$$

$$= (10\text{ N}\bullet\text{m})(432\text{ rad/s}) = 4320\text{ W}$$

The power out of the battery is

$$P = V_B i = (120\text{ V})(40\text{ A}) = 4800\text{ W}$$

This machine is operating as a *motor*, converting electric power to mechanical power.

(d) If a torque is applied in the direction of motion, the rotor accelerates. As the speed increases, the internal voltage e_{ind} increases and exceeds V_B, so the current flows out of the top of the bar and into the battery. This machine is now a *generator*. This current causes an induced torque opposite to the direction of motion. The induced torque opposes the external applied torque, and eventually $|\tau_{load}| = |\tau_{ind}|$ at a higher speed ω.

The current in the rotor will be

$$i = \frac{\tau_{ind}}{(2/\pi)\phi} = \frac{\tau_{ind}}{2rlB}$$

$$= \frac{7.5\text{ N}\bullet\text{m}}{(2)(0.5\text{ m})(1.0\text{ m})(0.25\text{ T})} = 30\text{ A}$$

The induced voltage e_{ind} is

$$\begin{aligned} e_{ind} &= V_B + iR \\ &= 120\text{ V} + (30\text{ A})(0.3\ \Omega) \\ &= 129\text{ V} \end{aligned}$$

Finally, the speed of the shaft is

$$\omega = \frac{e_{ind}}{(2/\pi)\phi} = \frac{e_{ind}}{2rlB}$$

$$= \frac{129\text{ V}}{(2)(0.5\text{ m})(1.0\text{ m})(0.25\text{ T})} = 516\text{ rad/s}$$

(e) Since the machine is initially unloaded at the original conditions, the speed $\omega =$ 480 rad/s. If the flux decreases, there is a transient. However, after the transient is over, the machine must again have zero torque, since there is still no load on its shaft. If $\tau_{ind} = 0$, then the current in the rotor must be zero, and $V_B = e_{ind}$. The shaft speed is thus

$$\omega = \frac{e_{ind}}{(2/\pi)\phi} = \frac{e_{ind}}{2rlB}$$

$$= \frac{120\text{ V}}{(2)(0.5\text{ m})(1.0\text{ m})(0.20\text{ T})} = 600\text{ rad/s}$$

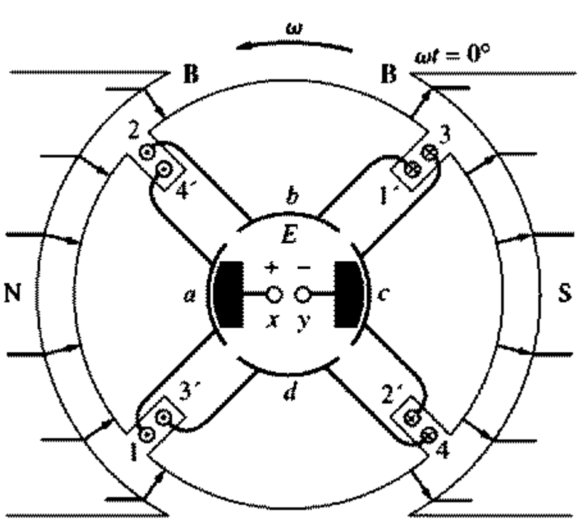
Notice that when the flux in the machine is decreased, its speed increases. This is the same behavior seen in the linear machine and the same way that real dc motors behave.

8.2 COMMUTATION IN A SIMPLE FOUR-LOOP DC MACHINE

Commutation is the process of converting the ac voltages and currents in the rotor of a dc machine to dc voltages and currents at its terminals. It is the most critical part of the design and operation of any dc machine. A more detailed study is necessary to determine just how this conversion occurs and to discover the problems associated with it. In this section, the technique of commutation will be explained for a machine more complex than the single rotating loop in Section 8.1 but less complex than a real dc machine. Section 8.3 will continue this development and explain commutation in real dc machines.

A simple four-loop, two-pole dc machine is shown in Figure 8–7. This machine has four complete loops buried in slots carved in the laminated steel of its rotor. The pole faces of the machine are curved to provide a uniform air-gap width and to give a uniform flux density everywhere under the faces.

The four loops of this machine are laid into the slots in a special manner. The "unprimed" end of each loop is the outermost wire in each slot, while the "primed" end of each loop is the innermost wire in the slot directly opposite. The winding's connections to the machine's commutator are shown in Figure 8–7b. Notice that loop 1 stretches between commutator segments a and b, loop 2 stretches between segments b and c, and so forth around the rotor.



(a)

FIGURE 8–7
(a) A four-loop two-pole dc machine shown at time $\omega t = 0°$. *(continues)*

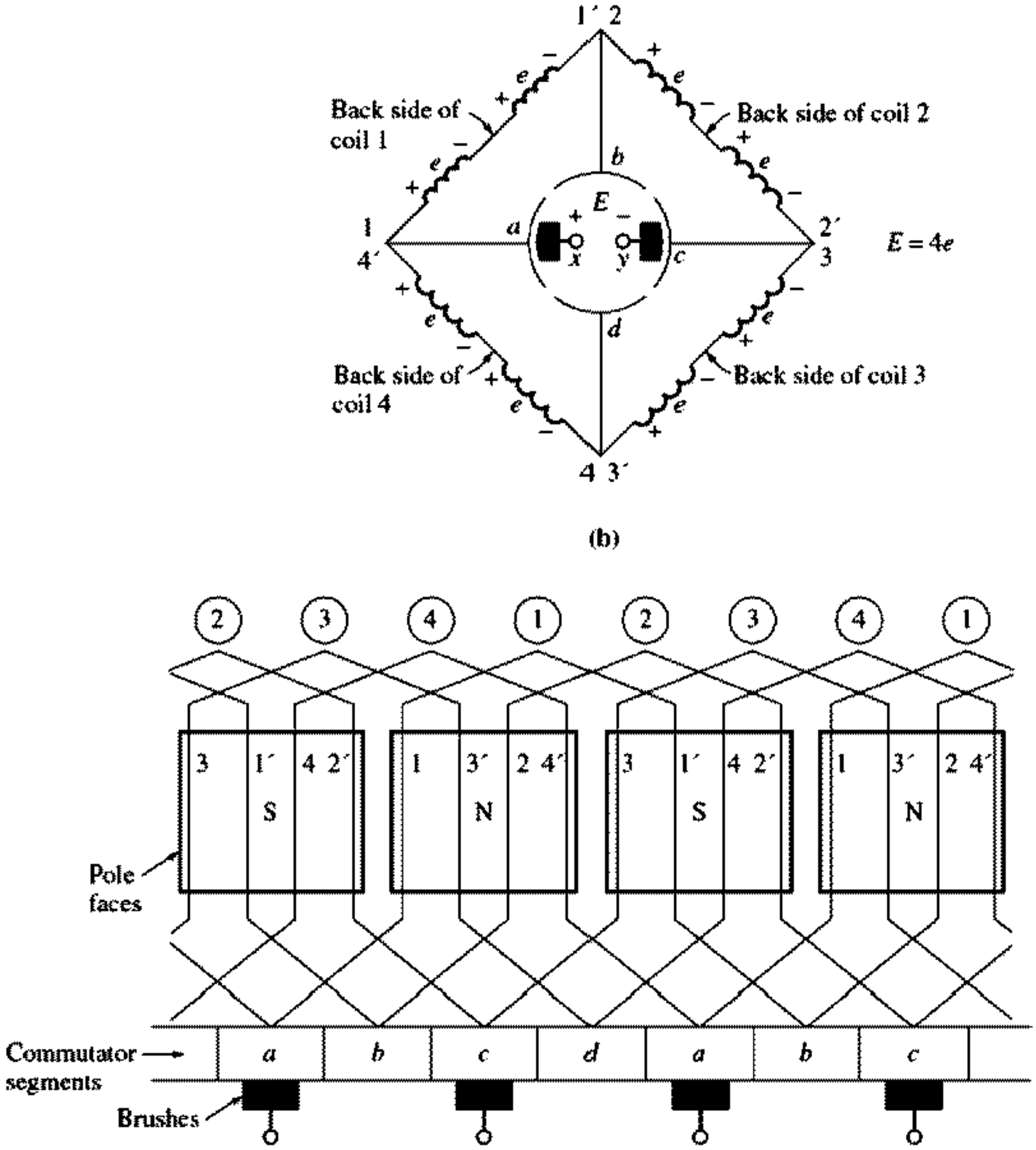


FIGURE 8–7 (concluded)
(b) The voltages on the rotor conductors at this time. (c) A winding diagram of this machine showing the interconnections of the rotor loops.

At the instant shown in Figure 8–7, the 1, 2, 3′, and 4′ ends of the loops are under the north pole face, while the 1′, 2′, 3, and 4 ends of the loops are under the south pole face. The voltage in each of the 1, 2, 3′, and 4′ ends of the loops is given by

$$e_{\text{ind}} = (\mathbf{v} \times \mathbf{B}) \cdot \mathbf{l} \qquad (1\text{–}45)$$

$$e_{\text{ind}} = vBl \qquad \text{positive out of page} \qquad (8\text{–}17)$$

The voltage in each of the 1′, 2′, 3, and 4 ends of the ends of the loops is given by

$$e_{\text{ind}} = (\mathbf{v} \times \mathbf{B}) \cdot \mathbf{l} \qquad (1\text{–}45)$$

$$= vBl \qquad \text{positive into the page} \qquad (8\text{–}18)$$

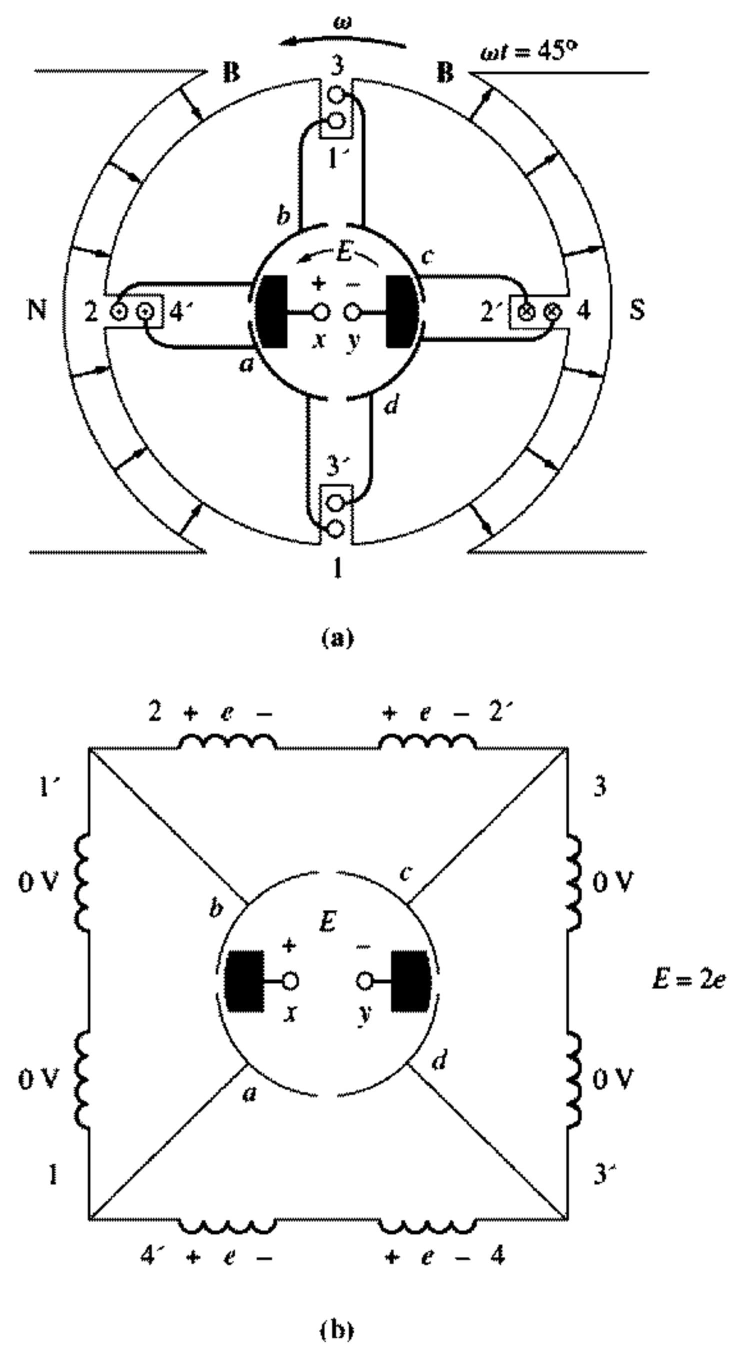


FIGURE 8–8
The same machine at time $\omega t = 45°$, showing the voltages on the conductors.

The overall result is shown in Figure 8–7b. In Figure 8–7b, each coil represents one side (or *conductor*) of a loop. If the induced voltage on any one side of a loop is called $e = vBl$, then the total voltage at the brushes of the machine is

$$\boxed{E = 4e \qquad \omega t = 0°} \tag{8–19}$$

Notice that there are two parallel paths for current through the machine. The existence of two or more parallel paths for rotor current is a common feature of all commutation schemes.

What happens to the voltage E of the terminals as the rotor continues to rotate? To find out, examine Figure 8–8. This figure shows the machine at time $\omega t = 45°$. At that time, loops 1 and 3 have rotated into the gap between the poles, so the voltage across each of them is zero. Notice that at this instant the brushes

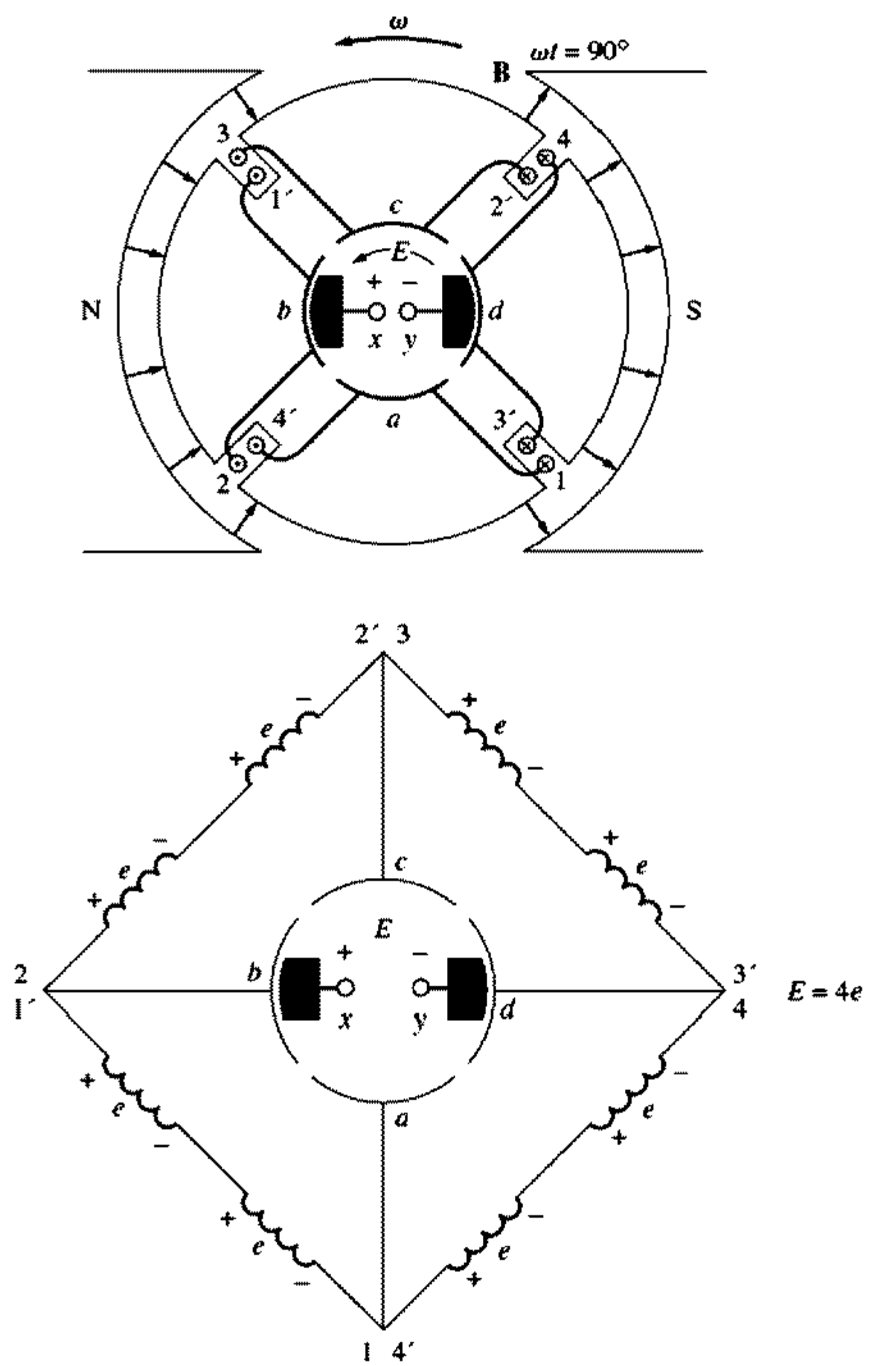


FIGURE 8–9
The same machine at time $\omega t = 90°$, showing the voltages on the conductors.

of the machine are shorting out commutator segments *ab* and *cd*. This happens just at the time when the loops between these segments have 0 V across them, so shorting out the segments creates no problem. At this time, only loops 2 and 4 are under the pole faces, so the terminal voltage *E* is given by

$$\boxed{E = 2e \qquad \omega t = 0°} \tag{8–20}$$

Now let the rotor continue to turn through another 45°. The resulting situation is shown in Figure 8–9. Here, the 1′, 2, 3, and 4′ ends of the loops are under

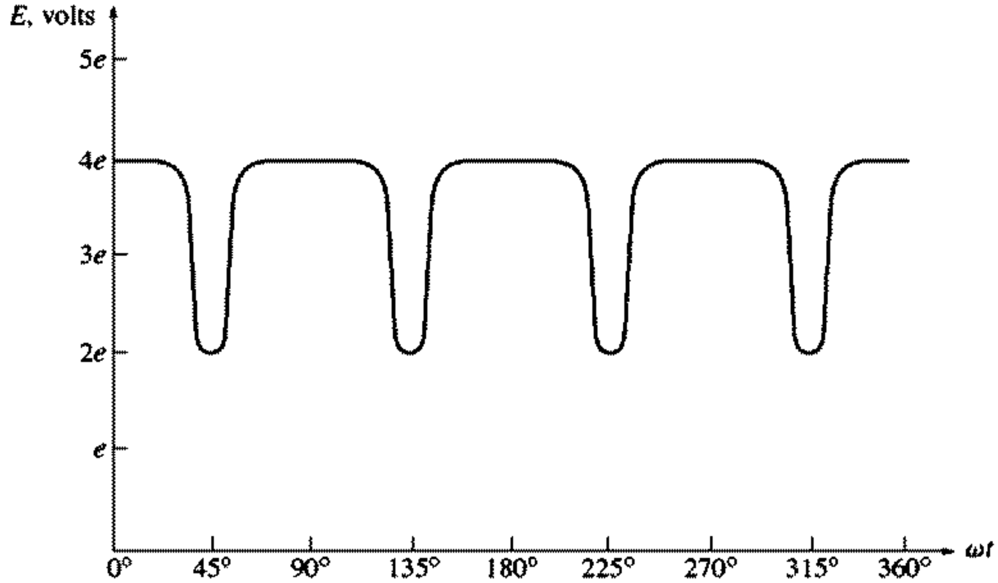


FIGURE 8–10
The resulting output voltage of the machine in Figure 8–7.

the north pole face, and the 1, 2′, 3′, and 4 ends of the loops are under the south pole face. The voltages are still built up out of the page for the ends under the north pole face and into the page for the ends under the south pole face. The resulting voltage diagram is shown in Figure 8–18*b*. There are now four voltage-carrying ends in each parallel path through the machine, so the terminal voltage E is given by

$$\boxed{E = 4e \qquad \omega t = 90^\circ} \tag{8–21}$$

Compare Figure 8–7 to Figure 8–9. Notice that *the voltages on loops 1 and 3 have reversed between the two pictures, but since their connections have also reversed, the total voltage is still being built up in the same direction as before.* This fact is at the heart of every commutation scheme. Whenever the voltage reverses in a loop, the connections of the loop are also switched, and the total voltage is still built up in the original direction.

The terminal voltage of this machine as a function of time is shown in Figure 8–10. It is a better approximation to a constant dc level than the single rotating loop in Section 8.1 produced. As the number of loops on the rotor increases, the approximation to a perfect dc voltage continues to get better and better.

In summary,

> *Commutation* is the process of switching the loop connections on the rotor of a dc machine just as the voltage in the loop switches polarity, in order to maintain an essentially constant dc output voltage.

As in the case of the simple rotating loop, the rotating segments to which the loops are attached are called *commutator segments*, and the stationary pieces that ride on top of the moving segments are called *brushes*. The commutator segments in real machines are typically made of copper bars. The brushes are made of a mixture containing graphite, so that they cause very little friction as they rub over the rotating commutator segments.

8.3 COMMUTATION AND ARMATURE CONSTRUCTION IN REAL DC MACHINES

In real dc machines, there are several ways in which the loops on the rotor (also called the *armature*) can be connected to its commutator segments. These different connections affect the number of parallel current paths within the rotor, the output voltage of the rotor, and the number and position of the brushes riding on the commutator segments. We will now examine the construction of the coils on a real dc rotor and then look at how they are connected to the commutator to produce a dc voltage.

The Rotor Coils

Regardless of the way in which the windings are connected to the commutator segments, most of the rotor windings themselves consist of diamond-shaped preformed coils which are inserted into the armature slots as a unit (see Figure 8–11). Each coil consists of a number of *turns* (loops) of wire, each turn taped and insulated from the other turns and from the rotor slot. Each side of a turn is called *a conductor*. The number of conductors on a machine's armature is given by

$$\boxed{Z = 2CN_C} \tag{8–22}$$

where Z = number of conductors on rotor

C = number of coils on rotor

N_C = number of turns per coil

Normally, a coil spans 180 electrical degrees. This means that when one side is under the center of a given magnetic pole, the other side is under the center of a pole of *opposite polarity*. The *physical* poles may not be located 180 mechanical degrees apart, but the magnetic field has completely reversed its polarity in traveling from under one pole to the next. The relationship between the electrical angle and mechanical angle in a given machine is given by

$$\boxed{\theta_e = \frac{P}{2}\theta_m} \tag{8–23}$$

where θ_e = electrical angle, in degrees

θ_m = mechanical angle, in degrees

P = number of magnetic poles on the machine

If a coil spans 180 electrical degrees, the voltages in the conductors on either side of the coil will be exactly the same in magnitude and opposite in direction at all times. Such a coil is called a *full-pitch coil.*

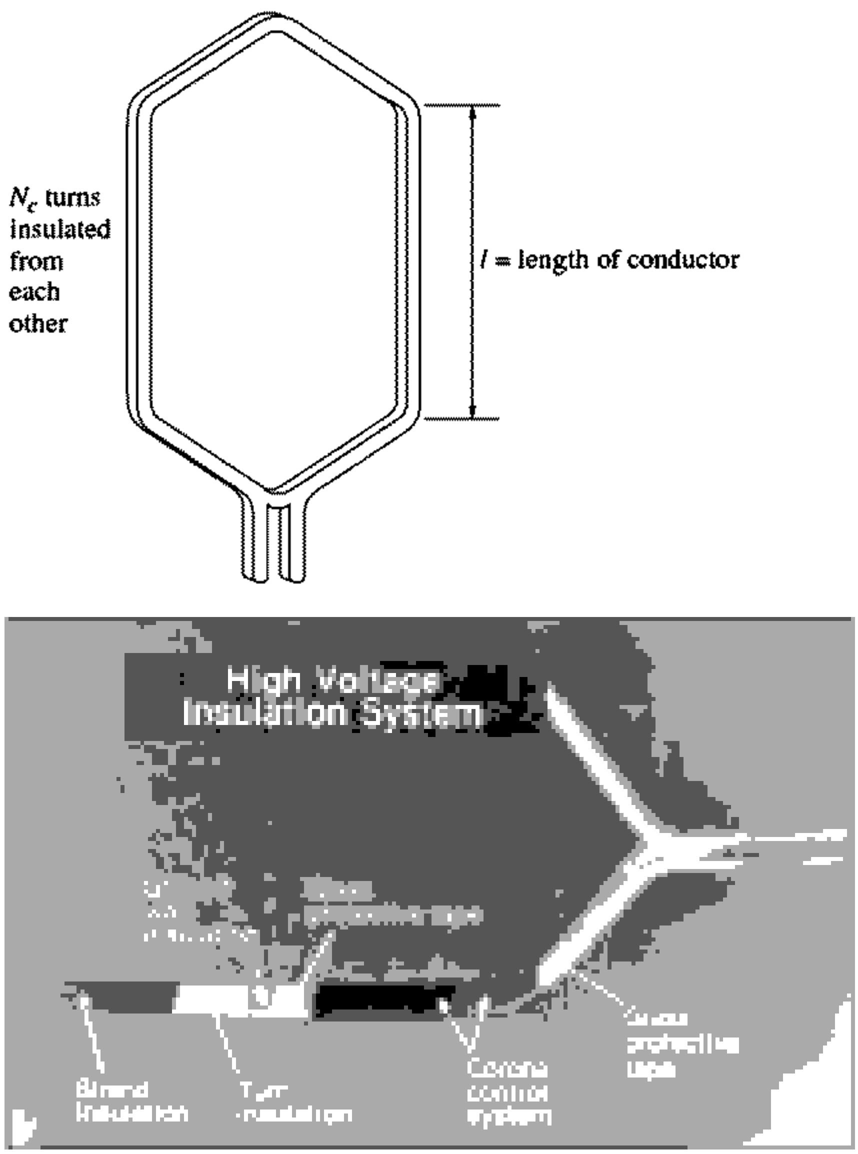


FIGURE 8–11
(a) The shape of a typical preformed rotor coil. (b) A typical coil insulation system showing the insulation between turns within a coil. (*Courtesy of General Electric Company.*)

Sometimes a coil is built that spans less than 180 electrical degrees. Such a coil is called a *fractional-pitch coil*, and a rotor winding wound with fractional-pitch coils is called a *chorded winding*. The amount of chording in a winding is described by a *pitch factor p*, which is defined by the equation

$$\boxed{p = \frac{\text{electrical angle of coil}}{180^\circ} \times 100\%} \tag{8–24}$$

Sometimes a small amount of chording will be used in dc rotor windings to improve commutation.

Most rotor windings are *two-layer windings*, meaning that sides from two different coils are inserted into each slot. One side of each coil will be at the bottom of its slot, and the other side will be at the top of its slot. Such a construction requires the individual coils to be placed in the rotor slots by a very elaborate

FIGURE 8–12
The installation of preformed rotor coils on a dc machine rotor. (*Courtesy of Westinghouse Electric Company.*)

procedure (see Figure 8–12). One side of each of the coils is placed in the bottom of its slot, and then after all the bottom sides are in place, the other side of each coil is placed in the top of its slot. In this fashion, all the windings are woven together, increasing the mechanical strength and uniformity of the final structure.

Connections to the Commutator Segments

Once the windings are installed in the rotor slots, they must be connected to the commutator segments. There are a number of ways in which these connections can be made, and the different winding arrangements which result have different advantages and disadvantages.

The distance (in number of segments) between the commutator segments to which the two ends of a coil are connected is called the *commutator pitch* y_c. If the end of a coil (or a set number of coils, for wave construction) is connected to a commutator segment ahead of the one its beginning is connected to, the winding is called a *progressive winding*. If the end of a coil is connected to a commutator segment behind the one its beginning is connected to, the winding is called a *retrogressive winding*. If everything else is identical, the direction of rotation of a progressive-wound rotor will be opposite to the direction of rotation of a retrogressive-wound rotor.

Rotor (armature) windings are further classified according to the *plex* of their windings. A *simplex* rotor winding is a single, complete, closed winding wound on a rotor. A *duplex* rotor winding is a rotor with *two complete and independent sets* of rotor windings. If a rotor has a duplex winding, then each of the windings will be associated with every other commutator segment: One winding

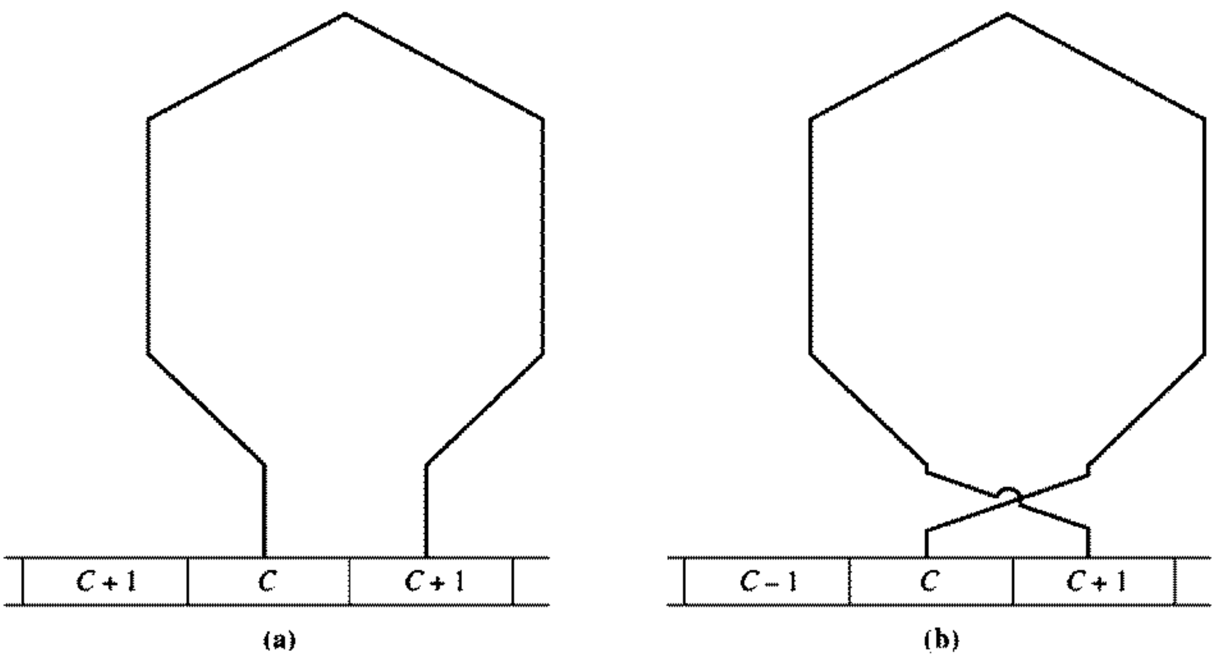


FIGURE 8–13
(a) A coil in a progressive rotor winding. (b) A coil in a retrogressive rotor winding.

will be connected to segments 1, 3, 5, etc., and the other winding will be connected to segments 2, 4, 6, etc. Similarly, a *triplex* winding will have three complete and independent sets of windings, each winding connected to every third commutator segment on the rotor. Collectively, all armatures with more than one set of windings are said to have *multiplex windings*.

Finally, armature windings are classified according to the sequence of their connections to the commutator segments. There are two basic sequences of armature winding connections—*lap windings* and *wave windings*. In addition, there is a third type of winding, called *a frog-leg winding*, which combines lap and wave windings on a single rotor. These windings will be examined individually below, and their advantages and disadvantages will be discussed.

The Lap Winding

The simplest type of winding construction used in modern dc machines is the simplex *series* or *lap winding*. A simplex lap winding is a rotor winding consisting of coils containing one or more turns of wire with the two ends of each coil coming out at *adjacent commutator segments* (Figure 8–13). If the end of the coil is connected to the segment after the segment that the beginning of the coil is connected to, the winding is a progressive lap winding and $y_c = 1$; if the end of the coil is connected to the segment before the segment that the beginning of the coil is connected to, the winding is a retrogressive lap winding and $y_c = -1$. A simple two-pole machine with lap windings is shown in Figure 8–14.

An interesting feature of simplex lap windings is that *there are as many parallel current paths through the machine as there are poles on the machine*. If C is the number of coils and commutator segments present in the rotor and P is the

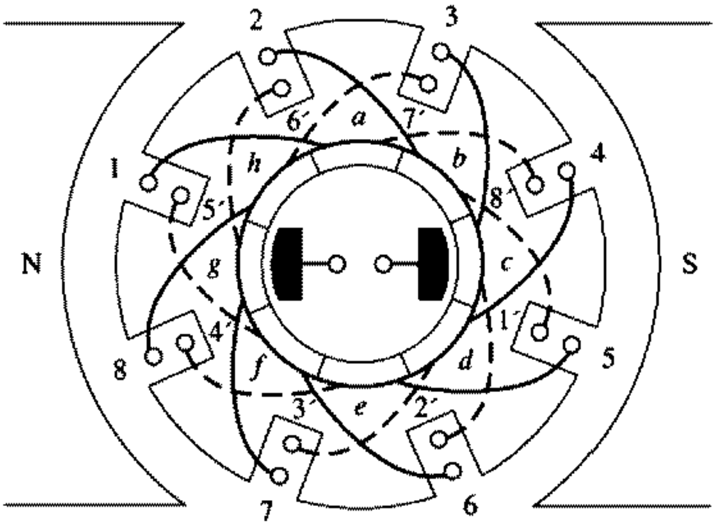


FIGURE 8–14
A simple two-pole lap-wound dc machine.

number of poles on the machine, then there will be *C/P* coils in each of the *P* parallel current paths through the machine. The fact that there are *P* current paths also requires that there be as many brushes on the machine as there are poles in order to tap all the current paths. This idea is illustrated by the simple four-pole motor in Figure 8–15. Notice that, for this motor, there are four current paths through the rotor, each having an equal voltage. The fact that there are many current paths in a multipole machine makes the lap winding an ideal choice for fairly low-voltage, high-current machines, since the high currents required can be split among the several different current paths. This current splitting permits the size of individual rotor conductors to remain reasonable even when the total current becomes extremely large.

The fact that there are many parallel paths through a multipole lap-wound machine can lead to a serious problem, however. To understand the nature of this problem, examine the six-pole machine in Figure 8–16. Because of long usage, there has been slight wear on the bearings of this machine, and the lower wires are closer to their pole faces than the upper wires are. As a result, there is a *larger* voltage in the current paths involving wires under the lower pole faces than in the paths involving wires under the upper pole faces. Since all the paths are connected in parallel, the result will be a circulating current flowing out some of the brushes in the machine and back into others, as shown in Figure 8–17. Needless to say, this is not good for the machine. Since the winding resistance of a rotor circuit is so small, a very tiny imbalance among the voltages in the parallel paths will cause large circulating currents through the brushes and potentially serious heating problems.

The problem of circulating currents within the parallel paths of a machine with four or more poles can never be entirely resolved, but it can be reduced somewhat by *equalizers* or *equalizing windings*. Equalizers are bars located on the rotor of a lap-wound dc machine that short together points at the same voltage

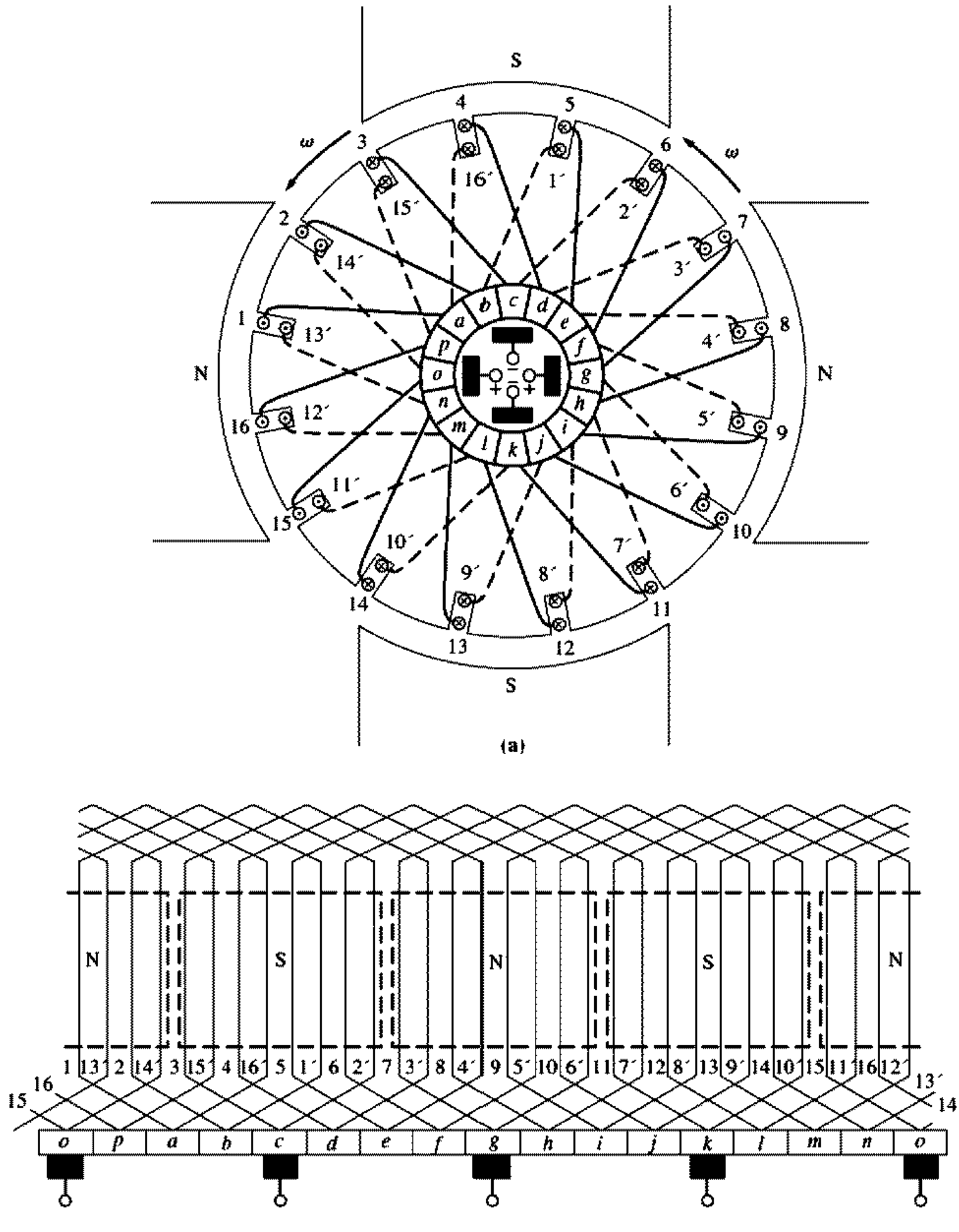


FIGURE 8–15
(a) A four-pole lap-wound dc motor. (b) The rotor winding diagram of this machine. Notice that each winding ends on the commutator segment just after the one it begins at. This is a progressive lap winding.

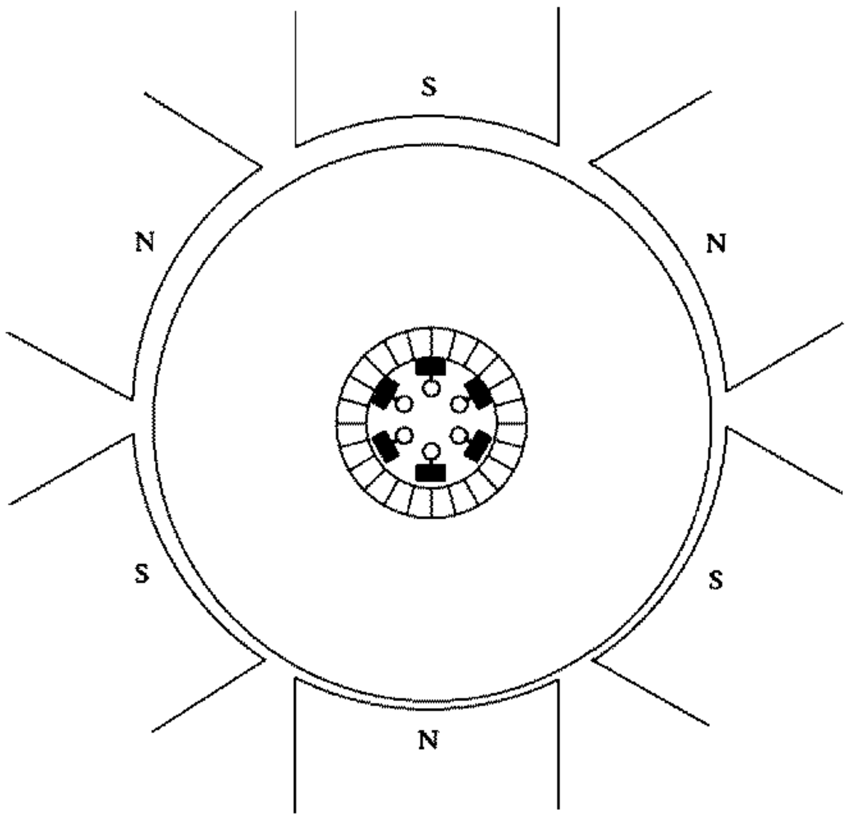


FIGURE 8–16
A six-pole dc motor showing the effects of bearing wear. Notice that the rotor is slightly closer to the lower poles than it is to the upper poles.

level in the different parallel paths. The effect of this shorting is to cause any circulating currents that occur to flow inside the small sections of windings thus shorted together and to prevent this circulating current from flowing through the brushes of the machine. These circulating currents even partially correct the flux imbalance that caused them to exist in the first place. An equalizer for the four-pole machine in Figure 8–15 is shown in Figure 8–18, and an equalizer for a large lap-wound dc machine is shown in Figure 8–19.

If a lap winding is duplex, then there are two completely independent windings wrapped on the rotor, and every other commutator segment is tied to one of the sets. Therefore, an individual coil ends on the second commutator segment down from where it started, and $y_c = \pm 2$ (depending on whether the winding is progressive or retrogressive). Since each set of windings has as many current paths as the machine has poles, there are *twice as many current paths* as the machine has poles in a duplex lap winding.

In general, for an m-plex lap winding, the commutator pitch y_c is

$$\boxed{y_c = \pm m} \qquad \text{lap winding} \tag{8–25}$$

and the number of current paths in a machine is

$$\boxed{a = mP} \qquad \text{lap winding} \tag{8–26}$$

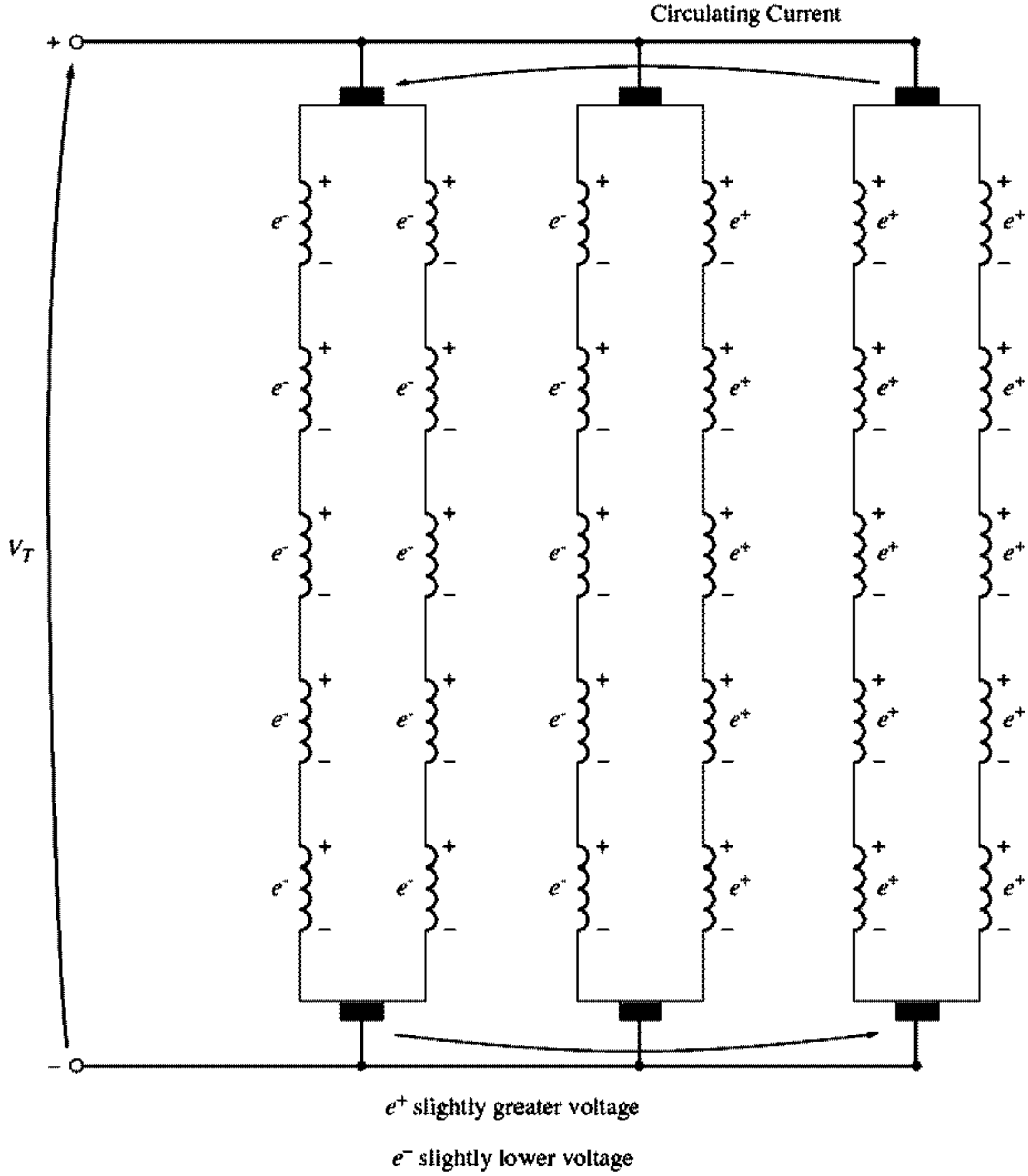
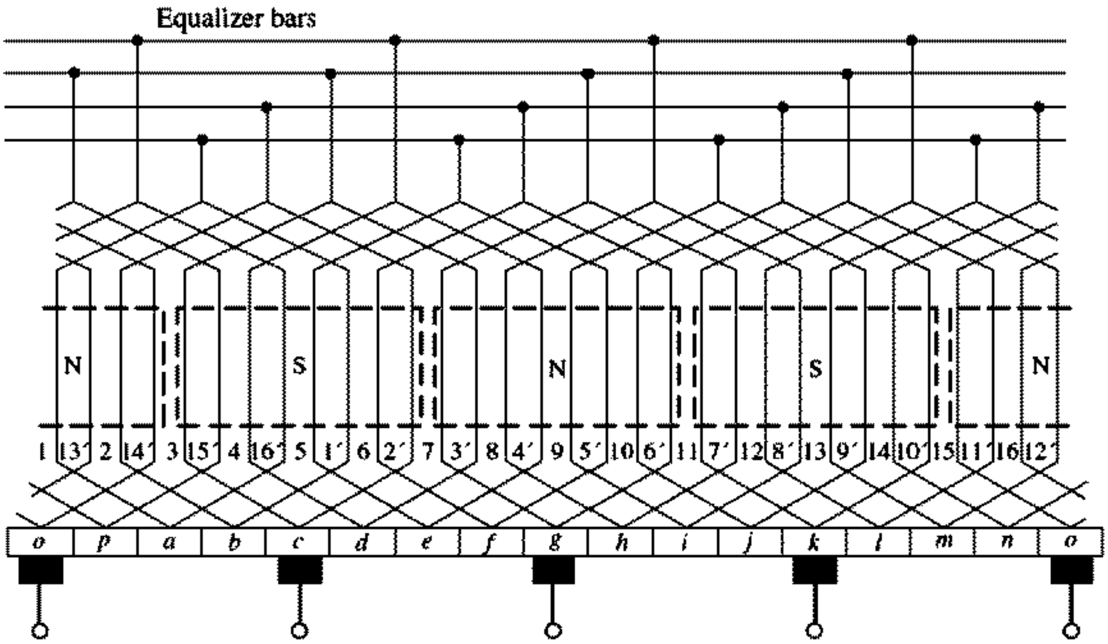


FIGURE 8–17
The voltages on the rotor conductors of the machine in Figure 8–16 are unequal, producing circulating currents flowing through its brushes.

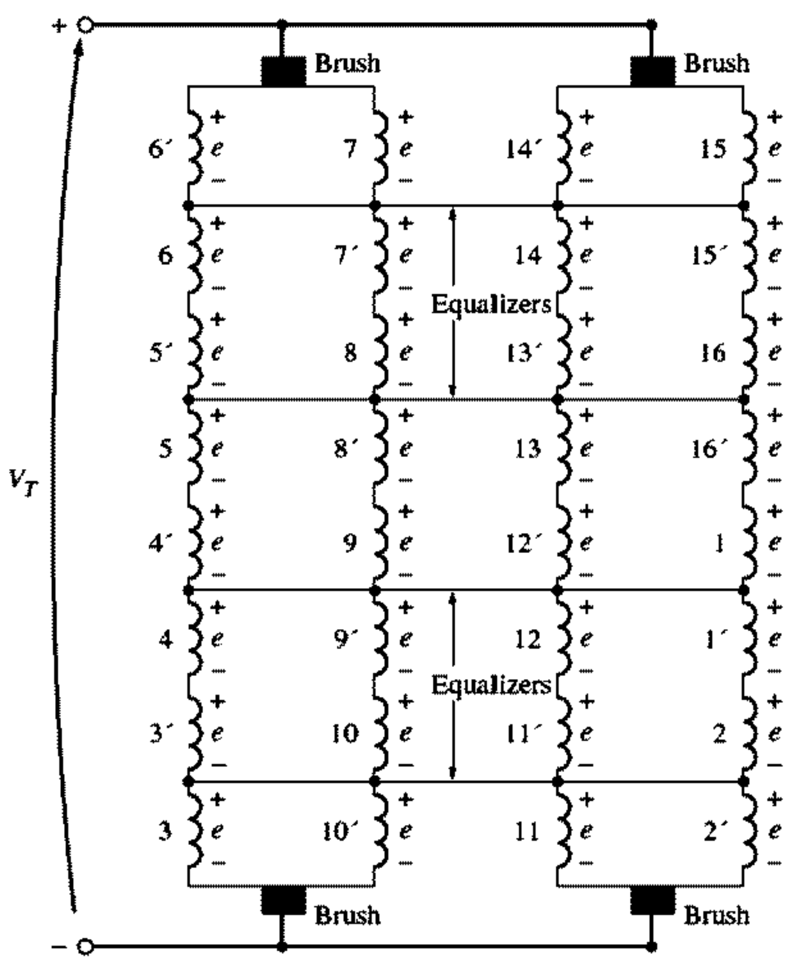
where a = number of current paths in the rotor
m = plex of the windings (1, 2, 3, etc.)
P = number of poles on the machine

The Wave Winding

The *series* or *wave winding* is an alternative way to connect the rotor coils to the commutator segments. Figure 8–20 shows a simple four-pole machine with a



(a)



(b)

FIGURE 8–18
(a) An equalizer connection for the four-pole machine in Figure 8–15. (b) A voltage diagram for the machine shows the points shorted by the equalizers.

FIGURE 8–19
A closeup of the commutator of a large lap-wound dc machine. The equalizers are mounted in the small ring just in front of the commutator segments. (*Courtesy of General Electric Company.*)

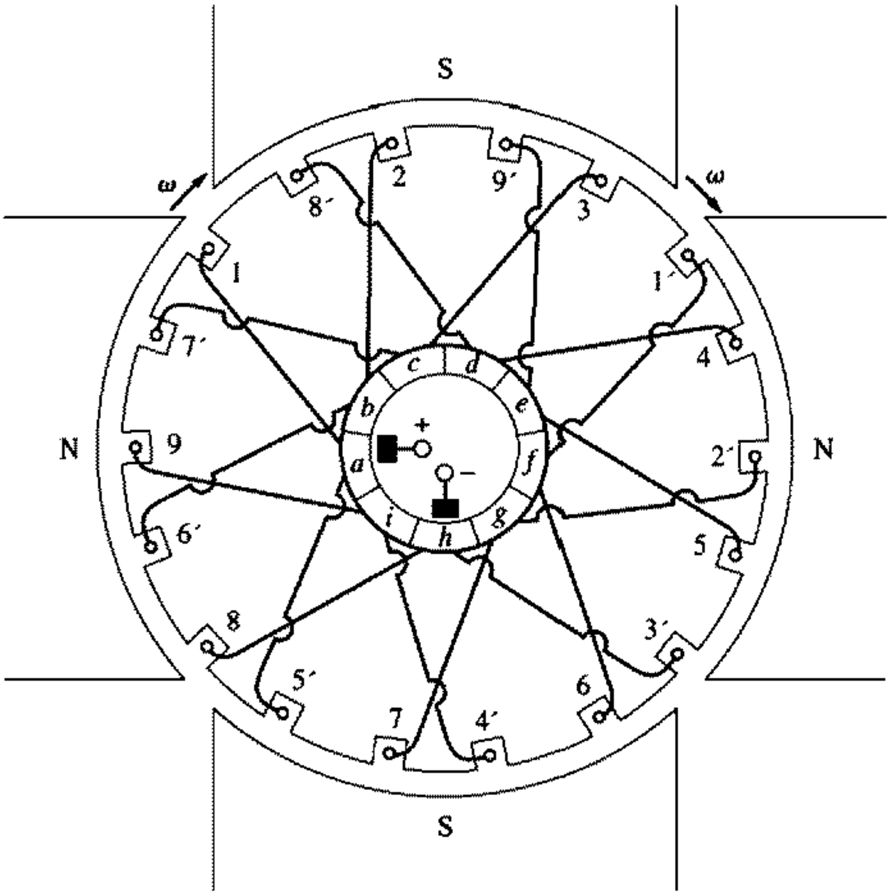


FIGURE 8–20
A simple four-pole wave-wound dc machine.

simplex wave winding. In this simplex wave winding, every *other* rotor coil connects back to a commutator segment adjacent to the beginning of the first coil. Therefore, *there are two coils in series* between the adjacent commutator segments. Furthermore, since each pair of coils between adjacent segments has a side under each pole face, all output voltages are the sum of the effects of every pole, and there can be no voltage imbalances.

The lead from the second coil may be connected to the segment either ahead of or behind the segment at which the first coil begins. If the second coil is connected to the segment ahead of the first coil, the winding is progressive; if it is connected to the segment behind the first coil, it is retrogressive.

In general, if there are P poles on the machine, then there are $P/2$ coils in series between adjacent commutator segments. If the $(P/2)$th coil is connected to the segment ahead of the first coil, the winding is progressive. If the $(P/2)$th coil is connected to the segment behind the first coil, the winding is retrogressive.

In a simplex wave winding, there are only two current paths. There are $C/2$ or one-half of the windings in each current path. The brushes in such a machine will be located a full pole pitch apart from each other.

What is the commutator pitch for a wave winding? Figure 8–20 shows a progressive nine-coil winding, and the end of a coil occurs five segments down from its starting point. In a retrogressive wave winding, the end of the coil occurs four segments down from its starting point. Therefore, the end of a coil in a four-pole wave winding must be connected just before or just after the point halfway around the circle from its starting point.

The general expression for commutator pitch in any simplex wave winding is

$$\boxed{y_c = \frac{2(C \pm 1)}{P}} \qquad \text{simplex wave} \qquad (8\text{–}27)$$

where C is the number of coils on the rotor and P is the number of poles on the machine. The plus sign is associated with progressive windings, and the minus sign is associated with retrogressive windings. A simplex wave winding is shown in Figure 8–21.

Since there are only two current paths through a simplex wave-wound rotor, only two brushes are needed to draw off the current. This is because the segments undergoing commutation connect the points with equal voltage under all the pole faces. More brushes can be added at points 180 electrical degrees apart if desired, since they are at the same potential and are connected together by the wires undergoing commutation in the machine. Extra brushes are usually added to a wave-wound machine, even though they are not necessary, because they reduce the amount of current that must be drawn through a given brush set.

Wave windings are well suited to building higher-voltage dc machines, since the number of coils in series between commutator segments permits a high voltage to be built up more easily than with lap windings.

A multiplex wave winding is a winding with multiple *independent* sets of wave windings on the rotor. These extra sets of windings have two current paths each, so the number of current paths on a multiplex wave winding is

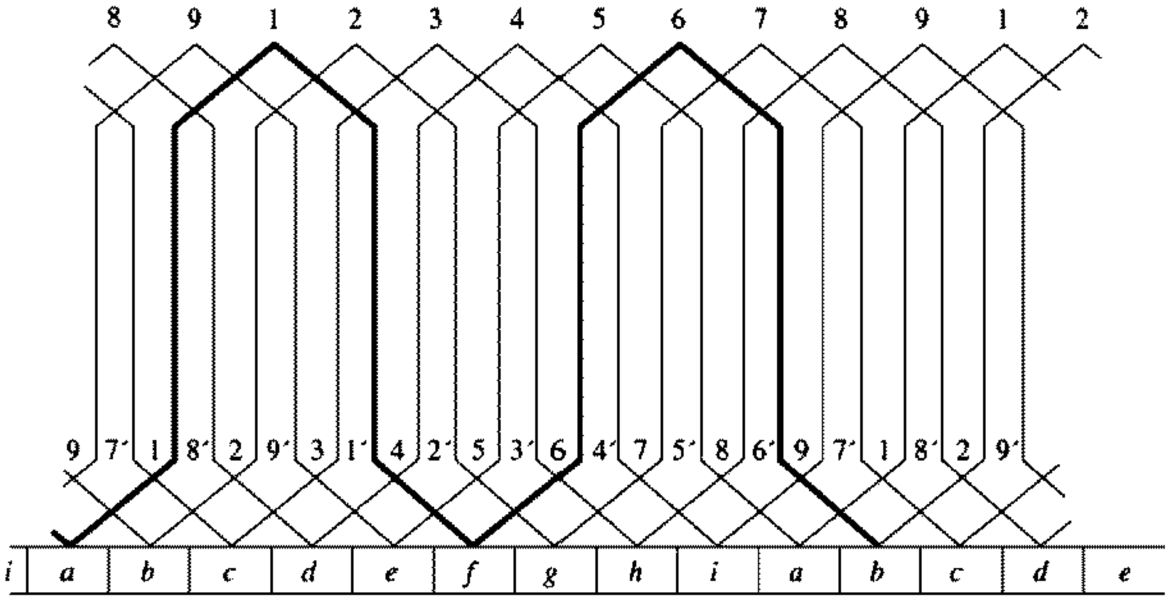


FIGURE 8–21
The rotor winding diagram for the machine in Figure 8–20. Notice that the end of every second coil in series connects to the segment after the beginning of the first coil. This is a progressive wave winding.

$$\boxed{a = 2m} \qquad \text{multiplex wave} \qquad (8\text{–}28)$$

The Frog-Leg Winding

The *frog-leg winding or self-equalizing winding* gets its name from the shape of its coils, as shown in Figure 8–22. It consists of a lap winding and a wave winding combined.

The equalizers in an ordinary lap winding are connected at points of equal voltage on the windings. Wave windings reach between points of essentially equal voltage under successive pole faces of the same polarity, which are the same locations that equalizers tie together. A frog-leg or self-equalizing winding combines a lap winding with a wave winding, so that the wave windings can function as equalizers for the lap winding.

The number of current paths present in a frog-leg winding is

$$\boxed{a = 2Pm_{\text{lap}}} \qquad \text{frog-leg winding} \qquad (8\text{–}29)$$

where P is the number of poles on the machine and m_{lap} is the plex of the lap winding.

Example 8–2. Describe the rotor winding arrangement of the four-loop machine in Section 8.2.

Solution

The machine described in Section 8.2 has four coils, each containing one turn, resulting in a total of eight conductors. It has a progressive lap winding.

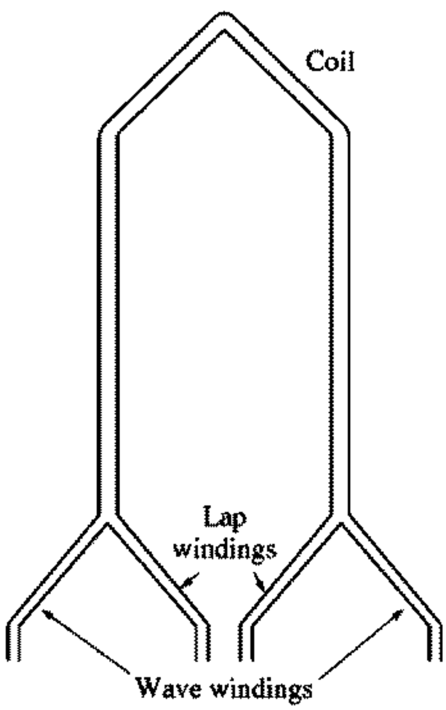


FIGURE 8–22
A frog-leg or self-equalizing winding coil.

8.4 PROBLEMS WITH COMMUTATION IN REAL MACHINES

The commutation process as described in Sections 8.2 and 8.3 is not as simple in practice as it seems in theory, because two major effects occur in the real world to disturb it:

1. Armature reaction
2. *L di/dt* voltages

This section explores the nature of these problems and the solutions employed to mitigate their effects.

Armature Reaction

If the magnetic field windings of a dc machine are connected to a power supply and the rotor of the machine is turned by an external source of mechanical power, then a voltage will be induced in the conductors of the rotor. This voltage will be rectified into a dc output by the action of the machine's commutator.

Now connect a load to the terminals of the machine, and a current will flow in its armature windings. This current flow will produce a magnetic field of its own, which will distort the original magnetic field from the machine's poles. This distortion of the flux in a machine as the load is increased is called *armature reaction*. It causes two serious problems in real dc machines.

The first problem caused by armature reaction is *neutral-plane shift*. The *magnetic neutral plane* is defined as the plane within the machine where the

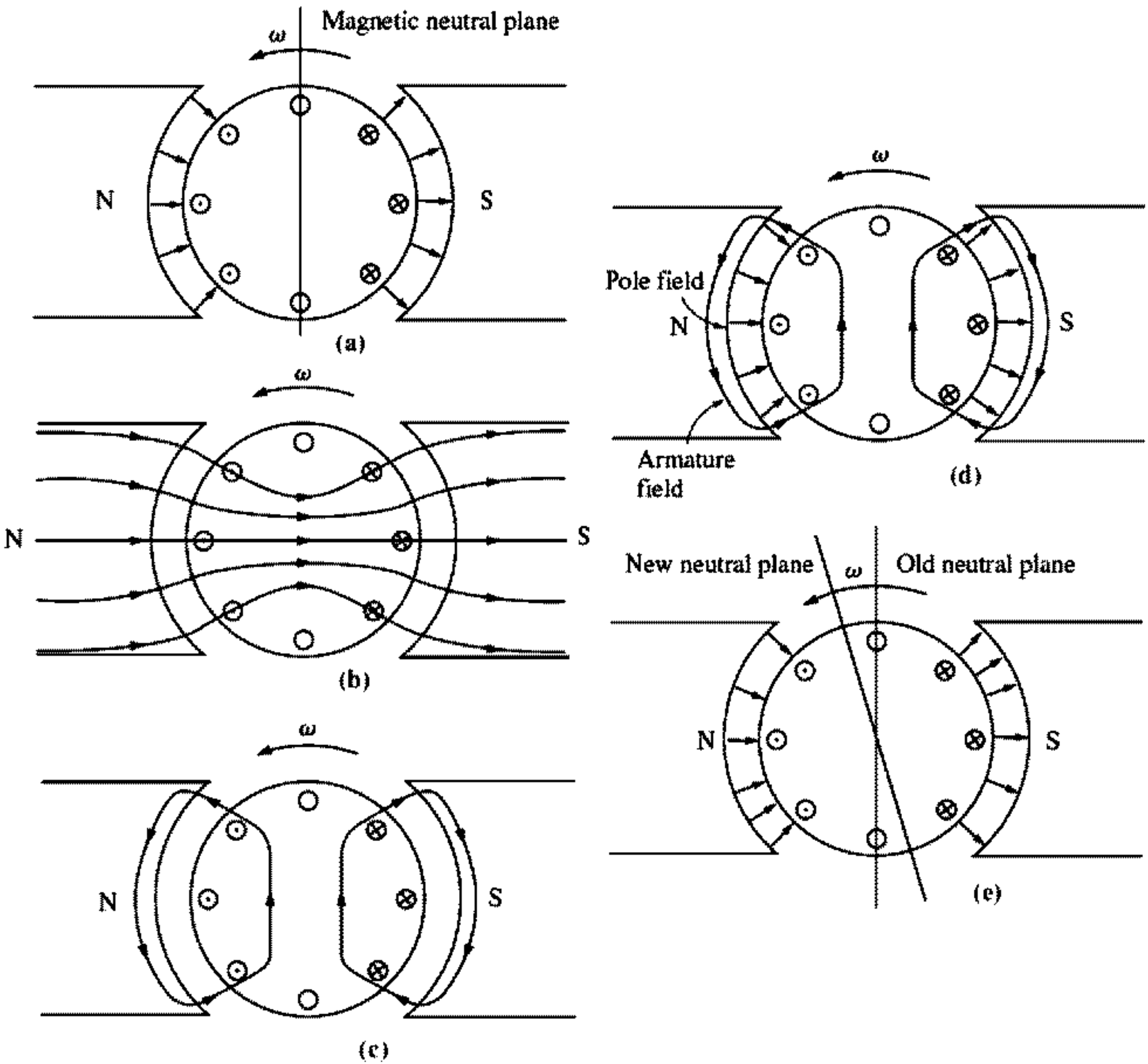


FIGURE 8–23
The development of armature reaction in a dc generator. (a) Initially the pole flux is uniformly distributed, and the magnetic neutral plane is vertical; (b) the effect of the air gap on the pole flux distribution; (c) the armature magnetic field resulting when a load is connected to the machine; (d) both rotor and pole fluxes are shown, indicating points where they add and subtract; (e) the resulting flux under the poles. The neutral plane has shifted in the direction of motion.

velocity of the rotor wires is exactly parallel to the magnetic flux lines, so that e_{ind} in the conductors in the plane is exactly zero.

To understand the problem of neutral-plane shift, examine Figure 8–23. Figure 8–23a shows a two-pole dc machine. Notice that the flux is distributed uniformly under the pole faces. The rotor windings shown have voltages built up out of the page for wires under the north pole face and into the page for wires under the south pole face. The neutral plane in this machine is exactly vertical.

Now suppose a load is connected to this machine so that it acts as a generator. Current will flow out of the positive terminal of the generator, so current will be flowing out of the page for wires under the north pole face and into the page for wires under the south pole face. This current flow produces a magnetic field from the rotor windings, as shown in Figure 8–23c. This rotor magnetic field affects the original magnetic field from the poles that produced the generator's voltage in the first place. In some places under the pole surfaces, it subtracts from the pole flux, and in other places it adds to the pole flux. The overall result is that the magnetic flux in the air gap of the machine is skewed as shown in Figure 8–23d and e. Notice that the place on the rotor where the induced voltage in a conductor would be zero (the neutral plane) has shifted.

For the generator shown in Figure 8–23, the magnetic neutral plane shifted in the direction of rotation. If this machine had been a motor, the current in its rotor would be reversed and the flux would bunch up in the opposite corners from the bunches shown in the figure. As a result, the magnetic neutral plane would shift the other way.

In general, the neutral-plane shifts in the direction of motion for a generator and opposite to the direction of motion for a motor. Furthermore, the amount of the shift depends on the amount of rotor current and hence on the load of the machine.

So what's the big deal about neutral-plane shift? It's just this: The commutator must short out commutator segments just at the moment when the voltage across them is equal to zero. If the brushes are set to short out conductors in the vertical plane, then the voltage between segments is indeed zero *until the machine is loaded.* When the machine is loaded, the neutral plane shifts, and the brushes short out commutator segments with a finite voltage across them. The result is a current flow circulating between the shorted segments and large sparks at the brushes when the current path is interrupted as the brush leaves a segment. The end result is *arcing and sparking at the brushes.* This is a very serious problem, since it leads to drastically reduced brush life, pitting of the commutator segments, and greatly increased maintenance costs. Notice that this problem cannot be fixed even by placing the brushes over the full-load neutral plane, because then they would spark at no load.

In extreme cases, the neutral-plane shift can even lead to *flashover* in the commutator segments near the brushes. The air near the brushes in a machine is normally ionized as a result of the sparking on the brushes. Flashover occurs when the voltage of adjacent commutator segments gets large enough to sustain an arc in the ionized air above them. If flashover occurs, the resulting arc can even melt the commutator's surface.

The second major problem caused by armature reaction is called *flux weakening.* To understand flux weakening, refer to the magnetization curve shown in Figure 8–24. Most machines operate at flux densities near the saturation point. Therefore, at locations on the pole surfaces where the rotor magnetomotive force adds to the pole magnetomotive force, only a small increase in flux occurs. But at locations on the pole surfaces where the rotor magnetomotive force subtracts from the pole magnetomotive force, there is a larger decrease in flux. The net result is that *the total average flux under the entire pole face is decreased* (see Figure 8–25).

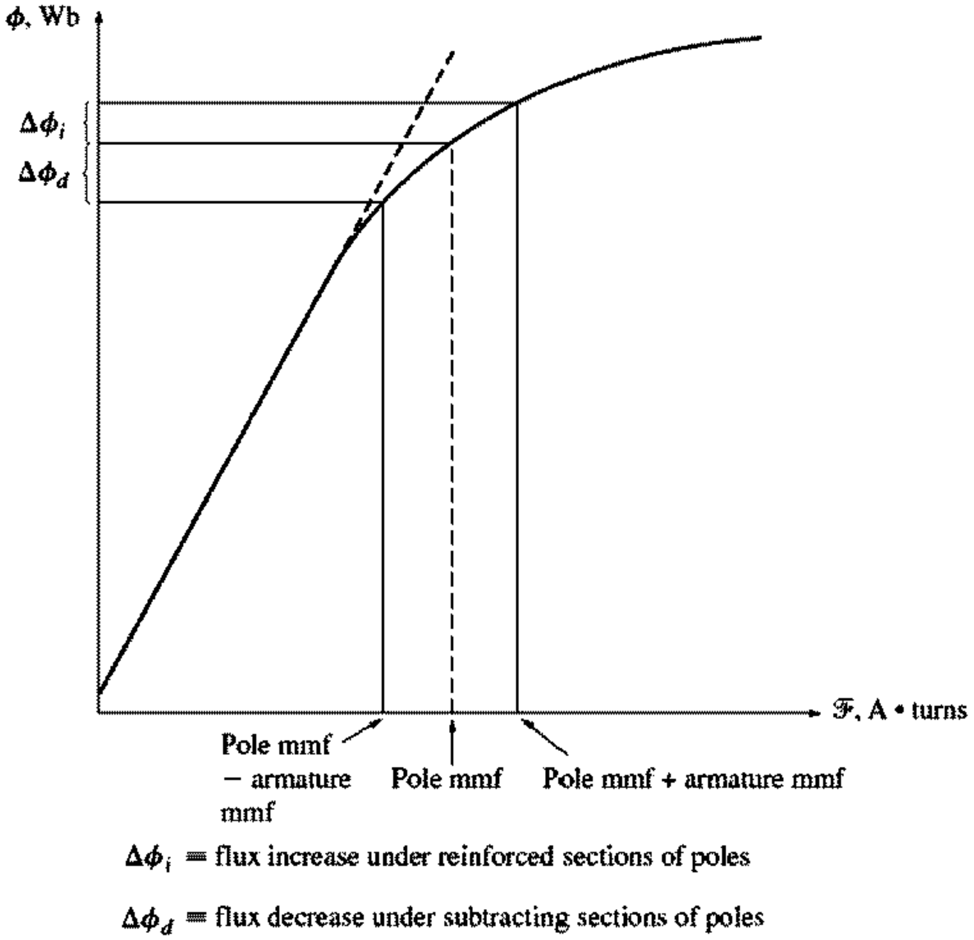


FIGURE 8–24
A typical magnetization curve shows the effects of pole saturation where armature and pole magnetomotive forces add.

Flux weakening causes problems in both generators and motors. In generators, the effect of flux weakening is simply to reduce the voltage supplied by the generator for any given load. In motors, the effect can be more serious. As the early examples in this chapter showed, when the flux in a motor is decreased, its speed increases. But increasing the speed of a motor can increase its load, resulting in more flux weakening. It is possible for some shunt dc motors to reach a runaway condition as a result of flux weakening, where the speed of the motor just keeps increasing until the machine is disconnected from the power line or until it destroys itself.

L di/dt Voltages

The second major problem is the *L di/dt* voltage that occurs in commutator segments being shorted out by the brushes, sometimes called *inductive kick*. To understand this problem, look at Figure 8–26. This figure represents a series of commutator segments and the conductors connected between them. Assuming that the current in the brush is 400 A, the current in each path is 200 A. Notice that when a commutator segment is shorted out, the current flow through that commutator

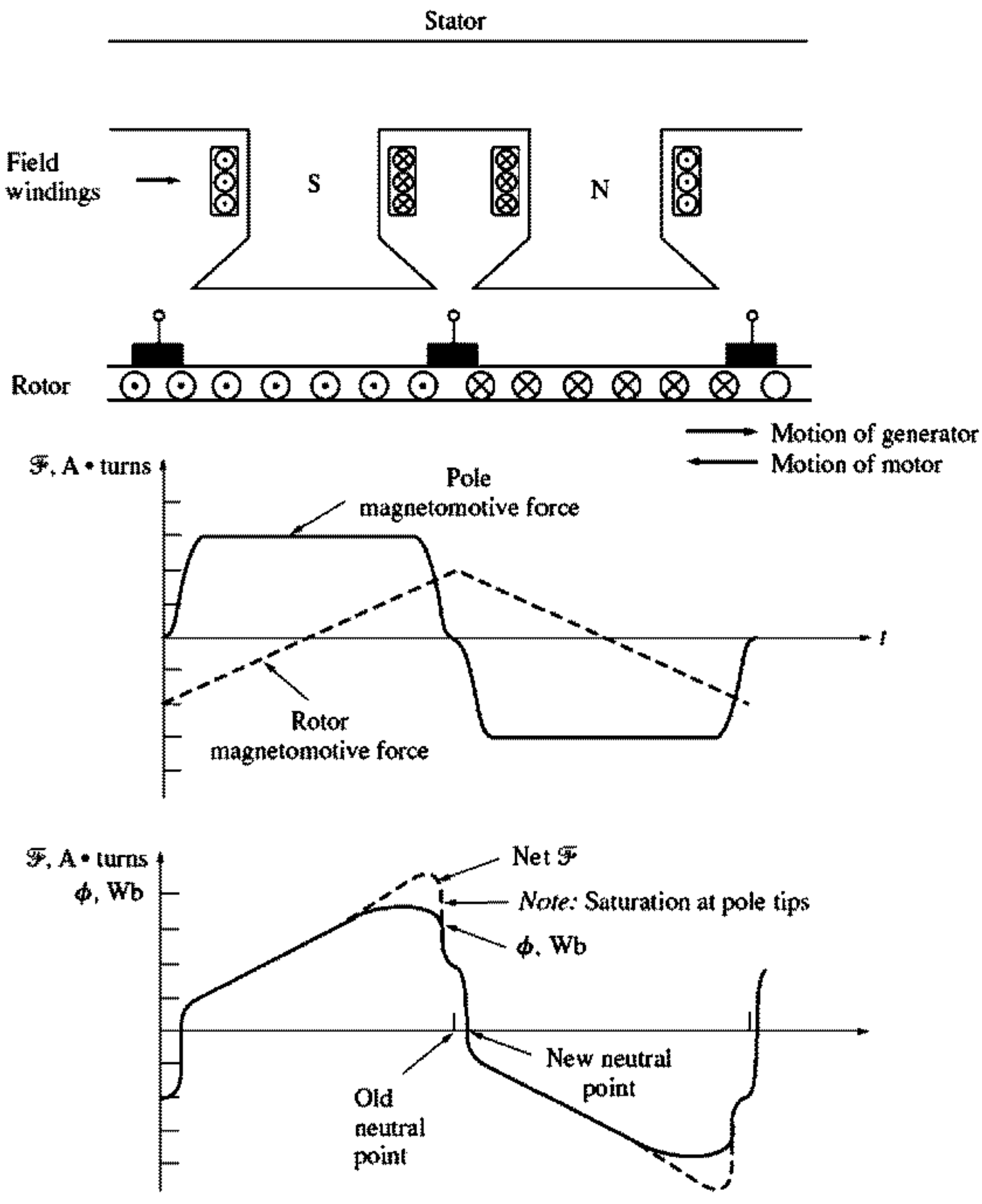


FIGURE 8–25
The flux and magnetomotive force under the pole faces in a dc machine. At those points where the magnetomotive forces subtract, the flux closely follows the net magnetomotive force in the iron; but at those points where the magnetomotive forces add, saturation limits the total flux present. Note also that the neutral point of the rotor has shifted.

segment must reverse. How fast must this reversal occur? Assuming that the machine is turning at 800 r/min and that there are 50 commutator segments (a reasonable number for a typical motor), each commutator segment moves under a brush and clears it again in $t = 0.0015$ s. Therefore, the rate of change in current with respect to time in the shorted loop must *average*

$$\frac{di}{dt} = \frac{400\text{ A}}{0.0015\text{ s}} = 266{,}667\text{ A/s} \qquad (8\text{–}30)$$

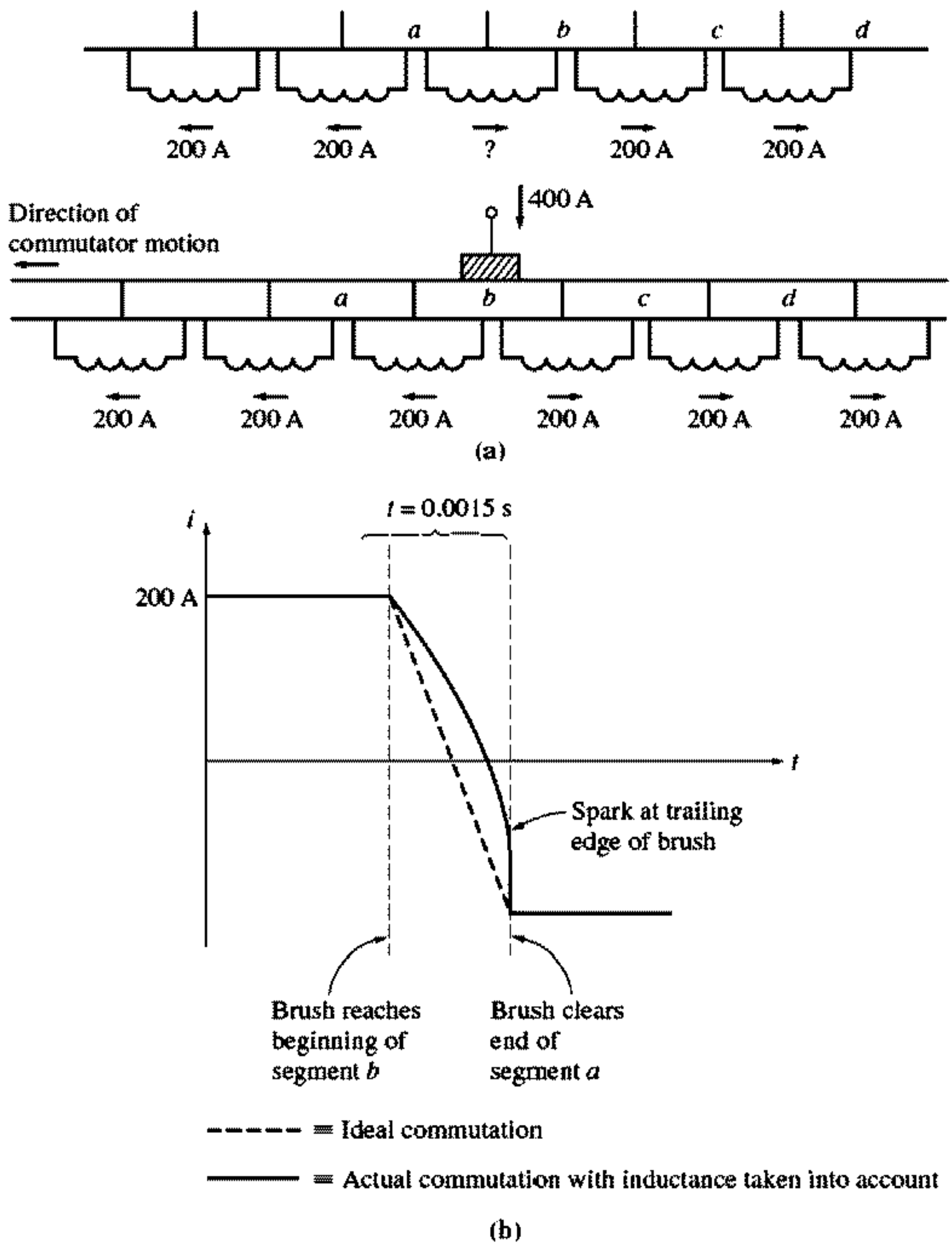


FIGURE 8–26

(a) The reversal of current flow in a coil undergoing commutation. Note that the current in the coil between segments a and b must reverse direction while the brush shorts together the two commutator segments. (b) The current reversal in the coil undergoing commutation as a function of time for both ideal commutation and real commutation, with the coil inductance taken into account.

With even a tiny inductance in the loop, a very significant inductive voltage kick $v = L\,di/dt$ will be induced in the shorted commutator segment. This high voltage naturally causes sparking at the brushes of the machine, resulting in the same arcing problems that the neutral-plane shift causes.

Solutions to the Problems with Commutation

Three approaches have been developed to partially or completely correct the problems of armature reaction and $L\,di/dt$ voltages:

1. Brush shifting
2. Commutating poles or interpoles
3. Compensating windings

Each of these techniques is explained below, together with its advantages and disadvantages.

BRUSH SHIFTING. Historically, the first attempts to improve the process of commutation in real dc machines started with attempts to stop the sparking at the brushes caused by the neutral-plane shifts and $L\,di/dt$ effects. The first approach taken by machine designers was simple: If the neutral plane of the machine shifts, why not shift the brushes with it in order to stop the sparking? It certainly seemed like a good idea, but there are several serious problems associated with it. For one thing, the neutral plane moves with every change in load, and the shift direction reverses when the machine goes from motor operation to generator operation. Therefore, someone had to adjust the brushes every time the load on the machine changed. In addition, shifting the brushes may have stopped the brush sparking, but it actually *aggravated* the flux-weakening effect of the armature reaction in the machine. This is true because of two effects:

1. The rotor magnetomotive force now has a vector component that opposes the magnetomotive force from the poles (see Figure 8–27).
2. The change in armature current distribution causes the flux to bunch up even more at the saturated parts of the pole faces.

Another slightly different approach sometimes taken was to fix the brushes in a compromise position (say, one that caused no sparking at two-thirds of full load). In this case, the motor sparked at no load and somewhat at full load, but if it spent most of its life operating at about two-thirds of full load, then sparking was minimized. Of course, such a machine could not be used as a generator at all—the sparking would have been horrible.

By about 1910, the brush-shifting approach to controlling sparking was already obsolete. Today, brush shifting is only used in very small machines that always run as motors. This is done because better solutions to the problem are simply not economical in such small motors.

COMMUTATING POLES OR INTERPOLES. Because of the disadvantages noted above and especially because of the requirement that a person must adjust the brush positions of machines as their loads change, another solution to the problem of brush sparking was developed. The basic idea behind this new approach is that

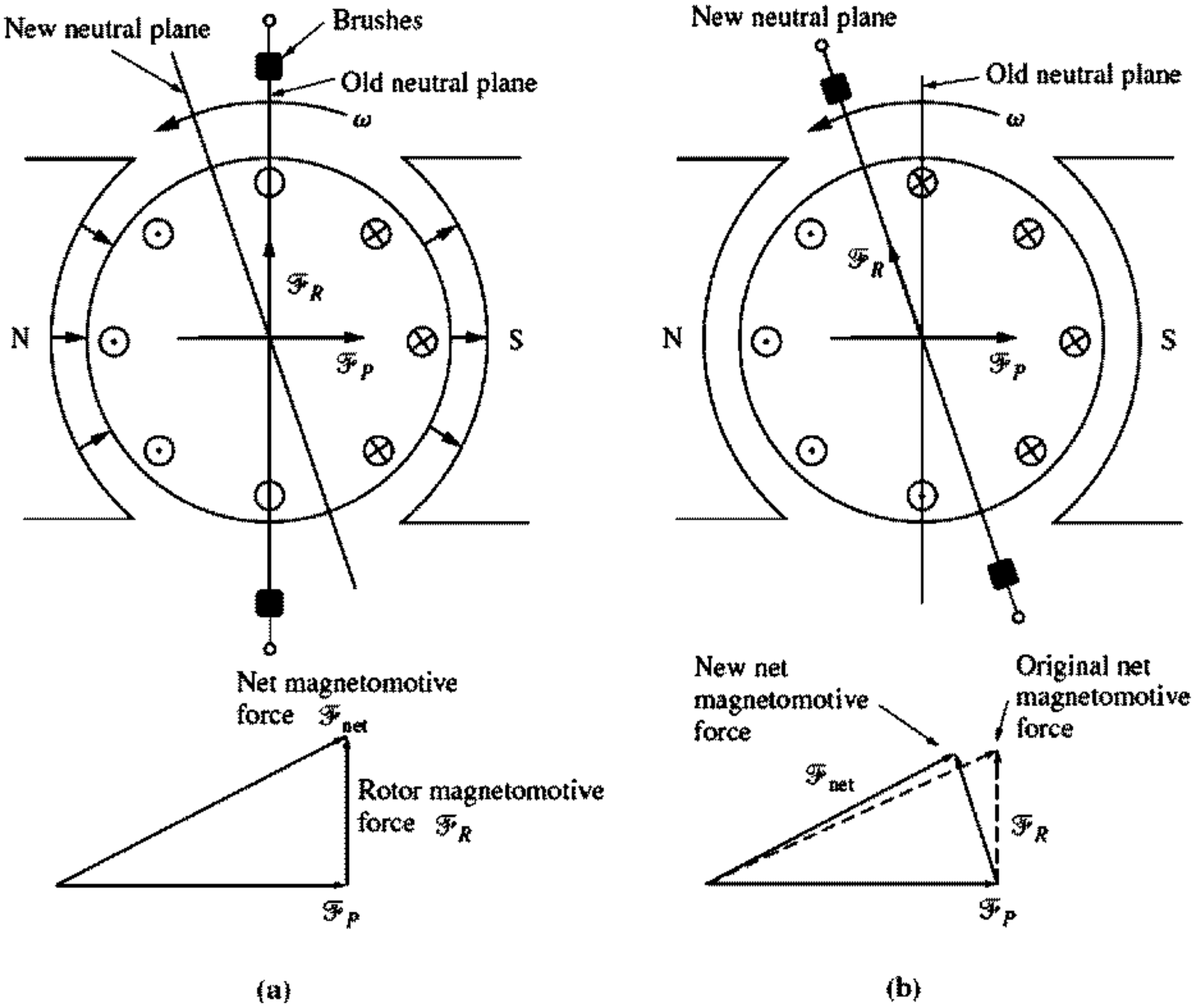


FIGURE 8–27
(a) The net magnetomotive force in a dc machine with its brushes in the vertical plane. (b) The net magnetomotive force in a dc machine with its brushes over the shifted neutral plane. Notice that now there is a component of armature magnetomotive force *directly opposing* the poles' magnetomotive force, and the net magnetomotive force in the machine is reduced.

if the voltage in the wires undergoing commutation can be made zero, then there will be no sparking at the brushes. To accomplish this, small poles, called *commutating poles* or *interpoles*, are placed midway between the main poles. These commutating poles are located *directly over* the conductors being commutated. By providing a flux from the commutating poles, the voltage in the coils undergoing commutation can be exactly canceled. If the cancellation is exact, then there will be no sparking at the brushes.

The commutating poles do not otherwise change the operation of the machine, because they are so small that they affect only the few conductors about to undergo commutation. Notice that the *armature reaction* under the main pole faces is unaffected, since the effects of the commutating poles do not extend that far. This means that the flux weakening in the machine is unaffected by commutating poles.

How is cancellation of the voltage in the commutator segments accomplished for all values of loads? This is done by simply connecting the interpole windings in *series* with the windings on the rotor, as shown in Figure 8–28. As the load increases and the rotor current increases, the magnitude of the neutral-plane shift and the size of the *L di/dt* effects increase too. Both these effects increase the voltage in the conductors undergoing commutation. However, the interpole flux increases too, producing a larger voltage in the conductors that opposes the voltage due to the neutral-plane shift. The net result is that their effects cancel over a broad range of loads. Note that interpoles work for both motor and generator operation, since when the machine changes from motor to generator, the current both in its rotor and in its interpoles reverses direction. Therefore, the voltage effects from them still cancel.

What polarity must the flux in the interpoles be? The interpoles must induce a voltage in the conductors undergoing commutation that is *opposite* to the voltage caused by neutral-plane shift and *L di/dt* effects. In the case of a generator, the neutral plane shifts in the direction of rotation, meaning that the conductors undergoing commutation have the same polarity of voltage as the pole they just left (see Figure 8–29). To oppose this voltage, the interpoles must have the opposite flux, which is the flux of the upcoming pole. In a motor, however, the neutral plane shifts opposite to the direction of rotation, and the conductors undergoing commutation have the same flux as the pole they are approaching. In order to oppose this voltage, the interpoles must have the same polarity as the previous main pole. Therefore,

1. The interpoles must be of the same polarity as the next upcoming main pole in a generator.

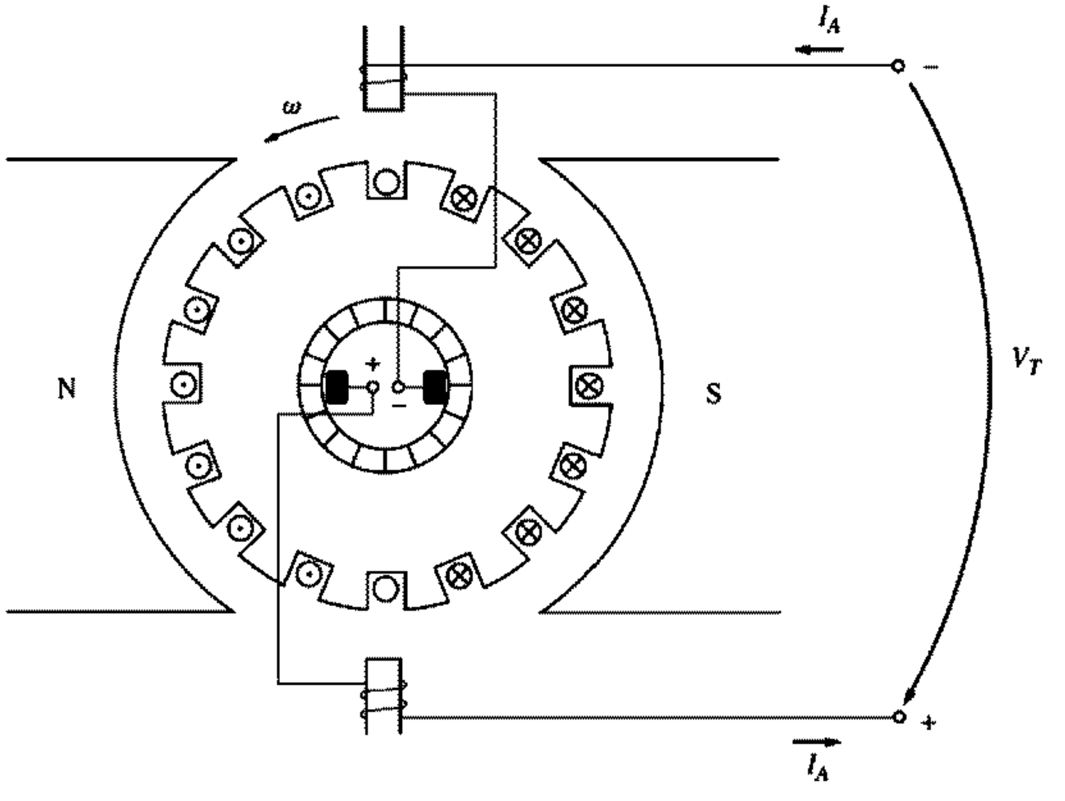


FIGURE 8–28
A dc machine with interpoles.

2. The interpoles must be of the same polarity as the previous main pole in a motor.

The use of commutating poles or interpoles is very common, because they correct the sparking problems of dc machines at a fairly low cost. They are almost always found in any dc machine of 1 hp or larger. It is important to realize, though, that they do *nothing* for the flux distribution under the pole faces, so the flux-weakening problem is still present. Most medium-size, general-purpose motors correct for sparking problems with interpoles and just live with the flux-weakening effects.

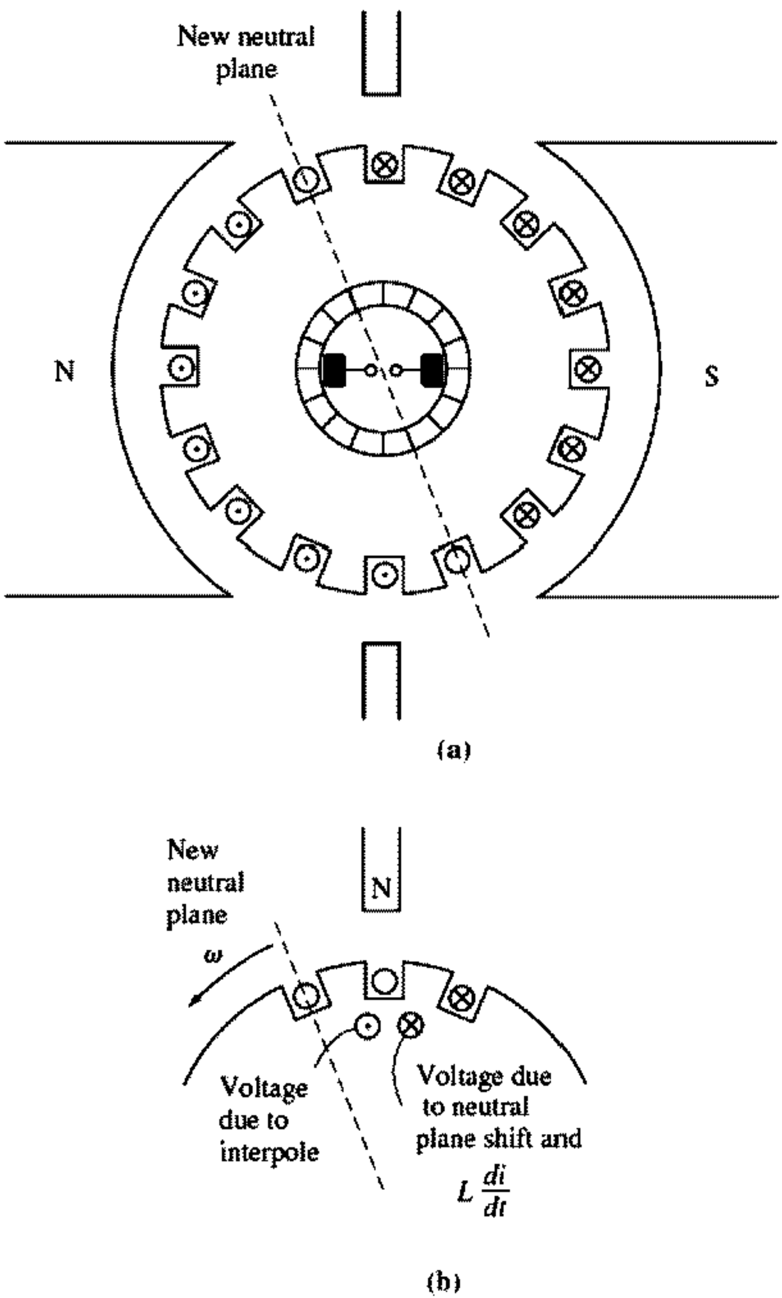


FIGURE 8–29
Determining the required polarity of an interpole. The flux from the interpole must produce a voltage that opposes the existing voltage in the conductor.

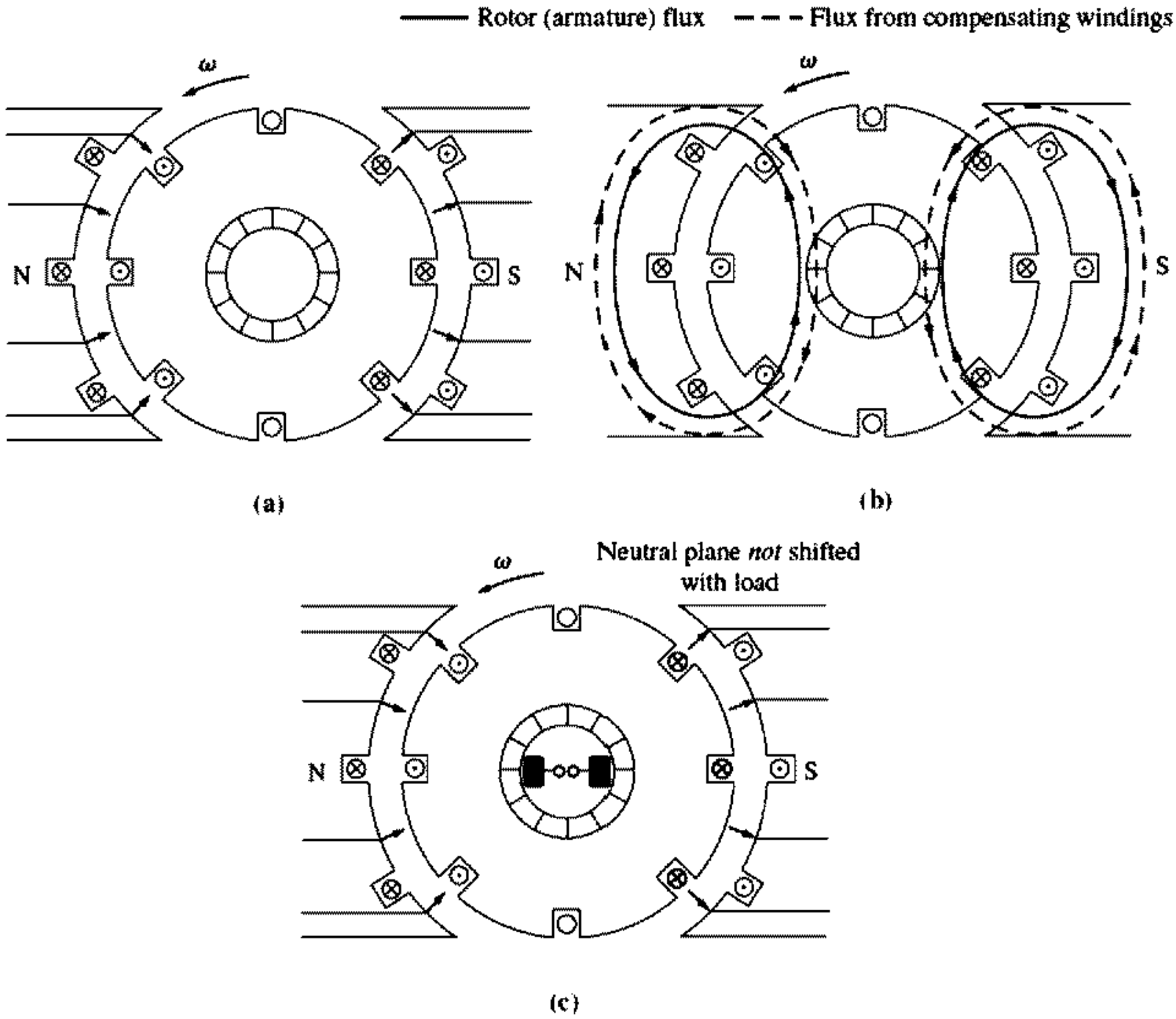


FIGURE 8–30
The effect of compensating windings in a dc machine. (a) The pole flux in the machine; (b) the fluxes from the armature and compensating windings. Notice that they are equal and opposite; (c) the net flux in the machine, which is just the original pole flux.

COMPENSATING WINDINGS. For very heavy, severe duty cycle motors, the flux-weakening problem can be very serious. To completely cancel armature reaction and thus eliminate both neutral-plane shift and flux weakening, a different technique was developed. This third technique involves placing *compensating windings* in slots carved in the faces of the poles parallel to the rotor conductors, to cancel the distorting effect of armature reaction. These windings are connected in series with the rotor windings, so that whenever the load changes in the rotor, the current in the compensating windings changes, too. Figure 8–30 shows the basic concept. In Figure 8–30a, the pole flux is shown by itself. In Figure 8–30b, the rotor flux and the compensating winding flux are shown. Figure 8–30c represents the sum of these three fluxes, which is just equal to the original pole flux by itself.

Figure 8–31 shows a more careful development of the effect of compensating windings on a dc machine. Notice that the magnetomotive force due to the

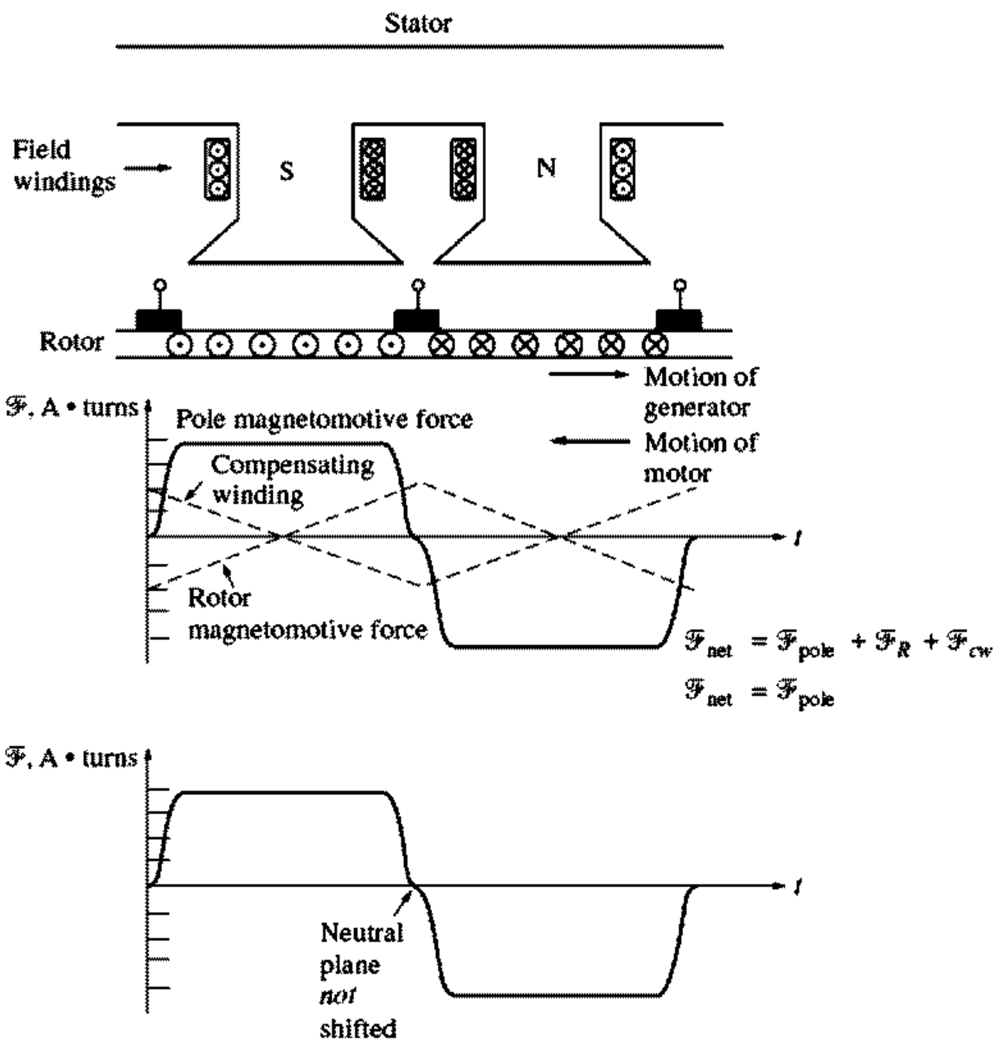


FIGURE 8–31
The flux and magnetomotive forces in a dc machine with compensating windings.

compensating windings is equal and opposite to the magnetomotive force due to the rotor at every point under the pole faces. The resulting net magnetomotive force is just the magnetomotive force due to the poles, so the flux in the machine is unchanged regardless of the load on the machine. The stator of a large dc machine with compensating windings is shown in Figure 8–32.

The major disadvantage of compensating windings is that they are expensive, since they must be machined into the faces of the poles. Any motor that uses them must also have interpoles, since compensating windings do not cancel *L di/dt* effects. The interpoles do not have to be as strong, though, since they are canceling only *L di/dt* voltages in the windings, and not the voltages due to neutral-plane shifting. Because of the expense of having both compensating windings and interpoles on such a machine, these windings are used only where the extremely severe nature of a motor's duty demands them.

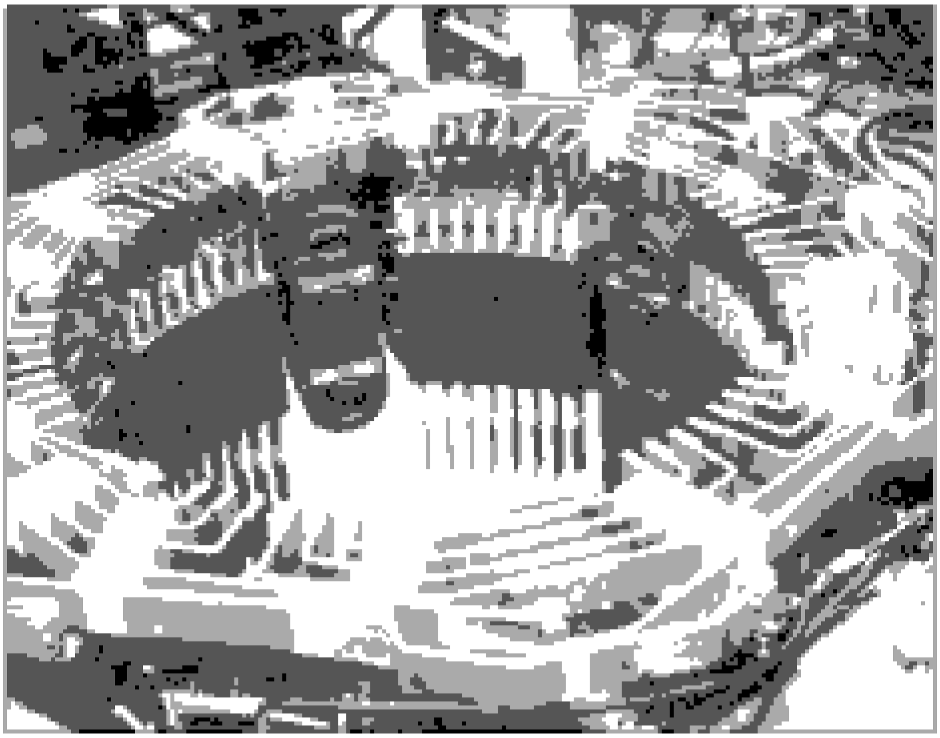


FIGURE 8–32
The stator of a six-pole dc machine with interpoles and compensating windings. (*Courtesy of Westinghouse Electric Company.*)

8.5 THE INTERNAL GENERATED VOLTAGE AND INDUCED TORQUE EQUATIONS OF REAL DC MACHINES

How much voltage is produced by a real dc machine? The induced voltage in any given machine depends on three factors:

1. The flux ϕ in the machine
2. The speed ω of the machine's rotor
3. A constant depending on the construction of the machine

How can the voltage in the rotor windings of a real machine be determined? The voltage out of the armature of a real machine is equal to the number of conductors per current path times the voltage on each conductor. The voltage in *any single conductor under the pole faces* was previously shown to be

$$e_{\text{ind}} = e = vBl \tag{8–31}$$

The voltage out of the armature of a real machine is thus

$$E_A = \frac{ZvBl}{a} \tag{8–32}$$

where Z is the total number of conductors and a is the number of current paths. The velocity of each conductor in the rotor can be expressed as $v = r\omega$, where r is the radius of the rotor, so

$$E_A = \frac{Zr\omega Bl}{a} \tag{8-33}$$

This voltage can be reexpressed in a more convenient form by noting that the flux of a pole is equal to the flux density under the pole times the pole's area:

$$\phi = BA_P$$

The rotor of the machine is shaped like a cylinder, so its area is

$$A = 2\pi rl \tag{8-34}$$

If there are P poles on the machine, then the portion of the area associated with each pole is the total area A divided by the number of poles P:

$$A_P = \frac{A}{P} = \frac{2\pi rl}{P} \tag{8-35}$$

The total *flux per pole* in the machine is thus

$$\phi = BA_P = \frac{B(2\pi rl)}{P} = \frac{2\pi rlB}{P} \tag{8-36}$$

Therefore, the internal generated voltage in the machine can be expressed as

$$E_A = \frac{Zr\omega Bl}{a} \tag{8-33}$$

$$= \left(\frac{ZP}{2\pi a}\right)\left(\frac{2\pi rlB}{P}\right)\omega$$

$$\boxed{E_A = \frac{ZP}{2\pi a}\phi\omega} \tag{8-37}$$

Finally,

$$\boxed{E_A = K\phi\omega} \tag{8-38}$$

where

$$\boxed{K = \frac{ZP}{2\pi a}} \tag{8-39}$$

In modern industrial practice, it is common to express the speed of a machine in revolutions per minute instead of radians per second. The conversion from revolutions per minute to radians per second is

$$\omega = \frac{2\pi}{60}n \tag{8-40}$$

so the voltage equation with speed expressed in terms of revolutions per minute is

$$\boxed{E_A = K'\phi n} \tag{8–41}$$

where

$$\boxed{K' = \frac{ZP}{60a}} \tag{8–42}$$

How much torque is induced in the armature of a real dc machine? The torque in any dc machine depends on three factors:

1. The flux ϕ in the machine
2. The armature (or rotor) current I_A in the machine
3. A constant depending on the construction of the machine

How can the torque on the rotor of a real machine be determined? The torque on the armature of a real machine is equal to the number of conductors Z times the torque on each conductor. The torque in *any single conductor under the pole faces* was previously shown to be

$$\tau_{\text{cond}} = rI_{\text{cond}}lB \tag{8–43}$$

If there are a current paths in the machine, then the total armature current I_A is split among the a current paths, so the current in a single conductor is given by

$$I_{\text{cond}} = \frac{I_A}{a} \tag{8–44}$$

and the torque in a single conductor on the motor may be expressed as

$$\tau_{\text{cond}} = \frac{rI_A lB}{a} \tag{8–45}$$

Since there are Z conductors, the total induced torque in a dc machine rotor is

$$\tau_{\text{ind}} = \frac{ZrlBI_A}{a} \tag{8–46}$$

The flux per pole in this machine can be expressed as

$$\phi = BA_P = \frac{B(2\pi rl)}{P} = \frac{2\pi rlB}{P} \tag{8–47}$$

so the total induced torque can be reexpressed as

$$\tau_{\text{ind}} = \frac{ZP}{2\pi a}\phi I_A \tag{8–48}$$

Finally,

$$\boxed{\tau_{\text{ind}} = K\phi I_A} \tag{8–49}$$

where

$$\boxed{K = \frac{ZP}{2\pi a}} \tag{8–39}$$

Both the internal generated voltage and the induced torque equations just given are only approximations, because not all the conductors in the machine are under the pole faces at any given time and also because the surfaces of each pole do not cover an entire $1/P$ of the rotor's surface. To achieve greater accuracy, the number of conductors under the pole faces could be used instead of the total number of conductors on the rotor.

Example 8–3. A duplex lap-wound armature is used in a six-pole dc machine with six brush sets, each spanning two commutator segments. There are 72 coils on the armature, each containing 12 turns. The flux per pole in the machine is 0.039 Wb, and the machine spins at 400 r/min.

(*a*) How many current paths are there in this machine?
(*b*) What is its induced voltage E_A?

Solution

(*a*) The number of current paths in this machine is

$$a = mP = 2(6) = 12 \text{ current paths} \tag{8–26}$$

(*b*) The induced voltage in the machine is

$$E_A = K'\phi n \tag{8–41}$$

and

$$K' = \frac{ZP}{60a} \tag{8–42}$$

The number of conductors in this machine is

$$Z = 2CN_C \tag{8–22}$$
$$= 2(72)(12) = 1728 \text{ conductors}$$

Therefore, the constant K' is

$$K' = \frac{ZP}{60a} = \frac{(1728)(6)}{(60)(12)} = 14.4$$

and the voltage E_A is

$$\begin{aligned} E_A &= K'\phi n \\ &= (14.4)(0.039 \text{ Wb})(400 \text{ r/min}) \\ &= 224.6 \text{ V} \end{aligned}$$

Example 8–4. A 12-pole dc generator has a simplex wave-wound armature containing 144 coils of 10 turns each. The resistance of each turn is 0.011 Ω. Its flux per pole is 0.05 Wb, and it is turning at a speed of 200 r/min.

(*a*) How many current paths are there in this machine?
(*b*) What is the induced armature voltage of this machine?
(*c*) What is the effective armature resistance of this machine?
(*d*) If a 1-kΩ resistor is connected to the terminals of this generator, what is the resulting induced countertorque on the shaft of the machine? (Ignore the internal armature resistance of the machine.)

Solution

(a) There are $a = 2m = 2$ current paths in this winding.

(b) There are $Z = 2CN_c = 2(144)(10) = 2880$ conductors on this generator's rotor. Therefore,

$$K' = \frac{ZP}{60a} = \frac{(2880)(12)}{(60)(2)} = 288$$

Therefore, the induced voltage is

$$\begin{aligned} E_A &= K'\phi n \\ &= (288)(0.05 \text{ Wb})(200 \text{ r/min}) \\ &= 2880 \text{ V} \end{aligned}$$

(c) There are two parallel paths through the rotor of this machine, each one consisting of $Z/2 = 1440$ conductors, or 720 turns. Therefore, the resistance in each current path is

$$\text{Resistance/path} = (720 \text{ turns})(0.011 \ \Omega/\text{turn}) = 7.92 \ \Omega$$

Since there are two parallel paths, the effective armature resistance is

$$R_A = \frac{7.92 \ \Omega}{2} = 3.96 \ \Omega$$

(d) If a 1000-Ω load is connected to the terminals of the generator, and if R_A is ignored, then a current of $I = 2880 \text{ V}/1000 \ \Omega = 2.88$ A flows. The constant K is given by

$$K = \frac{ZP}{2\pi a} = \frac{(2880)(12)}{(2\pi)(2)} = 2750.2$$

Therefore, the countertorque on the shaft of the generator is

$$\begin{aligned} \tau_{\text{ind}} &= K\phi I_A = (2750.2)(0.05 \text{ Wb})(2.88 \text{ A}) \\ &= 396 \text{ N} \bullet \text{m} \end{aligned}$$

8.6 THE CONSTRUCTION OF DC MACHINES

A simplified sketch of a dc machine is shown in Figure 8–33, and a more detailed cutaway diagram of a dc machine is shown in Figure 8–34.

The physical structure of the machine consists of two parts: the *stator* or stationary part and the *rotor* or rotating part. The stationary part of the machine consists of the *frame*, which provides physical support, and the *pole pieces*, which project inward and provide a path for the magnetic flux in the machine. The ends of the pole pieces that are near the rotor spread out over the rotor surface to distribute its flux evenly over the rotor surface. These ends are called the *pole shoes*. The exposed surface of a pole shoe is called a *pole face*, and the distance between the pole face and the rotor is called the *air gap*.

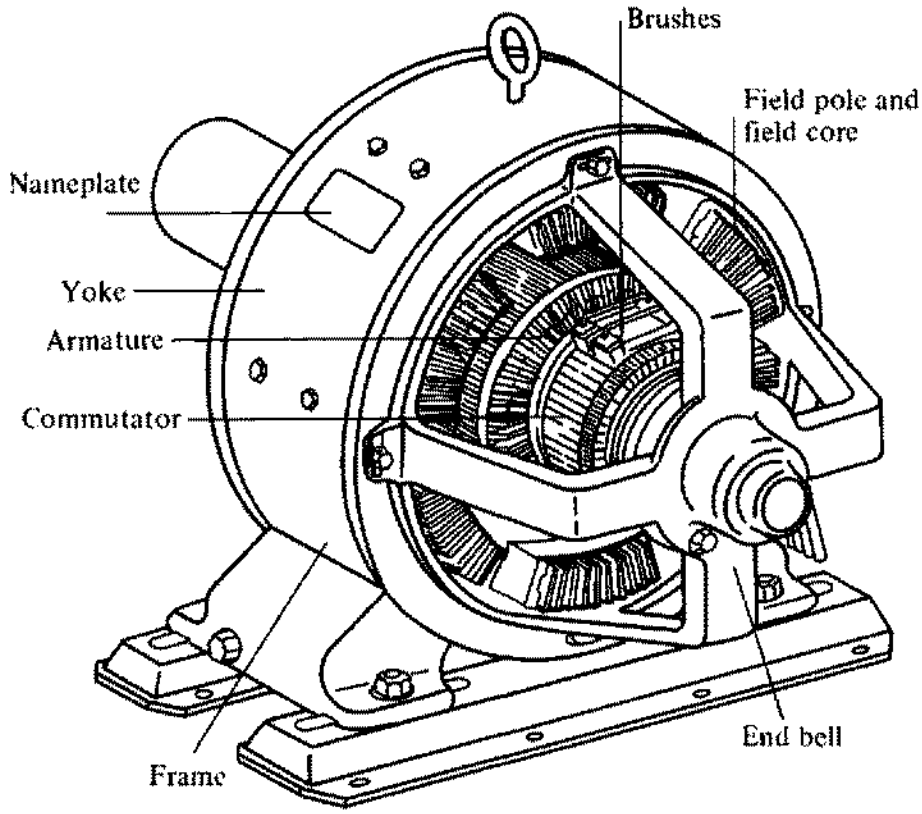
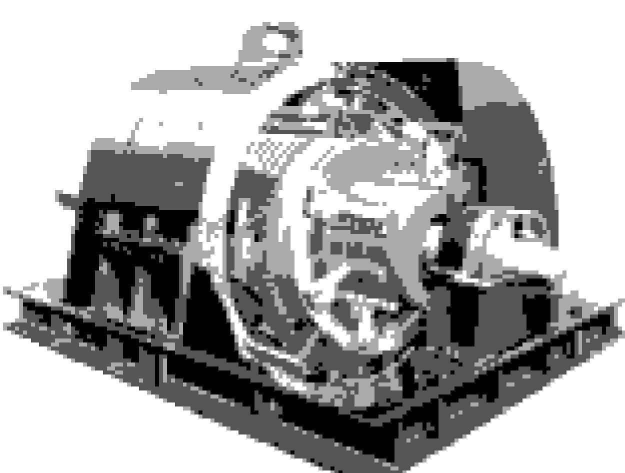
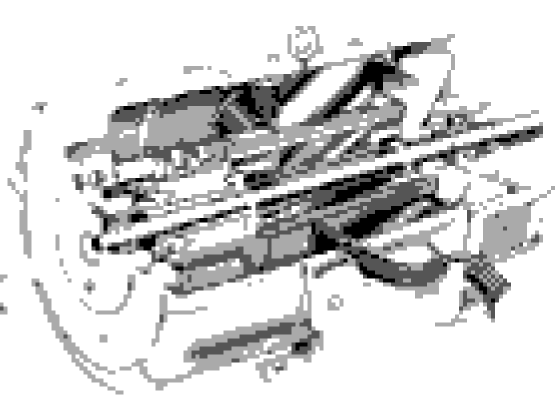


FIGURE 8–33
A simplified diagram of a dc machine.



(a)



(b)

FIGURE 8–34
(a) A cutaway view of a 4000-hp, 700-V, 18–pole dc machine showing compensating windings, interpoles, equalizer, and commutator. (*Courtesy of General Electric Company.*) (b) A cutaway view of a smaller four-pole dc motor including interpoles but without compensating windings. (*Courtesy of MagneTek Incorporated.*)

There are two principal windings on a dc machine: the armature windings and the field windings. The *armature windings* are defined as the windings in which a voltage is induced, and the *field windings* are defined as the windings that produce the main magnetic flux in the machine. In a normal dc machine, the armature windings are located on the rotor, and the field windings are located on the stator. Because the armature windings are located on the rotor, a dc machine's rotor itself is sometimes called an *armature.*

Some major features of typical dc motor construction are described below.

Pole and Frame Construction

The main poles of older dc machines were often made of a single cast piece of metal, with the field windings wrapped around it. They often had bolted-on laminated tips to reduce core losses in the pole faces. Since solid-state drive packages have become common, the main poles of newer machines are made entirely of laminated material (see Figure 8–35). This is true because there is a much higher ac content in the power supplied to dc motors driven by solid-state drive packages, resulting in much higher eddy current losses in the stators of the machines. The pole faces are typically either *chamfered* or *eccentric* in construction, meaning that the outer tips of a pole face are spaced slightly further from the rotor's surface than the center of the pole face is (see Figure 8–36). This action increases the reluctance at the tips of a pole face and therefore reduces the flux-bunching effect of armature reaction on the machine.

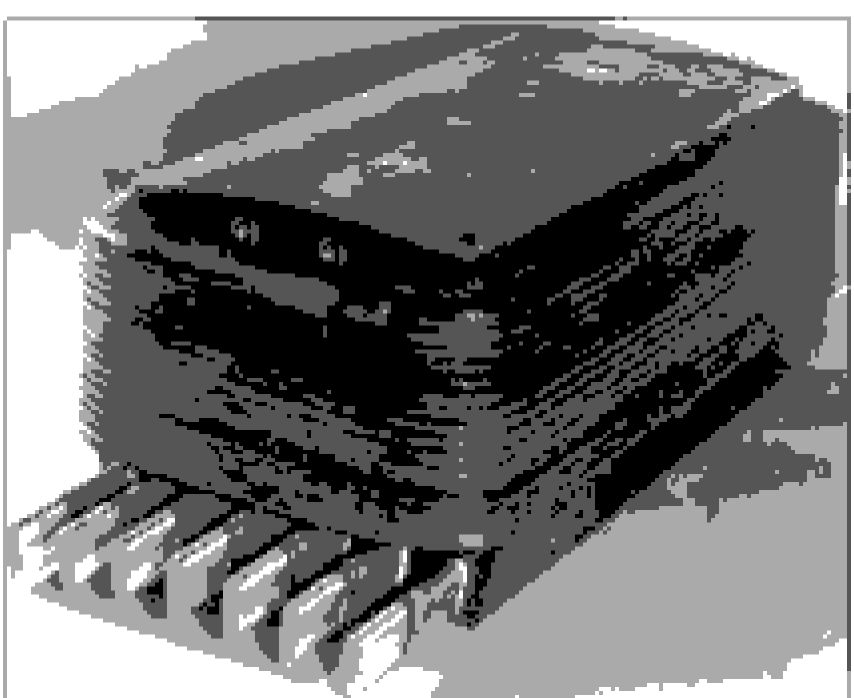


FIGURE 8–35
Main field pole assembly for a dc motor. Note the pole laminations and compensating windings. (*Courtesy of General Electric Company.*)

The poles on dc machines are called *salient poles*, because they stick out from the surface of the stator.

The interpoles in dc machines are located between the main poles. They are more and more commonly of laminated construction, because of the same loss problems that occur in the main poles.

Some manufacturers are even constructing the portion of the frame that serves as the magnetic flux's return path (the *yoke*) with laminations, to further reduce core losses in electronically driven motors.

Rotor or Armature Construction

The rotor or armature of a dc machine consists of a shaft machined from a steel bar with a core built up over it. The core is composed of many laminations stamped from a steel plate, with notches along its outer surface to hold the armature windings. The commutator is built onto the shaft of the rotor at one end of the core. The armature coils are laid into the slots on the core, as described in Section 8.4, and their ends are connected to the commutator segments. A large dc machine rotor is shown in Figure 8–37.

Commutator and Brushes

The commutator in a dc machine (Figure 8–38) is typically made of copper bars insulated by a mica-type material. The copper bars are made sufficiently thick to permit normal wear over the lifetime of the motor. The mica insulation between commutator segments is harder than the commutator material itself, so as a machine ages, it is often necessary to *undercut* the commutator insulation to ensure that it does not stick up above the level of the copper bars.

The brushes of the machine are made of carbon, graphite, metal graphite, or a mixture of carbon and graphite. They have a high conductivity to reduce electrical losses and a low coefficient of friction to reduce excessive wear. They are

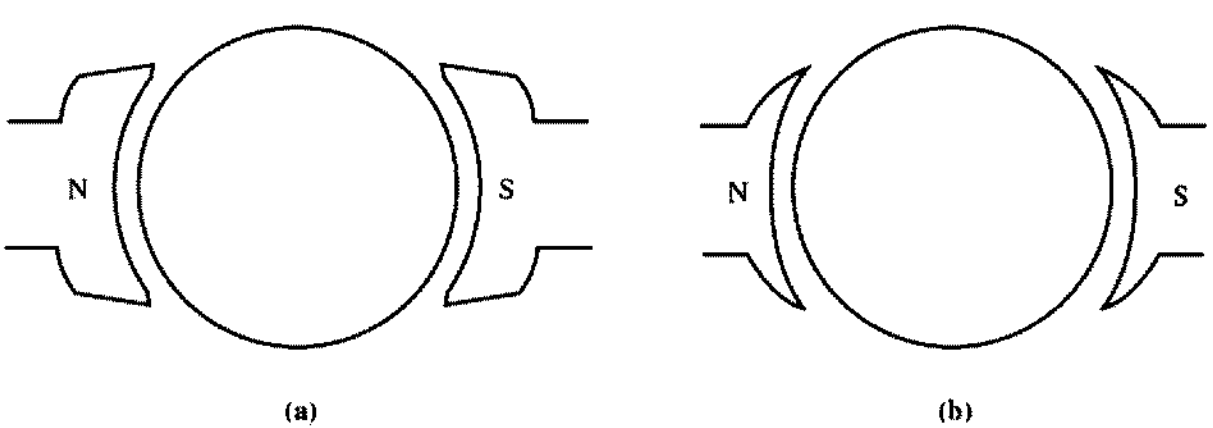


FIGURE 8–36
Poles with extra air-gap width at the tips to reduce armature reaction. (a) Chamfered poles; (b) eccentric or uniformly graded poles.

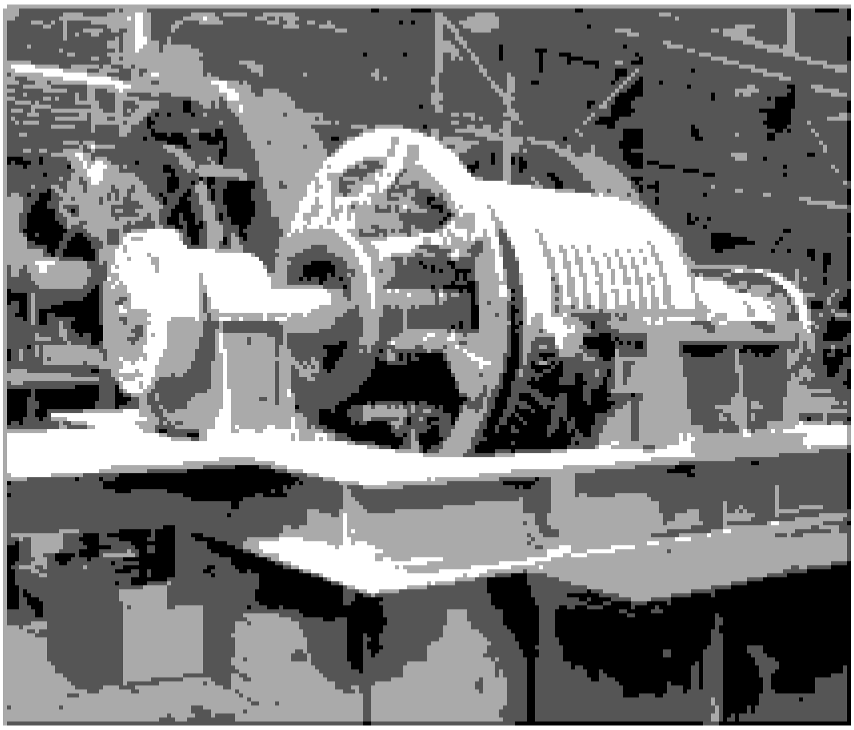


FIGURE 8–37
Photograph of a dc machine with the upper stator half removed shows the construction of its rotor. (*Courtesy of General Electric Company.*)

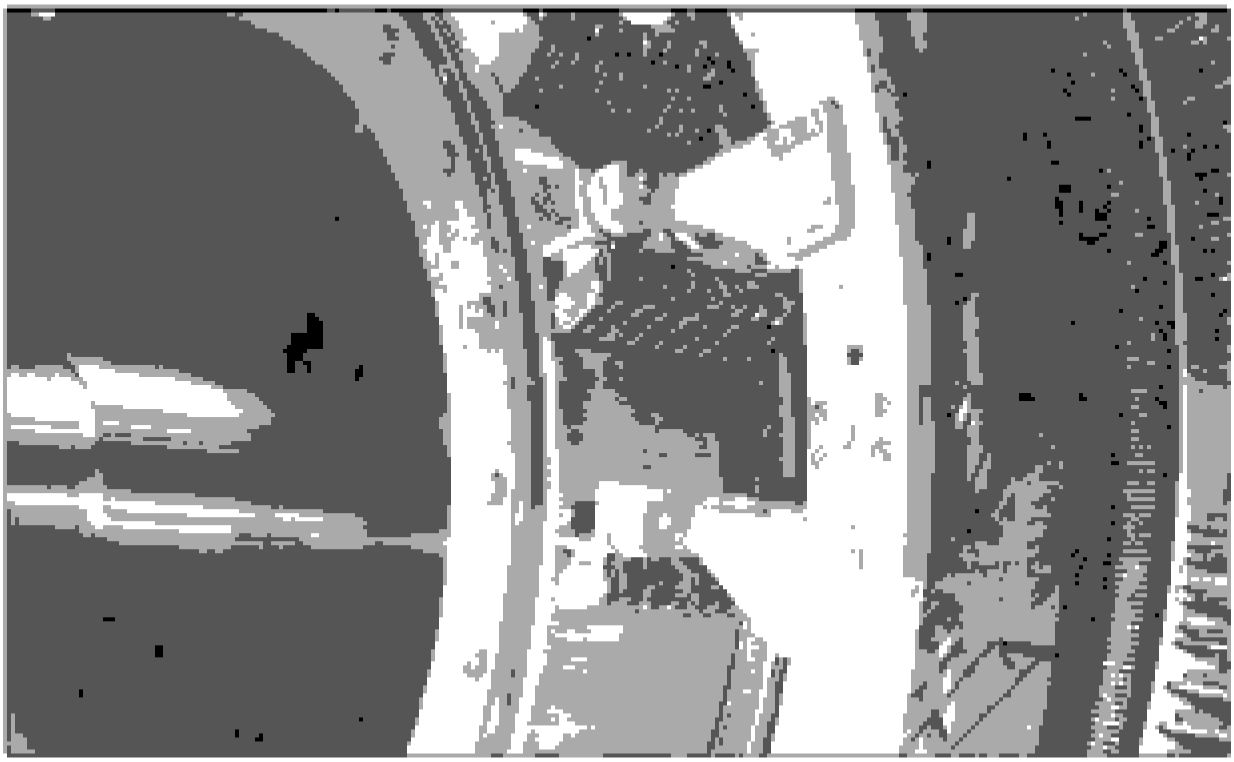


FIGURE 8–38
Close-up view of commutator and brushes in a large dc machine. (*Courtesy of General Electric Company.*)

deliberately made of much softer material than that of the commutator segments, so that the commutator surface will experience very little wear. The choice of brush hardness is a compromise: If the brushes are too soft, they will have to be replaced too often; but if they are too hard, the commutator surface will wear excessively over the life of the machine.

All the wear that occurs on the commutator surface is a direct result of the fact that the brushes must rub over them to convert the ac voltage in the rotor wires to dc voltage at the machine's terminals. If the pressure of the brushes is too great, both the brushes and commutator bars wear excessively. However, if the brush pressure is too small, the brushes tend to jump slightly and a great deal of sparking occurs at the brush-commutator segment interface. This sparking is equally bad for the brushes and the commutator surface. Therefore, the brush pressure on the commutator surface must be carefully adjusted for maximum life.

Another factor which affects the wear on the brushes and segments in a dc machine commutator is the amount of current flowing in the machine. The brushes normally ride over the commutator surface on a thin oxide layer, which lubricates the motion of the brush over the segments. However, if the current is very small, that layer breaks down, and the friction between the brushes and the commutator is greatly increased. This increased friction contributes to rapid wear. For maximum brush life, a machine should be at least partially loaded all the time.

Winding Insulation

Other than the commutator, the most critical part of a dc motor's design is the insulation of its windings. If the insulation of the motor windings breaks down, the motor shorts out. The repair of a machine with shorted insulation is quite expensive, if it is even possible. To prevent the insulation of the machine windings from breaking down as a result of overheating, it is necessary to limit the temperature of the windings. This can be partially done by providing a cooling air circulation over them, but ultimately the maximum winding temperature limits the maximum power that can be supplied continuously by the machine.

Insulation rarely fails from immediate breakdown at some critical temperature. Instead, the increase in temperature produces a gradual degradation of the insulation, making it subject to failure due to another cause such as shock, vibration, or electrical stress. There is an old rule of thumb which says that the life expectancy of a motor with a given insulation is halved for each 10 percent rise in winding temperature. This rule still applies to some extent today.

To standardize the temperature limits of machine insulation, the National Electrical Manufacturers Association (NEMA) in the United States has defined a series of *insulation system classes*. Each insulation system class specifies the maximum temperature rise permissible for each type of insulation. There are four standard NEMA insulation classes for integral-horsepower dc motors: A, B, F, and H. Each class represents a higher permissible winding temperature than the one before it. For example, if the armature winding temperature rise above ambient temperature in one type of continuously operating dc motor is measured by thermometer, it must be limited to 70°C for class A, 100°C for class B, 130°C for class F, and 155°C for class H insulation.

These temperature specifications are set out in great detail in NEMA Standard MG1-1993, *Motors and Generators*. Similar standards have been defined by the International Electrotechnical Commission (IEC) and by various national standards organizations in other countries.

8.7 POWER FLOW AND LOSSES IN DC MACHINES

DC generators take in mechanical power and produce electric power, while dc motors take in electric power and produce mechanical power. In either case, not all the power input to the machine appears in useful form at the other end—there is *always* some loss associated with the process.

The efficiency of a dc machine is defined by the equation

$$\boxed{\eta = \frac{P_{\text{out}}}{P_{\text{in}}} \times 100\%} \tag{8–50}$$

The difference between the input power and the output power of a machine is the losses that occur inside it. Therefore,

$$\boxed{\eta = \frac{P_{\text{out}} - P_{\text{loss}}}{P_{\text{in}}} \times 100\%} \tag{8–51}$$

The Losses in DC Machines

The losses that occur in dc machines can be divided into five basic categories:

1. Electrical or copper losses (I^2R losses)
2. Brush losses
3. Core losses
4. Mechanical losses
5. Stray load losses

ELECTRICAL OR COPPER LOSSES. Copper losses are the losses that occur in the armature and field windings of the machine. The copper losses for the armature and field windings are given by

$$\text{Armature loss:} \quad P_A = I_A^2 R_A \tag{8–52}$$

$$\text{Field loss:} \quad P_F = I_F^2 R_F \tag{8–53}$$

where P_A = armature loss

P_F = field circuit loss

I_A = armature current
I_F = field current
R_A = armature resistance
R_F = field resistance

The resistance used in these calculations is usually the winding resistance at normal operating temperature.

BRUSH LOSSES. The brush drop loss is the power lost across the contact potential at the brushes of the machine. It is given by the equation

$$\boxed{P_{BD} = V_{BD} I_A} \qquad (8\text{–}54)$$

where P_{BD} = brush drop loss
V_{BD} = brush voltage drop
I_A = armature current

The reason that the brush losses are calculated in this manner is that the voltage drop across a set of brushes is approximately constant over a large range of armature currents. Unless otherwise specified, the brush voltage drop is usually assumed to be about 2 V.

CORE LOSSES. The core losses are the hysteresis losses and eddy current losses occurring in the metal of the motor. These losses are described in Chapter 1. These losses vary as the square of the flux density (B^2) and, for the rotor, as the 1.5th power of the speed of rotation ($n^{1.5}$).

MECHANICAL LOSSES. The mechanical losses in a dc machine are the losses associated with mechanical effects. There are two basic types of mechanical losses: *friction* and *windage*. Friction losses are losses caused by the friction of the bearings in the machine, while windage losses are caused by the friction between the moving parts of the machine and the air inside the motor's casing. These losses vary as the cube of the speed of rotation of the machine.

STRAY LOSSES (OR MISCELLANEOUS LOSSES). Stray losses are losses that cannot be placed in one of the previous categories. No matter how carefully losses are accounted for, some always escape inclusion in one of the above categories. All such losses are lumped into stray losses. For most machines, stray losses are taken by convention to be 1 percent of full load.

The Power-Flow Diagram

One of the most convenient techniques for accounting for power losses in a machine is the *power-flow diagram*. A power-flow diagram for a dc generator is shown in Figure 8–39a. In this figure, mechanical power is input into the machine,

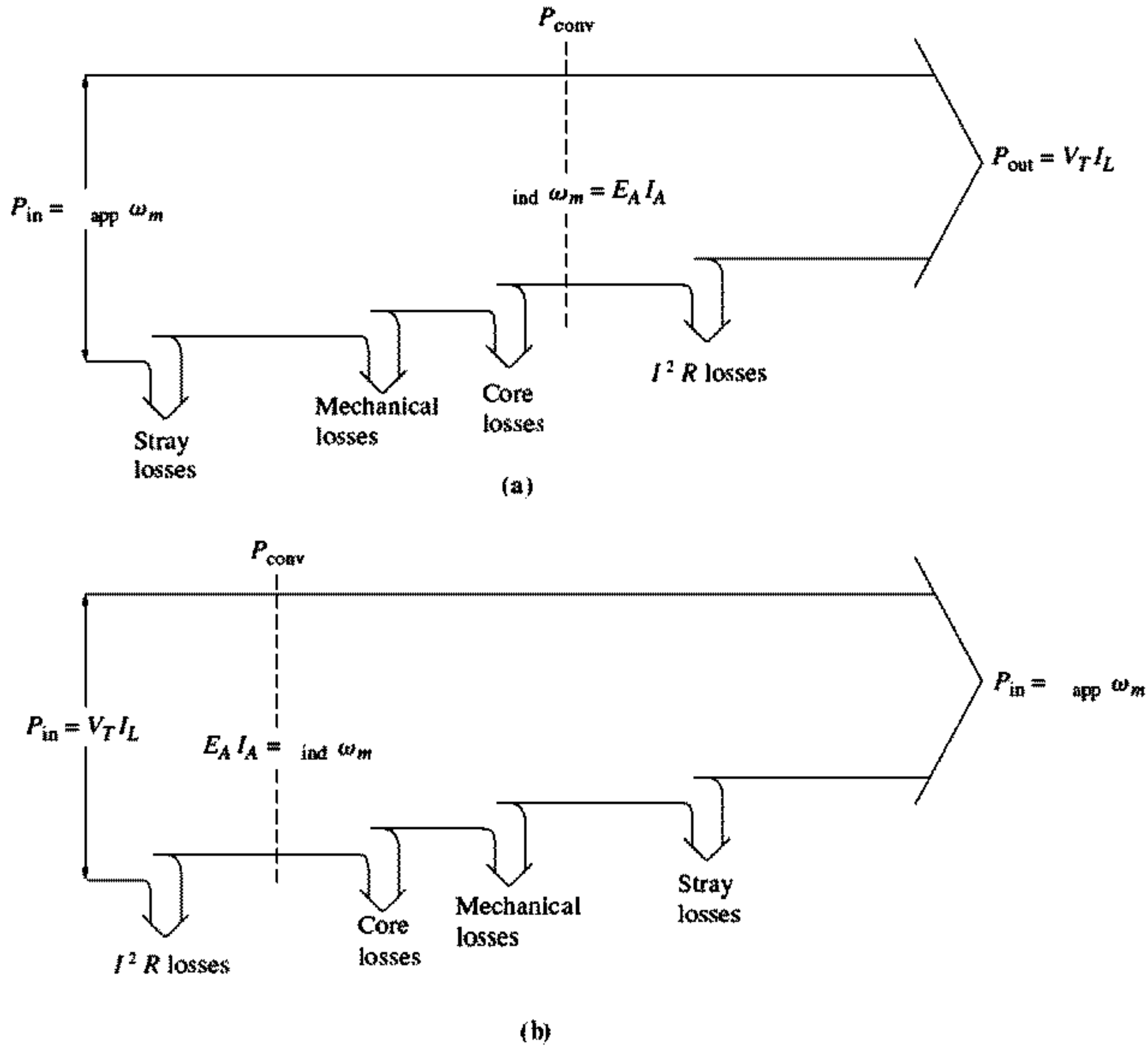


FIGURE 8–39
Power-flow diagrams for dc machine: (a) generator; (b) motor.

and then the stray losses, mechanical losses, and core losses are subtracted. After they have been subtracted, the remaining power is ideally converted from mechanical to electrical form at the point labeled P_{conv}. The mechanical power that is converted is given by

$$\boxed{P_{conv} = \tau_{ind}\omega_m} \tag{8–55}$$

and the resulting electric power produced is given by

$$\boxed{P_{conv} = E_A I_A} \tag{8–56}$$

However, this is not the power that appears at the machine's terminals. Before the terminals are reached, the electrical I^2R losses and the brush losses must be subtracted.

In the case of dc motors, this power-flow diagram is simply reversed. The power-flow diagram for a motor is shown in Figure 8–39b.

Example problems involving the calculation of motor and generator efficiencies will be given in the next two chapters.

8.8 SUMMARY

DC machines convert mechanical power to dc electric power, and vice versa. In this chapter, the basic principles of dc machine operation were explained first by looking at a simple linear machine and then by looking at a machine consisting of a single rotating loop.

The concept of commutation as a technique for converting the ac voltage in rotor conductors to a dc output was introduced, and its problems were explored. The possible winding arrangements of conductors in a dc rotor (lap and wave windings) were also examined.

Equations were then derived for the induced voltage and torque in a dc machine, and the physical construction of the machines was described. Finally, the types of losses in the dc machine were described and related to its overall operating efficiency.

QUESTIONS

8–1. What is commutation? How can a commutator convert ac voltages on a machine's armature to dc voltages at its terminals?

8–2. Why does curving the pole faces in a dc machine contribute to a smoother dc output voltage from it?

8–3. What is the pitch factor of a coil?

8–4. Explain the concept of electrical degrees. How is the electrical angle of the voltage in a rotor conductor related to the mechanical angle of the machine's shaft?

8–5. What is commutator pitch?

8–6. What is the plex of an armature winding?

8–7. How do lap windings differ from wave windings?

8–8. What are equalizers? Why are they needed on a lap-wound machine but not on a wave-wound machine?

8–9. What is armature reaction? How does it affect the operation of a dc machine?

8–10. Explain the $L\,di/dt$ voltage problem in conductors undergoing commutation.

8–11. How does brush shifting affect the sparking problem in dc machines?

8–12. What are commutating poles? How are they used?

8–13. What are compensating windings? What is their most serious disadvantage?

8–14. Why are laminated poles used in modern dc machine construction?

8–15. What is an insulation class?

8–16. What types of losses are present in a dc machine?

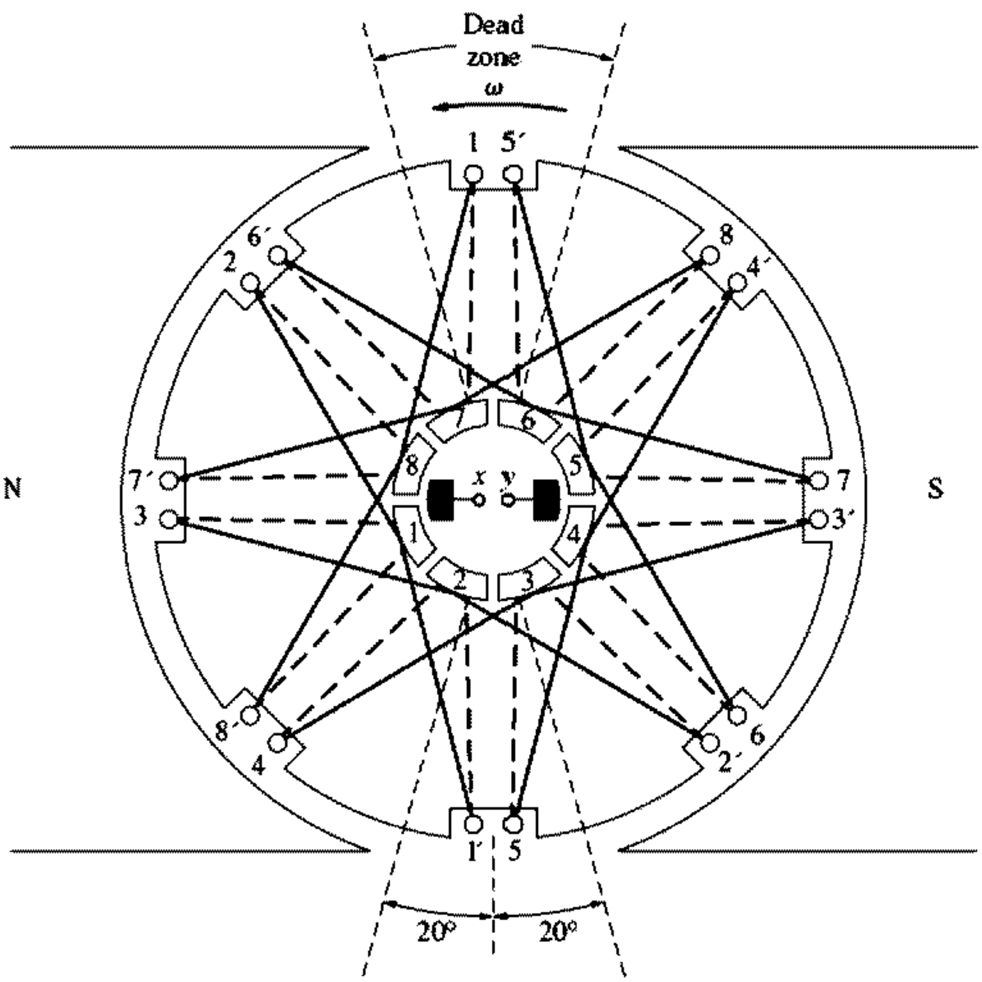
PROBLEMS

8–1. The following information is given about the simple rotating loop shown in Figure 8–6:

$$B = 0.8\text{ T} \qquad V_B = 24\text{ V}$$
$$l = 0.5\text{ m} \qquad R = 0.4\ \Omega$$
$$r = 0.125\text{ m} \qquad \omega = 250\text{ rad/s}$$

(*a*) Is this machine operating as a motor or a generator? Explain.
(*b*) What is the current i flowing into or out of the machine? What is the power flowing into or out of the machine?
(*c*) If the speed of the rotor were changed to 275 rad/s, what would happen to the current flow into or out of the machine?
(*d*) If the speed of the rotor were changed to 225 rad/s, what would happen to the current flow into or out of the machine?

8–2. Refer to the simple two-pole eight-coil machine shown in Figure P8–1. The following information is given about this machine:



Given: **B** = 1.0 T in the air gap
l = 0.3 m (length of sides)
r = 0.08 m (radius of coils)
n = 1700 r/min

——— Lines on this side of rotor
– – – – Lines on other side of rotor

FIGURE P8–1
The machine in Problem 8–2.

$B = 1.0$ T in air gap

$l = 0.3$ m (length of coil sides)

$r = 0.08$ m (radius of coils)

$n = 1700$ r/min CCW

The resistance of each rotor coil is 0.04 Ω.

(*a*) Is the armature winding shown a progressive or retrogressive winding?

(*b*) How many current paths are there through the armature of this machine?

(*c*) What are the magnitude and the polarity of the voltage at the brushes in this machine?

(*d*) What is the armature resistance R_A of this machine?

(*e*) If a 10-Ω resistor is connected to the terminals of this machine, how much current flows in the machine? Consider the internal resistance of the machine in determining the current flow.

(*f*) What are the magnitude and the direction of the resulting induced torque?

(*g*) Assuming that the speed of rotation and magnetic flux density are constant, plot the terminal voltage of this machine as a function of the current drawn from it.

8–3. Prove that the equation for the induced voltage of a single simple rotating loop

$$e_{ind} = \frac{2}{\pi}\phi\omega \qquad (8\text{–}6)$$

is just a special case of the general equation for induced voltage in a dc machine

$$E_A = K\phi\omega \qquad (8\text{–}38)$$

8–4. A dc machine has eight poles and a rated current of 100 A. How much current will flow in each path at rated conditions if the armature is (*a*) simplex lap-wound, (*b*) duplex lap-wound, (*c*) simplex wave-wound?

8–5. How many parallel current paths will there be in the armature of a 12-pole machine if the armature is (*a*) simplex lap-wound, (*b*) duplex wave-wound, (*c*) triplex lap-wound, (*d*) quadruplex wave-wound?

8–6. The power converted from one form to another within a dc motor was given by

$$P_{conv} = E_A I_A = \tau_{ind}\omega_m$$

Use the equations for E_A and τ_{ind} [Equations (8–38) and (8–49)] to prove that $E_A I_A = \tau_{ind}\omega_m$; that is, prove that the electric power disappearing at the point of power conversion is exactly equal to the mechanical power appearing at that point.

8–7. An eight-pole, 25-kW, 120-V dc generator has a duplex lap-wound armature which has 64 coils with 16 turns per coil. Its rated speed is 2400 r/min.

(*a*) How much flux per pole is required to produce the rated voltage in this generator at no-load conditions?

(*b*) What is the current per path in the armature of this generator at the rated load?

(*c*) What is the induced torque in this machine at the rated load?

(*d*) How many brushes must this motor have? How wide must each one be?

(*e*) If the resistance of this winding is 0.011 Ω per turn, what is the armature resistance R_A of this machine?

8–8. Figure P8–2 shows a small two-pole dc motor with eight rotor coils and four turns per coil. The flux per pole in this machine is 0.0125 Wb.

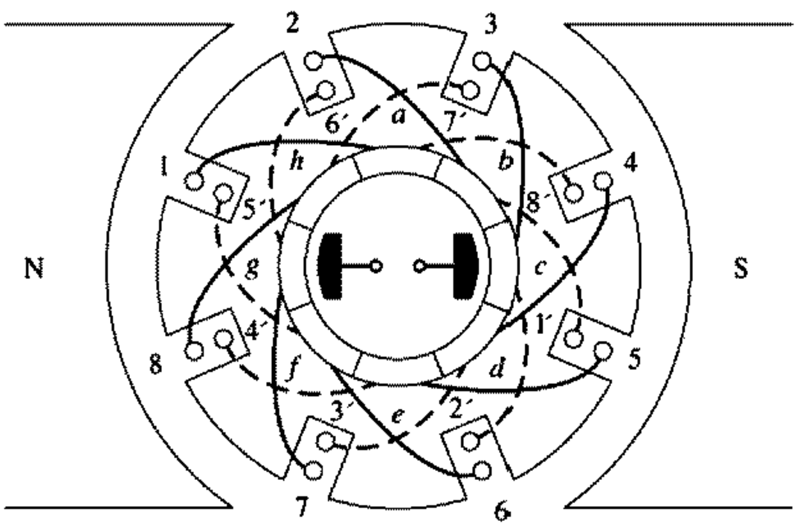


FIGURE P8–2
The machine in Problem 8–8.

(a) If this motor is connected to a 12-V dc car battery, what will the no-load speed of the motor be?
(b) If the positive terminal of the battery is connected to the rightmost brush on the motor, which way will it rotate?
(c) If this motor is loaded down so that it consumes 50 W from the battery, what will the induced torque of the motor be? (Ignore any internal resistance in the motor.)

8–9. Refer to the machine winding shown in Figure P8–3.
(a) How many parallel current paths are there through this armature winding?
(b) Where should the brushes be located on this machine for proper commutation? How wide should they be?
(c) What is the plex of this machine?
(d) If the voltage on any single conductor under the pole faces in this machine is *e*, what is the voltage at the terminals of this machine?

8–10. Describe in detail the winding of the machine shown in Figure P8–4. If a positive voltage is applied to the brush under the north pole face, which way will this motor rotate?

REFERENCES

1. Del Toro, V. *Electric Machines and Power Systems.* Englewood Cliffs, N.J.: Prentice-Hall, 1985.
2. Fitzgerald, A. E., C. Kingsley, Jr., and S. D. Umans. *Electric Machinery.* 5th ed. New York: McGraw-Hill, 1990.
3. Hubert, Charles I. *Preventative Maintenance of Electrical Equipment.* 2nd ed. New York: McGraw-Hill, 1969.
4. Kosow, Irving L. *Electric Machinery and Transformers.* Englewood Cliffs, N.J.: Prentice-Hall, 1972.
5. National Electrical Manufacturers Association. *Motors and Generators,* Publication MG1-1993, Washington, D.C., 1993.
6. Siskind, Charles. *Direct Current Machinery.* New York: McGraw-Hill, 1952.
7. Werninck, E. H. (ed.). *Electric Motor Handbook.* London: McGraw-Hill, 1978.

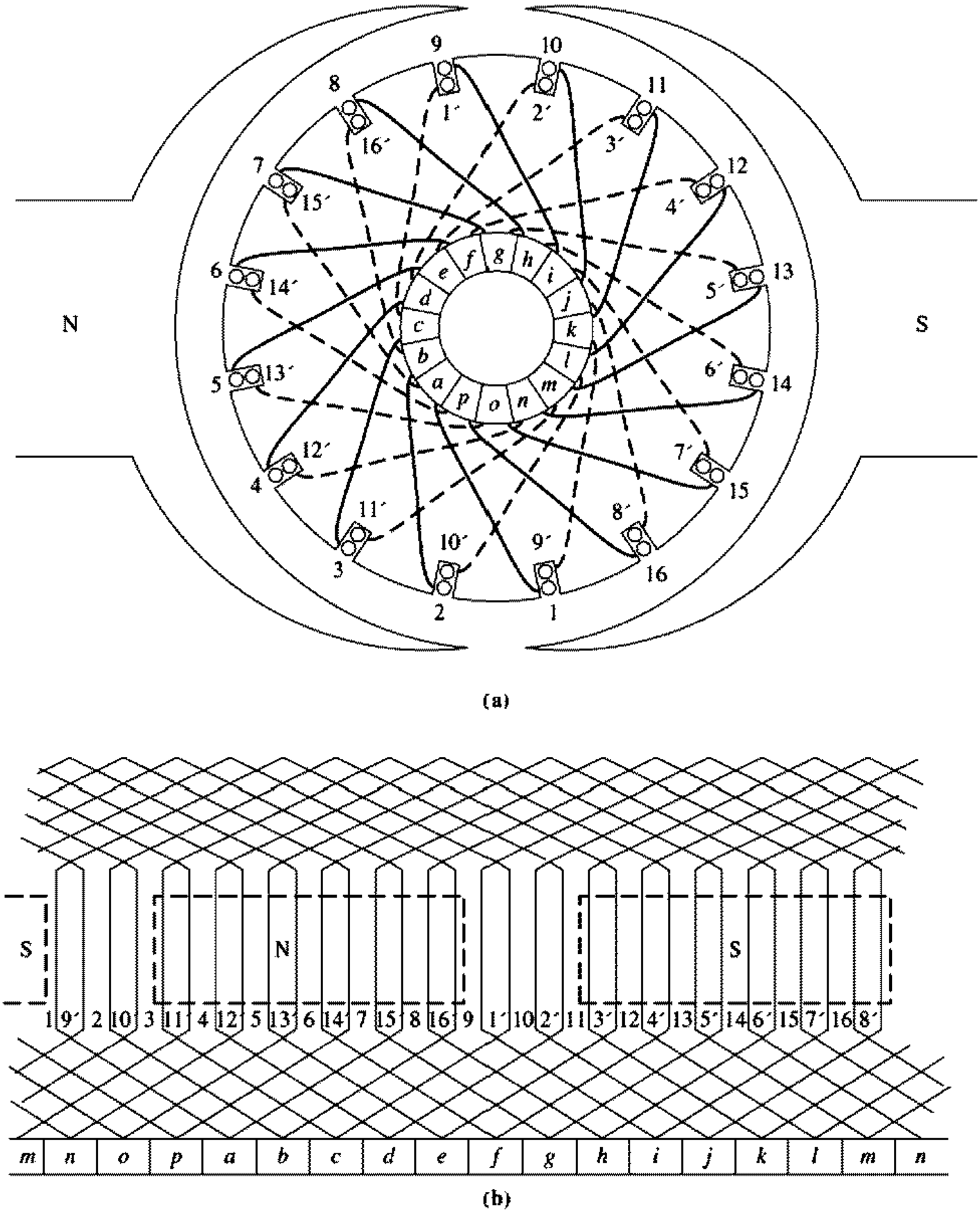


FIGURE P8–3
(a) The machine in Problem 8–9. (b) The armature winding diagram of this machine.

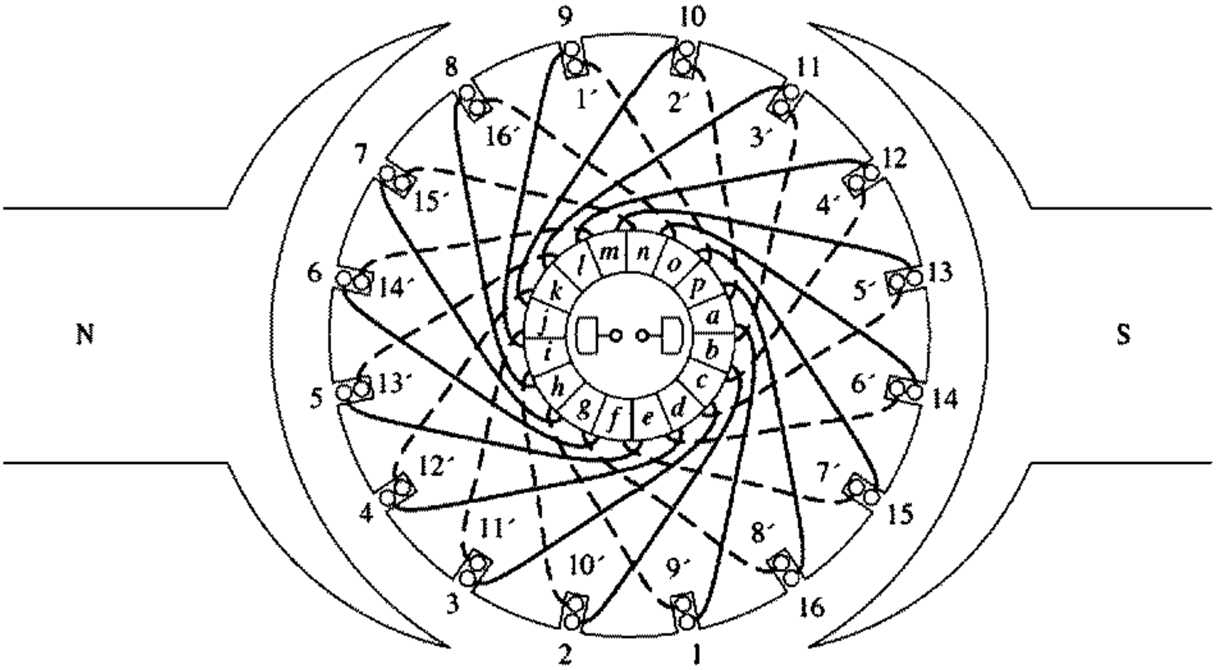


FIGURE P8–4
The machine in Problem 8–10.